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An Example of Interaction Between Two Gasdynamic Objects: a Shock Discontinuity and a Model of Turbulence

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Abstract. ∎ The context of the considered interaction assumes a *minimal* nonlinearity - in the form of a nonlinear subconscious. Consequently the interaction solution is essentially constructed as an admissible solution. The present analysis has essentially two objectives: (a) finding an *explicit optimal form* for the interaction solution, and (b) offering an *exhaustively classifying characterization* of this mentioned solution. ∎ Realising the objective (a) is connected with: (a_1) considering a *singular limit* of the interaction solution, (a_2) considering a *hierarchy of (natural) partitions* of the singular limit, (a_3) inserting some (natural) *gasdynamic factorizations* at a certain level of the mentioned hierarchy and noticing a *compatibility (coherence)* of these factorizations, (a_4) identifying some *inner connections* inside one of the mentioned partitions, (a_5) *predicting some exact details* of the interaction solution, (a_6) indicating some parasite singularities [= strictly depending on the method] to be compensated [= pseudosingularities], (a_7) *re-weighting* the singular limit of the interaction solution. Realising the objective (b) is connected with finding some *Lorentz arguments of criticity.* ∎ The interaction solution appears essentially to (exhaustively) include a *subcritical* and respectively a *supercritical* contribution distinguished by differences of a *"relativistic"* nature. Precisely: in the singular limit of the interaction solution [see (a_1)] the emergent sound is *singular* in the subcritical contribution and it is *regular* in the supercritical contribution (see Fig. 1). It can be shown that this "relativistic" discontinuity in the nature of the emergent sound, corresponding to the singular limit of the interaction solution appears to be dissembled (hidden) in the re-weighted interaction solution [mentioned in (a_7)].

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A Fourier-Snell representation of the *parallel* linearized interaction between a planar shock discontinuity and a planar compressible finite-core vortex the axis of which is parallel to the shock has been considered first time by Ribner [11] in a theoretical attempt consecutive to a pioneering and most suggestive experimental approach of Hollingworth and Richards [8] concerning the mentioned interaction. An ample and significant series of theoretical and experimental developments has followed the two mentioned works [see Ribner [12] for a thorough review]. A planar compressible finite-core vortex the axis of which is parallel to the shock has the representation

$$[\widetilde{u}(\underset{\sim}{x},\underset{\sim}{y}),\widetilde{v}(\underset{\sim}{x},\underset{\sim}{y})]=\frac{\widetilde{\varepsilon}}{2\pi}\begin{cases}(1/r_*^2)[-\underset{\sim}{y},\underset{\sim}{x}] & \text{for } r\le r_*\\ (1/r^2)[-\underset{\sim}{y},\underset{\sim}{x}] & \text{for } r_*\le r\end{cases},\ \widetilde{s}\equiv\widetilde{p}\equiv 0 \qquad (1)$$

where we use the Lagrangian reference frames $\underset{\sim}{x}$, $\underset{\sim}{y}$ (fixed on the undisturbed flow ahead of the shock) and $\widetilde{x}$, $\widetilde{y}$ (fixed on the undisturbed flow behind of the shock) in addition to the frame X, Y fixed on the shock discontinuity; we have

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$$\underset{\sim}{x} = X - \overline{M}T, \;\; \widetilde{x} = X - MT = \underset{\sim}{x} + (\overline{M} - M)\,\underset{\sim}{t}\,; \;\; \underset{\sim}{y} = \widetilde{y} = Y; \;\; \underset{\sim}{t} = \widetilde{t} = T,$$

where M and $\overline{M}$ are the Mach numbers respectively associated to the regions behind or ahead of the shock discontinuity [both are taken with respect to the sound velocity behind].

The limit $r_* \to 0$ of (1) results in

$$[\widetilde{u}(\underset{\sim}{x}, \underset{\sim}{y}), \widetilde{v}(\underset{\sim}{x}, \underset{\sim}{y})] = \frac{\widetilde{\varepsilon}}{2\pi} \cdot \frac{[-\underset{\sim}{y}, \underset{\sim}{x}]}{\underset{\sim}{x}^2 + \underset{\sim}{y}^2}, \;\; \widetilde{s} \equiv \widetilde{p} \equiv 0 \tag{2}$$

$$\frac{\partial \widetilde{v}}{\partial \underset{\sim}{x}} - \frac{\partial \widetilde{u}}{\partial \underset{\sim}{y}} = \widetilde{\varepsilon}\delta(\underset{\sim}{x})\delta(\underset{\sim}{y}). \tag{3}$$

The present paper (see Dinu [1], [2]) has three main objectives:

• (i) to notice a *gasdynamic factorization* of the vorticity-shock interaction; via

$$E(\mathfrak{z}^2) \equiv (d_{01}\mathfrak{z}^2 + d_{02})^2 + (d_{03}\mathfrak{z}^2 + d_{04})^2(\mathfrak{z}^2 - \mathfrak{z}_c^2) \equiv d_{03}^2(\mathfrak{z}^2 + a^2)(\mathfrak{z}^2 - b^2)(\mathfrak{z}^2 - c^2)$$

where

$$a \stackrel{\text{def}}{=} \frac{\overline{M}}{M}, \;\; \varpi_\pm^2 \stackrel{\text{def}}{=} \frac{\overline{M}}{M}\left[(2M\overline{M} - 1) \pm 2M\sqrt{\frac{\gamma - 1}{\gamma + 1}M\overline{M}}\right], \;\; b^2 \stackrel{\text{def}}{=} \varpi_+^2, \;\; c^2 \stackrel{\text{def}}{=} \varpi_-^2$$

$$a > 1, \; \begin{cases} b^2 > 0 \quad \text{for } -1 < \gamma < \dfrac{5}{3}; \quad c^2 > 0 \\ 0 < |b| < |c| < \mathfrak{z}_c \end{cases}$$

[the coefficients d_{0j} should be presented below] and to make use of this factorization to give an *explicit, closed form* to Ribner's representation;

• (ii) to identify a sequence of *other five gasdynamic factorizations* in the explicit form of the vortex-shock interaction solution [since a vortex represents a *structured* vorticity, the present factorizations appear to be induced by that mentioned in (i) by structuring] and to take into account the reality of a *factoring compatibility* of these factorizations in order to select an *extensible* (to the case of the oblique interactions) structure of the mentioned explicit form; an *optimal simplicity* [see (5)-(10)] is seen to be induced in the extensible structure by this factoring compatibility; we use the *Lorentz transform*

$$x = \frac{\widetilde{x} + M\tilde{t}}{\sqrt{1 - M^2}} = \frac{X}{\sqrt{1 - M^2}}, \;\; y = \widetilde{y}, \;\; t = \frac{\tilde{t} + M\widetilde{x}}{\sqrt{1 - M^2}}. \tag{4}$$

in order to present the mentioned factorizations by

$$(\xi^2 + \eta^2 + \zeta_i)^2 - 4\xi^2\zeta_i$$
$$= \frac{1}{(x^2 + y^2)^2}[(\mathfrak{z}_c t - x\sqrt{\mathfrak{z}_c^2 - \zeta_i})^2 - \zeta_i y^2][(\mathfrak{z}_c t + x\sqrt{\mathfrak{z}_c^2 - \zeta_i})^2 - \zeta_i y^2]$$

with

$$\mathfrak{z}_c = \frac{\overline{M}}{\sqrt{1 - M^2}}, \;\; \xi = \frac{\mathfrak{z}_c t y}{x^2 + y^2}, \;\; \eta = \frac{\mathfrak{z}_c x\sqrt{t^2 - x^2 - y^2}}{x^2 + y^2}$$

$$\begin{aligned}&\mathcal{E}_1^p(\zeta_i)[2(\mathfrak{z}_c^2-\zeta_i)x\xi+2\mathfrak{z}_c\sqrt{\mathfrak{z}_c^2-\zeta_i}\,t\xi-\sqrt{\mathfrak{z}_c^2-\zeta_i}\,y(\xi^2+\eta^2+\zeta_i)]\\&+\mathcal{E}_2^p(\zeta_i)[-2\mathfrak{z}_c t\xi+y(\xi^2+\eta^2+\zeta_i)-2\sqrt{\mathfrak{z}_c^2-\zeta_i}\,x\xi\\&=[\sqrt{\mathfrak{z}_c^2-\zeta_i}\mathcal{E}_1^p(\zeta_i)-\mathcal{E}_2^p(\zeta_i)][2\sqrt{\mathfrak{z}_c^2-\zeta_i}\,x\xi+2\mathfrak{z}_c t\xi-y(\xi^2+\eta^2+\zeta_i)]\\&=[\sqrt{\mathfrak{z}_c^2-\zeta_i}\mathcal{E}_1^p(\zeta_i)-\mathcal{E}_2^p(\zeta_i)]\left\{[y/(x^2+y^2)][(\mathfrak{z}_c t+x\sqrt{\mathfrak{z}_c^2-\zeta_i})^2-\zeta_i y^2]\right\}\end{aligned}$$

$$\begin{aligned}&\sqrt{\mathfrak{z}_c^2-\zeta_i}\,t[\mathfrak{z}_c^2(t^2-x^2)-\zeta_i(x^2+y^2)]\pm\mathfrak{z}_c x[\mathfrak{z}_c^2(t^2-x^2)-\zeta_i(2t^2-x^2-y^2)]\\&\equiv(t\sqrt{\mathfrak{z}_c^2-\zeta_i}\mp x\mathfrak{z}_c)[(\mathfrak{z}_c t\pm x\sqrt{\mathfrak{z}_c^2-\zeta_i})^2-\zeta_i y^2]\end{aligned}$$

$$\begin{aligned}&-\left\{2\xi\zeta_i\mathcal{T}_1^v(\zeta_i)+(\xi^2+\eta^2+\zeta_i)\mathcal{T}_2^v(\zeta_i)\right\}\\&+\sqrt{\mathfrak{z}_c^2-\zeta_i}\cdot\left\{2\xi\zeta_i[-y\mathcal{E}_1^v(\zeta_i)]+(\xi^2+\eta^2+\zeta_i)[\mathfrak{z}_c t\mathcal{E}_1^v(\zeta_i)-x\mathcal{E}_2^v(\zeta_i)]\right\}\\&=-[\mathcal{E}_2^v(\zeta_i)-\mathcal{E}_1^v(\zeta_i)\sqrt{\mathfrak{z}_c^2-\zeta_i}][(\xi^2+\eta^2+\zeta_i)(\mathfrak{z}_c t+x\sqrt{\mathfrak{z}_c^2-\zeta_i})-2\zeta_i y\xi]\\&=-[\mathcal{E}_2^v(\zeta_i)-\mathcal{E}_1^v(\zeta_i)\sqrt{\mathfrak{z}_c^2-\zeta_i}][1/(x^2+y^2)](\mathfrak{z}_c t-x\sqrt{\mathfrak{z}_c^2-\zeta_i})[(\mathfrak{z}_c t+x\sqrt{\mathfrak{z}_c^2-\zeta_i})^2-\zeta_i y^2]\end{aligned}$$

$$\begin{aligned}&\mathfrak{z}_c[\mathfrak{z}_c^2(t^4-t^2x^2-t^2y^2-x^2y^2)-\zeta_i(t^2y^2-x^2y^2-y^4-t^2x^2)]\\&\pm\sqrt{\mathfrak{z}_c^2-\zeta_i}\,tx[\mathfrak{z}_c^2(t^2-y^2)-(\mathfrak{z}_c^2-\zeta_i)(x^2+y^2)]\\&=[t(\mathfrak{z}_c t\mp x\sqrt{\mathfrak{z}_c^2-\zeta_i})-\mathfrak{z}_c^2y^2][(\mathfrak{z}_c t\pm x\sqrt{\mathfrak{z}_c^2-\zeta_i})^2-\zeta_i y^2]\end{aligned}$$

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where

$$Q_1(\mathfrak{z}^2)\overset{\text{def}}{=}d_{11}\mathfrak{z}^2+d_{12},\ Q_2(\mathfrak{z}^2)\overset{\text{def}}{=}d_{01}\mathfrak{z}^2+d_{02},\ Q_3(\mathfrak{z}^2)\overset{\text{def}}{=}d_{03}\mathfrak{z}^2+d_{04},$$

$$\mathcal{E}_1^p(\zeta_i)\overset{\text{def}}{=}MQ_2(\zeta_i)+\mathfrak{z}_cQ_3(\zeta_i),\ \mathcal{E}_2^p(\zeta_i)\overset{\text{def}}{=}\mathfrak{z}_cQ_2(\zeta_i)+M(\mathfrak{z}_c^2-\zeta_i)Q_3(\zeta_i)$$

$$\mathcal{E}_1^v(\zeta_i)\overset{\text{def}}{=}Q_3(\zeta_i),\ \mathcal{E}_2^v(\zeta_i)\overset{\text{def}}{=}Q_2(\zeta_i)$$

$$\mathcal{T}_1^v(\zeta_i)\overset{\text{def}}{=}-y\mathcal{E}_2^v(\zeta_i),\ \mathcal{T}_2^v(\zeta_i)\overset{\text{def}}{=}-x(\mathfrak{z}_c^2-\zeta_i)\mathcal{E}_1^v(\zeta_i)+t\mathfrak{z}_c\mathcal{E}_2^v(\zeta_i),$$

with

$$d_{01}=\frac{2}{\gamma+1}\frac{\overline{M}}{M}(1-2M^2),\ d_{02}=\frac{\overline{M}}{M}d_{01}-\frac{8}{(\gamma+1)^2}\frac{\overline{M}^2}{M^2}(1-M^2),$$

$$d_{03}=-\frac{2}{\gamma+1}\sqrt{1-M^2},\ d_{04}=\frac{\overline{M}}{M}d_{03},$$

$$d_{11}=\frac{8}{(\gamma+1)^2}(1-M^2),\ d_{12}=-\frac{\overline{M}}{M}d_{11}$$

• (*iii*) to use the mentioned extensible structure in order to indicate (see Fig. 1) an *exhaustively classifying, deterministic* and *explicit* characterization of Lighthill's *statistic* and *implicit* approach [10] concerning the turbulence − planar shock interaction. The incident turbulence, regarded as a perturbation, is modelled by a nonstatistical/noncorrelative superposition of some compressible finite core (or point core) planar vortices.

The *linearized* context implies the taking into consideration of a linear problem with a *nonlinear subconscious*; the resultant perturbation is regarded as a solution ("interaction solution") of such a *linearized* problem. A nonlinear subconscious results when the nonlinearity is allowed only at the zeroth order of a perturbation expansion: we linearize the perturbation of a *piecewise constant* admissible solution and prove that the requirement of admissibility is still active at the first order and essentially structures the linearized description. The turbulence − planar shock interaction is associated with a class of interaction elements. An interaction element models the interaction between a planar shock and a *single* incident vortex corresponding to a

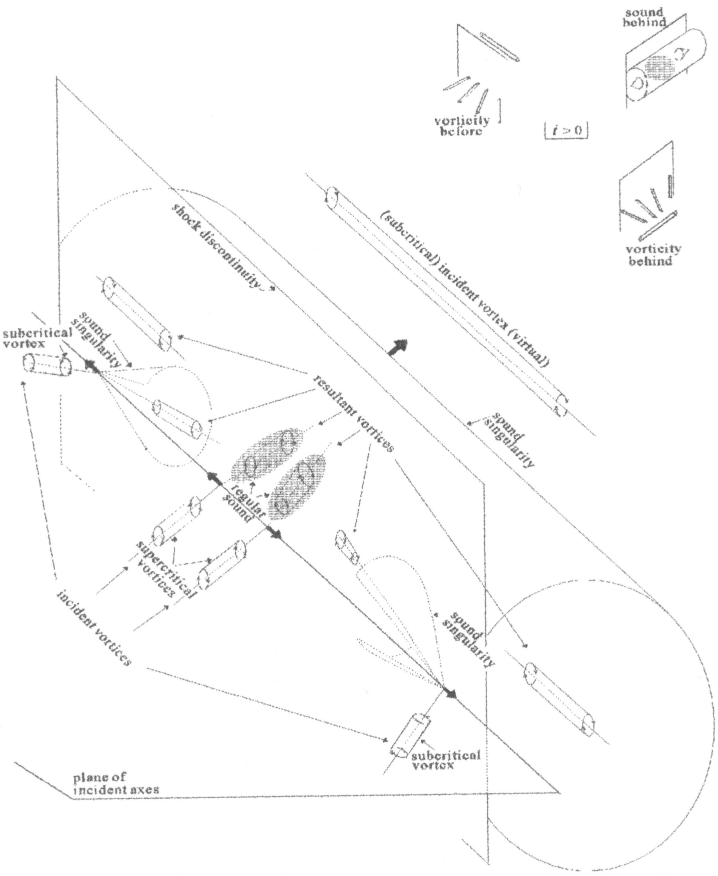

FIGURE 1. The simplest nonstatistical model of turbulence refraction ($\tilde{t} > 0$)

certain inclination of the vortex axis with respect to the shock. The resulting "relativistically motivated" classification, which is *essentially* oblique, takes into account the importance of some *subcritical* or *supercritical* inclinations of the incident vortices with respect to the shock in the mentioned interaction.

A final (extensive) version of the above mentioned analysis consists in replacing the present *vorticity* incident perturbation by a *general gasdynamic* incident perturbation (Dinu [3]). In fact, it may be proven (see Dinu [3]) that the structure (*i*)–(*iii*) of the above mentioned interaction analysis *persists* in this final version.

The approach of the present paper (see Dinu [1], [2]) corresponds to a *minimal* nonlinearity [associated to the presence of nonlinear subconscious]; still coupled with a *"maximal"* (exhaustive; explicit and oblique) classifying characterization of the turbulence – shock interaction.

This approach could be set in contrast with a lot of recent studies which allow (analytically or numerically) a *more complete* considering of the nonlinearity contribution yet in presence of the *"minimal"* case of a (strictly) parallel interaction; see for example Grove and Menikoff [6], Han and Yin [7] or Inoue et all [9].

The work of Han and Yin allows *more* nonlinearity yet in presence of a *set of (approximating) restrictions* [cf. its pag. 188]. These authors classify the context of their work to be "complicated" [pag. 189]. Still, from such a ("complicated") context an analogue of the maximal (exhaustive; explicit and oblique) characterization included in this paper (also see Dinu [1], [2]) does not emerge. A possible cause for such an issue appears to be the absence of some structuring arguments (needed to replace a "complicated" context by a *complex* context).

More nonlinearity is (numerically) allowed in the *parallel* interactions considered in the papers by Inoue et all or Grove and Menikoff.

A first aspect of the complex character of the interaction solution concerns the *modal* (entropy / vorticity / sound) structure involved.

Remark. Even in presence of a suitable set of structuring arguments we may need a bit of "chance" in order to get a successful calculation. For example, the attempt to obtain an explicit/closed form for the *parallel* interaction solution may be fruitless if we are not aware of the presence of a lot of *"traps"*:

• (a) the emergent sound contribution cannot be computed *directly*; in fact, if r_* is the radius of the core of the planar incident vortex, this contribution can be put in an explicit form directly *only* in the singular limit $r_* \to 0$ [which replaces the incidence (1) by (2)] and *only* in the interior points of the sonic cylinder $[X > 0, \widetilde{x}^2 + \widetilde{y}^2 < \widetilde{t}^2]$; incidentally it can be predicted (and verified) in the exterior points of the sonic cylinder too; cf.

$$\widetilde{p} = \widetilde{p}_r + \widetilde{p}_s,\ \widetilde{u} = \widetilde{u}_r + \widetilde{u}_s,\ \widetilde{v} = \widetilde{v}_r + \widetilde{v}_s, \tag{5}$$

$$\begin{aligned}&[\widetilde{p}_r(\widetilde{x},\widetilde{y},\widetilde{t}), \widetilde{u}_r(\widetilde{x},\widetilde{y},\widetilde{t}), \widetilde{v}_r(\widetilde{x},\widetilde{y},\widetilde{t})]\\ &\quad = -\overline{K}\sum_{i=1}^{4}\frac{k_i^r(\zeta)Q^-(\zeta_i)}{[\widetilde{t}\hat{k}^-(\zeta_i)+\widetilde{x}\check{k}^-(\zeta_i)]^2-\zeta_i\widetilde{y}^2}[\hat{k}^-(\zeta_i)\widetilde{y}, -\check{k}^-(\zeta_i)\widetilde{y},\ \widetilde{t}\hat{k}^-(\zeta_i)+\widetilde{x}\check{k}^-(\zeta_i)]\end{aligned} \tag{6}$$

and

$$\begin{aligned}\widetilde{p}_s(\widetilde{x},\widetilde{y},\widetilde{t}) \quad &= \quad -\frac{\overline{K}}{\sqrt{\widetilde{t}^2-\widetilde{x}^2-\widetilde{y}^2}}\cdot H(\widetilde{t}-\sqrt{\widetilde{x}^2+\widetilde{y}^2})\cdot\\ &\left\{\sum_{i=1}^{4}\overline{k}_i(\zeta)Q^-(\zeta_i)\hat{k}^-(\zeta_i)\frac{\widetilde{y}[\widetilde{t}\check{k}^-(\zeta_i)+\widetilde{x}\hat{k}^-(\zeta_i)]}{[\widetilde{t}\hat{k}^-(\zeta_i)+\widetilde{x}\check{k}^-(\zeta_i)]^2-\zeta_i\widetilde{y}^2}\ +\right.\\ &\qquad\left.\sum_{i=1}^{4}\overline{k}_i(\zeta)Q^+(\zeta_i)\hat{k}^+(\zeta_i)\frac{\widetilde{y}[\widetilde{t}\check{k}^+(\zeta_i)+\widetilde{x}\hat{k}^-(\zeta_i)]}{[\widetilde{t}\hat{k}^+(\zeta_i)+\widetilde{x}\check{k}^+(\zeta_i)]^2-\zeta_i\widetilde{y}^2}\right\}\end{aligned} \tag{7}$$

$$\begin{aligned}\widetilde{u}_s(\widetilde{x},\widetilde{y},\widetilde{t}) \quad &= \quad \frac{\overline{K}}{\sqrt{\widetilde{t}^2-\widetilde{x}^2-\widetilde{y}^2}}\cdot H(\widetilde{t}-\sqrt{\widetilde{x}^2+\widetilde{y}^2})\cdot\\ &\left\{\sum_{i=1}^{4}\overline{k}_i(\zeta)Q^-(\zeta_i)\check{k}^-(\zeta_i)\frac{\widetilde{y}[\widetilde{t}\check{k}^-(\zeta_i)+\widetilde{x}\hat{k}^-(\zeta_i)]}{[\widetilde{t}\hat{k}^-(\zeta_i)+\widetilde{x}\check{k}^-(\zeta_i)]^2-\zeta_i\widetilde{y}^2}\ +\right.\\ &\qquad\left.\sum_{i=1}^{4}\overline{k}_i(\zeta)Q^+(\zeta_i)\check{k}^+(\zeta_i)\frac{\widetilde{y}[\widetilde{t}\check{k}^+(\zeta_i)+\widetilde{x}\hat{k}^+(\zeta_i)]}{[\widetilde{t}\hat{k}^+(\zeta_i)+\widetilde{x}\check{k}^+(\zeta_i)]^2-\zeta_i\widetilde{y}^2}\right\}\end{aligned} \tag{8}$$

$$\begin{aligned}\widetilde{v}_s(\widetilde{x},\widetilde{y},\widetilde{t}) &= -\frac{\overline{K}}{\sqrt{\widetilde{t}^2-\widetilde{x}^2-\widetilde{y}^2}}\cdot H(\widetilde{t}-\sqrt{\widetilde{x}^2+\widetilde{y}^2})\cdot \\ &\left\{\sum_{i=1}^{4}\overline{k}_i(\zeta)Q^-(\zeta_i)\,\overset{\circ}{k}(\zeta_i)\frac{(\widetilde{t}+M\widetilde{x})[\widetilde{t}\hat{k}^-(\zeta_i)+\widetilde{x}\check{k}^-(\zeta_i)]-\overline{M}\widetilde{y}^2}{[\widetilde{t}\hat{k}^-(\zeta_i)+\widetilde{x}\check{k}^-(\zeta_i)]^2-\zeta_i\widetilde{y}^2}+\right.\\ &\left.\sum_{i=1}^{4}\overline{k}_i(\zeta)Q^+(\zeta_i)\,\overset{\circ}{k}(\zeta_i)\frac{(\widetilde{t}+M\widetilde{x})[\widetilde{t}\hat{k}^+(\zeta_i)+\widetilde{x}\check{k}^+(\zeta_i)]-\overline{M}\widetilde{y}^2}{[\widetilde{t}\hat{k}^+(\zeta_i)+\widetilde{x}\check{k}^+(\zeta_i)]^2-\zeta_i\widetilde{y}^2}\right\}\end{aligned} \tag{9}$$

where

$$\zeta_1=-a^2,\ \zeta_2=b^2,\ \zeta_3=c^2,\ \zeta_4=-1,$$

$$Q^{\pm}(\zeta_i)=Q_1(\zeta_i)[Q_2(\zeta_i)\pm Q_3(\zeta_i)\sqrt{\mathfrak{z}_c^2-\zeta_i}];$$

$$\overline{K}=\frac{\widetilde{\varepsilon}}{2\pi}\cdot\frac{1}{d_{03}^2},\ \hat{k}^{\pm}(\zeta_i)=\frac{\mathfrak{z}_c\pm M\sqrt{\mathfrak{z}_c^2-\zeta_i}}{\sqrt{1-M^2}},\ \check{k}^{\pm}(\zeta_i)=\frac{M\mathfrak{z}_c\pm\sqrt{\mathfrak{z}_c^2-\zeta_i}}{\sqrt{1-M^2}},$$

$$\overset{\circ}{k}(\zeta_i)=\frac{\zeta_i}{\sqrt{1-M^2}\sqrt{\mathfrak{z}_c^2-\zeta_i}},\ \overline{k}_i(\zeta)=\left[\prod_{j\neq i}(\zeta_i-\zeta_j)\right]^{-1},\ k_i^r(\zeta)=\frac{(2-i)(3-i)}{2}\overline{k}_i(\zeta)\sqrt{|\zeta_i|};$$

$$k_2^r(\zeta)=0,\ k_3^r(\zeta)=0;\ \hat{k}^-(\zeta_1)=0;\ Q^+(\zeta_1)=0,\ Q^-(\zeta_2)=0,\ Q^-(\zeta_3)=0.$$

- (b) the emergent vorticity contribution cannot be computed directly even in the singular limit $r_*\to 0$; its explicit form results by taking into account its connection with the emergent sound contribution:

$$\begin{aligned}\widetilde{u}_{\text{vorticity}}(\widetilde{x},\widetilde{y},\tilde{t}) &= \widetilde{u}_-\left(\underset{\sim}{y},\underset{\sim}{t}=T-\frac{X}{M}\right)-M^*\widetilde{p}_+\left(\widetilde{y},\tilde{t}=T-\frac{X}{M}\right)\\ &-\int_{T-\frac{X}{M}}^{T}\frac{\partial\widetilde{p}}{\partial\widetilde{x}}(\widetilde{x},\widetilde{y},\theta)\mathrm{d}\theta-\widetilde{u}_{\text{sound}}(\widetilde{x},\widetilde{y},\widetilde{t})\\ \widetilde{v}_{\text{vorticity}}(\widetilde{x},\widetilde{y},\tilde{t}) &= \widetilde{v}_-\left(\underset{\sim}{y},\underset{\sim}{t}=T-\frac{X}{M}\right)+(\overline{M}-M)\frac{\partial\psi}{\partial\widetilde{y}}\left(\widetilde{y},\tilde{t}=T-\frac{X}{M}\right)\\ &-\int_{T-\frac{X}{M}}^{T}\frac{\partial\widetilde{p}}{\partial\widetilde{y}}(\widetilde{x},\widetilde{y},\theta)\mathrm{d}\theta-\widetilde{v}_{\text{sound}}(\widetilde{x},\widetilde{y},\widetilde{t})\end{aligned}$$

where we have to insert,

$$M^*=\frac{\gamma+1}{4M}\left[\frac{3-\gamma}{\gamma+1}+\frac{M}{\overline{\overline{M}}}\right];\ T=\tilde{t},\ T-\frac{X}{M}=-\frac{\widetilde{x}}{M},\ \tilde{y}=\widetilde{y}.$$

The *entropy* component (in the sense of Carrier) of the resultant solution is:

$$\widetilde{s}(\widetilde{x},\widetilde{y},\tilde{t})\equiv\frac{\gamma^2-1}{4M\overline{\overline{M}}}(\overline{M}-M)^2\widetilde{p}_+\left(-\frac{\widetilde{x}}{M},\widetilde{y}\right).$$

We end this paragraph by presenting the expression of the shock discontinuity perturbation ψ. Since $\lim\limits_{T\to-\infty}\psi=0$, we obtain

$$\psi(\tilde{y},\tilde{t})=\int\limits_{-\infty}^{\tilde{t}}\left[-\frac{\gamma+1}{4M}\tilde{p}_+(\tilde{y},\theta)+\tilde{u}_-(\tilde{y},\theta)\right]\mathrm{d}\theta$$

where the subscripts + or − correspond respectively to the sides behind or ahead of the shock discontinuity.

• (c) finally, the explicit form of the nonsingular parallel interaction solution which corresponds to the incidence (1) results from a *re-weighting* {a re-set of the weight lost in the singular limit $r_*\to 0$; cf. Dinu and Dinu [4]}. □

We have to *strictly* follow this recipe in order to reach an extensible Lorentz type arguments structure of the mentioned parallel representation [the arguments structure (10) may be regarded (see pag. 8) as being a code ("cipher") which filters out the passage to an oblique approach]:

$$\begin{cases}\tilde{p}_r+\tilde{p}_s & \equiv & \tilde{p}_{\|}(x,y,t;\ \zeta_1,\zeta_2,\zeta_3,\zeta_4;\ \mathfrak{z}_c;\ Q_1,Q_2,Q_3)\\ \tilde{u}_r+\tilde{u}_s & \equiv & \tilde{u}_{\|}(x,y,t;\ \zeta_1,\zeta_2,\zeta_3,\zeta_4;\ \mathfrak{z}_c;\ Q_1,Q_2,Q_3)\\ \tilde{v}_r+\tilde{v}_s & \equiv & \tilde{v}_{\|}(x,y,t;\ \zeta_1,\zeta_2,\zeta_3,\zeta_4;\ \mathfrak{z}_c;\ Q_1,Q_2,Q_3).\end{cases} \tag{10}$$

The structure of the limit $r_*\to 0$ of the parallel interaction solution reflects: • the shape of the incident vortex [the emergent sound singularities are distributed along a (circular) sonic arc], • the details of the modal [vorticity-shock] interaction {some pseudosingularities [= compensated singularities: they are singularities for the components (6)–(9) taken separately still they appear to be compensated in the sums (5)] are present}, • the presence of a singularity in the incident contribution, • a memory of the various inner connections [cf. the compatibility of the mentioned factorizations], • a gasdynamic specificity [in most cases the above mentioned factorizations become *immaterial* if the gasdynamic context is extended/lost; cf. Dinu and Dinu [5].

We could abstract these key phrases by saying that the details of the interaction analysis in this paper (see Dinu [1], [2]) allow a *structural* characterization of the "prodigious memory" of the interaction solution.

A second aspect of the complex character of the interaction solution considered appears to be connected with the presence of a *"relativistic"* structure. Modelling the incident turbulence by a superposition of compressible planar vortices appears to correspond to a *first level* of decomposition; next, in order to proceed, each incident vortex is decomposed (by a Fourier representation) into planar monochromatic waves – a *second level* of decomposition; finally, each incident planar monochromatic wave is Snell passed through the shock discontinuity. The composition of the mentioned levels leads to a Fourier–Snell representation of the interaction solution. The main point of the present paper is that the result of the passage through the shock can again be presented by two levels of *recombination* so that each incident level of decomposition has a correspondent in the emergent solution.

A *"relativistic"* character appears to reflect, at the first level of decomposition, the importance of a critical ("relativistic") inclination corresponding to $\Theta=\Theta_c$ with

$$\tan\Theta_c=\mathfrak{z}_c$$

(cf. Dinu [1], [2]) where Θ is the the inclination of the vortex axis with respect to the plane of the discontinuity. The singular limit of the interaction solution essentially appears to (exhaustively) include a *subcritical* and, respectively, a *supercritical* contribution distinguished by differences of a "relativistic" nature. Precisely: in the singular limit of the interaction solution [see Remark (a) here above] the emergent sound is *singular* in the subcritical contribution and it is *regular* in the supercritical contribution (see Fig. 1). In the mentioned singular limit the contributions Dinu [1], [2] present, particularly, the following *oblique* subcritical extension of the sound contribution (10)

$$\begin{aligned}
\widetilde{p}(x,y,t) &= \{1+\mathfrak{z}_c^-(\Theta)\}\cdot\widetilde{p}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*] \\
&\quad +M\mathfrak{z}_c^-(\Theta)\cdot\widetilde{u}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*] \\
\widetilde{u}(x,y,t) &= \overline{M}\left\{1+\frac{M^2}{\overline{M}^2}\mathfrak{z}_c\left[\mathfrak{z}_c^-(\Theta)-\frac{1}{\mathfrak{z}_c}\tan^2\Theta\right]\right\}\cdot\frac{\cos\Theta}{\sqrt{M_+(\Theta)}} \\
&\quad \cdot\widetilde{u}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*] \\
&\quad +\frac{M}{\overline{M}}\mathfrak{z}_c\left[\mathfrak{z}_c^-(\Theta)-\frac{1}{\mathfrak{z}_c}\tan^2\Theta\right]\cdot\frac{\cos\Theta}{\sqrt{M_+(\Theta)}} \\
&\quad \cdot\widetilde{p}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*] \\
\widetilde{v}(x,y,t) &= \widetilde{v}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*] \\
\widetilde{w}(x,y,t) &= M\left(2+\frac{M^2}{\overline{M}^2}\cdot\mathfrak{z}_c\cdot\mathfrak{z}_c^-(\Theta)\right)\cdot\frac{(\operatorname{sign}\theta)\sin\Theta}{\sqrt{M_+(\Theta)}} \\
&\quad \cdot\widetilde{u}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*] \\
&\quad +\left(1+\frac{M^2}{\overline{M}^2}\cdot\mathfrak{z}_c\cdot\mathfrak{z}_c^-(\Theta)\right)\cdot\frac{(\operatorname{sign}\theta)\sin\Theta}{\sqrt{M_+(\Theta)}} \\
&\quad \cdot\widetilde{p}_{\|}[x,y,t;a^{*2},\varepsilon_b b^{*2},\varepsilon_c c^{*2},\upsilon^{*2};\mathfrak{z}_c^*(\Theta);Q_1^*,Q_2^*,Q_3^*]
\end{aligned}$$

where

$$\left\{\begin{array}{lcl}
M_+(\Theta) & \stackrel{\text{def}}{=} & \overline{M}^2+(M^2-\overline{M}^2)\sin^2\Theta \\
\mathfrak{z}_c^*(\Theta) & \stackrel{\text{def}}{=} & \sqrt{\mathfrak{z}_c^2-\tan^2\Theta} \\
\mathfrak{z}_c^-(\Theta) & \stackrel{\text{def}}{=} & \mathfrak{z}_c^*(\Theta)-\mathfrak{z}_c
\end{array}\right.$$

$$\left\{\begin{array}{c}
a^{*2}=a^2+\tan^2\Theta,\ b^{*2}=|b^2-\tan^2\Theta|, \\
c^{*2}=|c^2-\tan^2\Theta|,\ \upsilon^{*2}=1+\tan^2\Theta \\
\varepsilon_b=\operatorname{sign}(\tan^2\Theta-b^2),\ \varepsilon_c=\operatorname{sign}(\tan^2\Theta-c^2)
\end{array}\right.$$

$$\left\{\begin{array}{l}
Q_1^*(\mathfrak{z}^{*2})\stackrel{\text{def}}{=}d_{11}\mathfrak{z}^{*2}+(d_{11}\tan^2\Theta+d_{12}) \\
Q_2^*(\mathfrak{z}^{*2})\stackrel{\text{def}}{=}d_{01}\mathfrak{z}^{*2}+(d_{01}\tan^2\Theta+d_{02}) \\
Q_3^*(\mathfrak{z}^{*2})\stackrel{\text{def}}{=}d_{03}\mathfrak{z}^{*2}+(d_{03}\tan^2\Theta+d_{04})
\end{array}\right.$$

and x, y, t have the *extended subcritic Lorentz expressions* [which reduce to (4) in the limit $\Theta \to 0$]

$$\begin{cases} x &= \dfrac{\mathfrak{z}_c \cos\Theta}{\sqrt{M_+(\Theta)}}\widetilde{x} + \dfrac{M\mathfrak{z}_c}{M}\widetilde{t} + \dfrac{M\mathfrak{z}_c}{M}\cdot\dfrac{(\operatorname{sign}\theta)\sin\Theta}{\sqrt{M_+(\Theta)}}\widetilde{z} = \dfrac{X}{\sqrt{1-M^2}}; \\ y &= \widetilde{y}\ ;\ z = \widetilde{z} \\ t &= \dfrac{M\mathfrak{z}_c^*(\Theta)\cos\Theta}{\sqrt{M_+(\Theta)}}\widetilde{x} + \dfrac{\mathfrak{z}_c^2}{M\mathfrak{z}_c^*(\Theta)}\widetilde{t} + \dfrac{\mathfrak{z}_c^2}{M\mathfrak{z}_c^*(\Theta)}\cdot\dfrac{(\operatorname{sign}\theta)\sin\Theta}{\sqrt{M_+(\Theta)}}\widetilde{z} \end{cases}$$

In Dinu [2] it is shown that the "relativistic" discontinuity in the nature of the emergent sound, corresponding to the singular limit of the interaction solution, appears to be dissembled (hidden) in the re-weighted interaction solution.

We finally notice that the singular limit of the interaction solution appears to be, cf. (3), an example of *fundamental* solution in presence of a nonlinear subconscious. The contribution of this (minimal) nonlinearity results in a subcritical *widening* of the incident singularities of the mentioned singular limit.

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