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New concepts of irregular-intuitionistic fuzzy graphs with applications

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Abstract. In this paper, some types of edge irregular intuitionistic fuzzy graphs such as neighbourly edge totally irregular intuitionistic fuzzy graphs, strongly edge irregular intuitionistic fuzzy graphs and strongly edge totally irregular intuitionistic fuzzy graphs are introduced. A comparative study between neighbourly edge irregular intuitionistic fuzzy graphs and neighbourly edge totally irregular intuitionistic fuzzy graphs is done. Likewise some properties of them are studied. Finally, we have given some interesting results a bout edge irregular IFGs that are very useful in computer science and networks.

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1. Introduction

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as logic, geometry, algebra, topology, analysis, number theory, information theory, artificial intelligence, operations research, optimization, neural networks, planning, computer science and etc [5], [7], [8], [10], [13].

Fuzzy set theory, introduced by Zadeh in 1965, is a mathematical tool for handling uncertainties like vagueness, ambiguity and imprecision in linguistic variables [30]. Research on theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its application. Fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest.

In 1983, Atanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [30]. Atanassov added a new component(which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the nonmembership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of

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fields including computer science, engineering, mathematics, medicine, chemistry and economics [4], [6], [11].

In 1975, Rosenfeld [23] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [12].

A. Nagoor Gani et al. introduced the concept of regular fuzzy graphs and defined degree of a vertex in fuzzy graphs [15], [16], [19]. A. Nagoor Gani and S.R.Latha [18] introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008. S.P.Nandhini and E.Nandhini introduced the concept of strongly irregular fuzzy graphs and discussed about its properties [20].

K. Radha and N. Kumaravel [22] introduced the concept of edge degree, total edge degree in fuzzy graph and edge regular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs. N.R. Santhi Maheswari and C. Sekar introduced the concept of edge irregular fuzzy graphs and edge totally irregular fuzzy graphs and discussed about its properties [25]. Also, N.R. Santhi Maheswari and C. Sekar introduced the concept of neighbourly edge irregular fuzzy graphs, neighbourly edge totally irregular fuzzy graphs, strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and discussed about its properties [26], [27].

In the literature, many extensions of fuzzy graphs and their properties have been deeply studied by several researchers, such as intuitionistic fuzzy graphs, interval-valued fuzzy graphs, interval-valued intuitionistic fuzzy graphs, bipolar fuzzy graphs and etc [1], [2], [9], [14], [17], [21], [24], [28], [29].

This is the background to introduce neighbourly edge irregular intuitionistic fuzzy graphs, neighbourly edge totally irregular intuitionistic fuzzy graphs, strongly edge irregular intuitionistic fuzzy graphs, strongly edge totally irregular intuitionistic fuzzy graphs and discussed some of their properties. Also neighbourly edge irregularity and strongly edge irregularity on some intuitionistic fuzzy graphs whose underlying crisp graphs are a path, a cycle and a star are studied.

2. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1. A graph is an ordered pair $G^* = (V, E)$, where V is the set of vertices of G^* and E is the set of edges of G^*. A graph G^* is finite if its vertex set and edge set are finite.

Definition 2.2. The degree $d_{G^*}(v)$ of a vertex v in G^* or simply $d(v)$ is the number of edges of G^* incident with vertex v.

Definition 2.3. A fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V, E)$ is a pair of functions (σ, μ) where $\sigma : V \longrightarrow [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : E \longrightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u and v in V the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.4. An intuitionistic fuzzy graph (IFG) is of the form $G : (\sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2)$ and $\mu = (\mu_1, \mu_2)$ such that

(i) The functions $\sigma_1 : V \longrightarrow [0,1]$ and $\sigma_2 : V \longrightarrow [0,1]$ denote the degree of membership and nonmembership of the element $u \in V$, respectively, and $0 \leq \sigma_1(u)+\sigma_2(u) \leq 1$ for every $u \in V$;
(ii) The functions $\mu_1 : V \times V \longrightarrow [0,1]$ and $\mu_2 : V \times V \longrightarrow [0,1]$ are the degree of membership and nonmembership of the edge $uv \in E$, respectively, such that $\mu_1(uv) \leq \wedge[\sigma_1(u), \sigma_1(v)]$ and $\mu_2(uv) \geq \vee[\sigma_2(u), \sigma_2(v)]$ and $0 \leq \mu_1(uv) + \mu_2(uv) \leq 1$ for every uv in E.

Definition 2.5. Let $G : (\sigma, \mu)$ be an IFG on $G^* : (V, E)$. Then the degree of a vertex u is defined as $d_G(u) = (d_{\sigma_1}(u), d_{\sigma_2}(u))$ where $d_{\sigma_1}(u) = \Sigma_{v\neq u}\mu_1(u, v)$ and $d_{\sigma_2}(u) = \Sigma_{v\neq u}\mu_2(u, v)$. The minimum degree of G is $\delta(G) = (\delta_{\sigma_1}(G), \delta_{\sigma_2}(G))$ where $\delta_{\sigma_1}(G) = \wedge\{d_{\sigma_1}(u) : u \in V\}$ and $\delta_{\sigma_2}(G) = \wedge\{d_{\sigma_2}(u) : u \in V\}$. The maximum degree of G is $\triangle(G) = (\triangle_{\sigma_1}(G), \triangle_{\sigma_2}(G))$ where $\triangle_{\sigma_1}(G) = \vee\{d_{\sigma_1}(u) : u \in V\}$ and $\triangle_{\sigma_2}(G) = \vee\{d_{\sigma_2}(u) : u \in V\}$.

Definition 2.6. Let $G : (\sigma, \mu)$ be an IFG on $G^* : (V, E)$. Then the total degree of a vertex u is defined by $td_G(u) = (td_{\sigma_1}(u), td_{\sigma_2}(u))$ where $td_{\sigma_1}(u) = \Sigma_{v\neq u}\mu_1(u, v) + \sigma_1(u)$ and $td_{\sigma_2}(u) = \Sigma_{v\neq u}\mu_2(u, v) + \sigma_2(u)$.

Definition 2.7. Let $G : (\sigma, \mu)$ be an IFG on $G^* : (V, E)$. Then:
(i) G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.
(ii) G is totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

Definition 2.8. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. Then:
(i) G is said to be a neighbourly irregular intuitionistic fuzzy graph if every pair of adjacent vertices have distinct degrees.
(ii) G is said to be a neighbourly totally irregular intuitionistic fuzzy graph if every pair of adjacent vertices have distinct total degrees.
(iii) G is said to be a strongly irregular intuitionistic fuzzy graph if every pair of vertices have distinct degrees.
(iv) G is said to be a strongly totally irregular intuitionistic fuzzy graph if every pair of vertices have distinct total degrees.
(v) G is said to be a highly irregular intuitionistic fuzzy graph if every vertex in G is adjacent to the vertices having distinct degrees.
(vi) G is said to be a highly totally irregular intuitionistic fuzzy graph if every vertex in G is adjacent to the vertices having distinct total degrees.

Definition 2.9. Let $G : (\sigma, \mu)$ be an IFG on $G^* : (V, E)$. The degree of an edge uv is defined as $d_G(uv) = (d_{\mu_1}(uv), d_{\mu_2}(uv))$ where $d_{\mu_1}(uv) = d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2\mu_1(uv)$ and $d_{\mu_2}(uv) = d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2\mu_2(uv)$.

Definition 2.10. Let $G : (\sigma, \mu)$ be an IFG on $G^* : (V, E)$. The total degree of an edge uv is defined as $td_G(uv) = (td_{\mu_1}(uv), td_{\mu_2}(uv))$ where $td_{\mu_1}(uv) = d_{\sigma_1}(u) + d_{\sigma_1}(v) - \mu_1(uv) = d_{\mu_1}(uv) + \mu_1(uv)$ and $td_{\mu_2}(uv) = d_{\sigma_2}(u) + d_{\sigma_2}(v) - \mu_2(uv) = d_{\mu_2}(uv) + \mu_2(uv)$. The minimum total degree of an edge G is $\delta_{tE}(G) = (\delta_{t\mu_1}(G), \delta_{t\mu_2}(G))$ where $\delta_{t\mu_1}(G) = \wedge\{td_{\mu_1}(uv) : uv \in E\}$ and $\delta_{t\mu_2}(G) = \wedge\{td_{\mu_2}(uv) : uv \in E\}$. The maximum total degree of G is $\triangle_{tE}(G) = (\triangle_{t\mu_1}(G), \triangle_{t\mu 2}(G))$ where $\triangle_{t\mu_1}(G) = \vee\{td_{\mu_1}(uv) : uv \in E\}$ and $\triangle_{t\mu_2}(G) = \vee\{td_{\mu_2}(uv) : uv \in E\}$.

3. Neighbourly edge irregular intuitionistic fuzzy graphs, neighbourly edge totally irregular intuitionistic fuzzy graphs, Strongly edge irregular intuitionistic fuzzy graphs and strongly edge totally irregular intuitionistic fuzzy graphs

In this section, neighbourly edge irregular intuitionistic fuzzy graphs and neighbourly edge totally irregular intuitionistic fuzzy graphs are introduced.

Definition 3.1. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. Then G is said to be:
(i) A neighbourly edge irregular intuitionistic fuzzy graph if every pair of adjacent edges have distinct degrees.
(ii) A neighbourly edge totally irregular intuitionistic fuzzy graph if every pair of adjacent edges have distinct total degrees.

Example 3.1. Graph which is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.

Consider $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.
From Figure 1, $d_G(u) = d_G(v) = d_G(w) = d_G(x) = (0.5, 1.0)$.

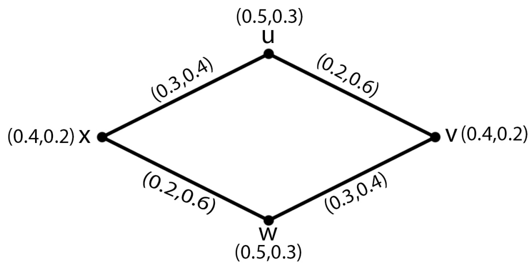

FIGURE 1. Both neighbourly edge irregular IFG and neighbourly edge totally irregular IFG.

Degrees of the edges are calculated as follows: $d_G(uv) = d_G(wx) = (0.6, 0.8)$, $d_G(vw) = d_G(xu) = (0.4, 1.2)$.

It is noted that every pair of adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph.

Total degrees of the edges are calculated as follows: $td_G(uv) = td_G(wx) = (0.8, 1.4)$, $td_G(vw) = td_G(xu) = (0.7, 1.6)$.

It is observed that every pair of adjacent edges having distinct total degrees. So, G is a neighbourly edge totally irregular intuitionistic fuzzy graph.

Hence G is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.

Example 3.2. Neighbourly edge irregular intuitionistic fuzzy graph don't need to be neighbourly edge totally irregular intuitionistic fuzzy graph.

Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is a star on four vertices. From Figure 2, $d_G(u) = (0.3, 0.4), d_G(v) = (0.2, 0.5), d_G(w) = (0.1, 0.6), d_G(x) = (0.6, 1.5)$;

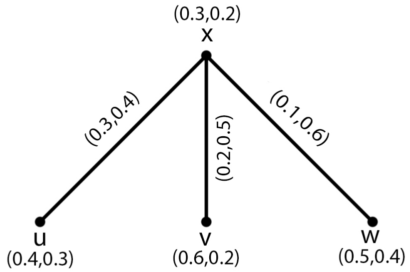

FIGURE 2. Neighbourly edge irregular IFG, not neighbourly edge totally irregular IFG

$d_G(ux) = (0.3, 1.1), d_G(vx) = (0.4, 1.0), d_G(wx) = (0.5, 0.9)$;
$td_G(ux) = td_G(vx) = td_G(wx) = (0.6, 1.5)$.

Here, $d_G(ux) \neq d_G(vx) \neq d_G(wx)$. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph. But G is not a neighbourly edge totally irregular intuitionistic fuzzy graph, since all edges have same total degrees.

Example 3.3. Neighbourly edge totally irregular intuitionistic fuzzy graphs don't need to be neighbourly edge irregular intuitionistic fuzzy graphs. Following shows this subject:

Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

From Figure 3, $d_G(u) = d_G(x) = (0.2, 0.3), d_G(v) = d_G(w) = (0.6, 0.9)$;

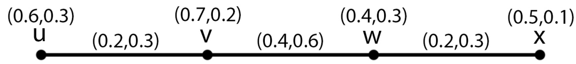

FIGURE 3. Neighbourly edge totally irregular IFG, not neighbourly edge irregular IFG.

$d_G(uv) = d_G(vw) = d_G(wx) = (0.4, 0.6)$;
$td_G(uv) = td_G(wx) = (0.6, 0.9), td_G(vw) = (0.8, 1.2)$.

Here, $d_G(uv) = d_G(vw) = d_G(wx)$. Hence G is not a neighbourly edge irregular intuitionistic fuzzy graph. But G is a neighbourly edge totally irregular intuitionistic fuzzy graph, since $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$.

Theorem 3.1. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. Then G is a neighbourly edge irregular intuitionistic fuzzy graph, if and only if G is a neighbourly edge totally irregular intuitionistic fuzzy graph.*

Proof. Assume that $\mu : (\mu_1, \mu_2)$ is a constant function, let $\mu(uv) = c$, for all uv in E, where $c = (c_1, c_2)$ is constant.

Let uv and vw be pair of adjacent edges in E, then we have
$d_G(uv) \neq d_G(vw)$

$\Longleftrightarrow d_G(uv) + c \neq d_G(vw) + c$
$\Longleftrightarrow (d_{\mu_1}(uv), d_{\mu_2}(uv)) + (c_1, c_2) \neq (d_{\mu_1}(vw), d_{\mu_2}(vw)) + (c_1, c_2)$
$\Longleftrightarrow (d_{\mu_1}(uv) + c_1, d_{\mu_2}(uv) + c_2) \neq (d_{\mu_1}(vw) + c_1, d_{\mu_2}(vw) + c_2)$
$\Longleftrightarrow (d_{\mu_1}(uv) + \mu_1(uv), d_{\mu_2}(uv) + \mu_2(uv)) \neq (d_{\mu_1}(vw) + \mu_1(vw), d_{\mu_2}(vw) + \mu_2(vw))$
$\Longleftrightarrow (td_{\mu_1}(uv), td_{\mu_2}(uv)) \neq (td_{\mu_1}(vw), td_{\mu_2}(vw))$
$\Longleftrightarrow td_G(uv) \neq td_G(vw)$.

Therefore every pair of adjacent edges have distinct degrees if and only if have distinct total degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph if and only if G is a neighbourly edge totally irregular intuitionistic fuzzy graph. □

Theorem 3.2. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. If G is a strongly irregular intuitionistic fuzzy graph, then G is a neighbourly edge irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. Assume that $\mu : (\mu_1, \mu_2)$ is a constant function, let $\mu(uv) = c$, for all uv in E, where $c = (c_1, c_2)$ is constant.

Let uv and vw be any two adjacent edges in G. Let us suppose that G is a strongly irregular intuitionistic fuzzy graph. Then every pair of vertices in G having distinct degrees, and hence $d_G(u) \neq d_G(w)$
$\Rightarrow (d_{\sigma_1}(u), d_{\sigma_2}(u)) \neq (d_{\sigma_1}(v), d_{\sigma_2}(v)) \neq (d_{\sigma_1}(w), d_{\sigma_2}(w))$
$\Rightarrow (d_{\sigma_1}(u), d_{\sigma_2}(u)) + (d_{\sigma_1}(v), d_{\sigma_2}(v)) - 2(c_1, c_2) \neq (d_{\sigma_1}(v), d_{\sigma_2}(v)) + (d_{\sigma_1}(w), d_{\sigma_2}(w)) - 2(c_1, c_2)$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2c_1, d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2c_2) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w) - 2c_1, d_{\sigma_2}(v) + d_{\sigma_2}(w) - 2c_2)$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2\mu_1(uv), d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2\mu_2(uv)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w) - 2\mu_1(vw), d_{\sigma_2}(v) + d_{\sigma_2}(w) - 2\mu_2(vw))$
$\Rightarrow (d_{\mu_1}(uv), d_{\mu_2}(uv)) \neq (d_{\mu_1}(vw), d_{\mu_2}(vw))$
$\Rightarrow d_G(uv) \neq d_G(vw)$.

Therefore every pair of adjacent edges have distinct degrees, hence G is a neighbourly edge irregular intuitionistic fuzzy graph. □

Similar to the above theorem can be considered the following theorem:

Theorem 3.3. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. If G is a strongly irregular intuitionistic fuzzy graph, then G is a neighbourly edge totally irregular intuitionistic fuzzy graph.*

Remark 3.1. Converse of the above theorems don't need to be true.

Example 3.4. Consider $G : (\sigma, \mu)$ be a intuitionistic fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

From Figure 4, $d_G(u) = d_G(x) = (0.3, 0.4), d_G(v) = d_G(w) = (0.6, 0.8)$.

Here, G is not a strongly irregular intuitionistic fuzzy graph.
$d_G(uv) = d_G(wx) = (0.3, 0.4), d_G(vw) = (0.6, 0.8)$;
$td_G(uv) = td_G(wx) = (0.6, 0.8), td_G(vw) = (0.9, 1.2)$.

It is noted that $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph. Also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence G is both neighbourly edge irregular intuitionistic fuzzy graph and

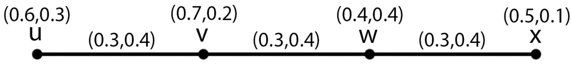

FIGURE 4. Both neighbourly edge irregular IFG and neighbourly edge totally irregular IFG, not strongly irregular IFG.

neighbourly edge totally irregular intuitionistic fuzzy graph. But G is not a strongly irregular intuitionistic fuzzy graph.

Theorem 3.4. *Let $G:(\sigma,\mu)$ be a connected intuitionistic fuzzy graph on $G^*:(V,E)$ and $\mu:(\mu_1,\mu_2)$ is a constant function. Then G is a highly irregular intuitionistic fuzzy graph if and only if G is a neighbourly edge irregular intuitionistic fuzzy graph.*

Proof. Let $G:(\sigma,\mu)$ be a connected intuitionistic fuzzy graph on $G^*:(V,E)$. Assume that $\mu:(\mu_1,\mu_2)$ is a constant function, let $\mu(uv)=c$, for all uv in E, where $c=(c_1,c_2)$ is constant.

Let uv and vw be any two adjacent edges in G. Then we have
$d_G(u)\neq d_G(w)$
$\Longleftrightarrow (d_{\sigma_1}(u),d_{\sigma_2}(u))\neq(d_{\sigma_1}(w),d_{\sigma_2}(w))$
$\Longleftrightarrow (d_{\sigma_1}(u),d_{\sigma_2}(u))+(d_{\sigma_1}(v),d_{\sigma_2}(v))-2(c_1,c_2)\neq(d_{\sigma_1}(v),d_{\sigma_2}(v))+(d_{\sigma_1}(w),d_{\sigma_2}(w))-2(c_1,c_2)$
$\Longleftrightarrow (d_{\sigma_1}(u)+d_{\sigma_1}(v)-2c_1,d_{\sigma_2}(u)+d_{\sigma_2}(v)-2c_2)\neq(d_{\sigma_1}(v)+d_{\sigma_1}(w)-2c_1,d_{\sigma_2}(v)+d_{\sigma_2}(w)-2c_2)$
$\Longleftrightarrow (d_{\sigma_1}(u)+d_{\sigma_1}(v)-2\mu_1(uv),d_{\sigma_2}(u)+d_{\sigma_2}(v)-2\mu_2(uv))\neq(d_{\sigma_1}(v)+d_{\sigma_1}(w)-2\mu_1(vw),d_{\sigma_2}(v)+d_{\sigma_2}(w)-2\mu_2(vw))$
$\Longleftrightarrow (d_{\mu_1}(uv),d_{\mu_2}(uv))\neq(d_{\mu_1}(vw),d_{\mu_2}(vw))$
$\Longleftrightarrow d_G(uv)\neq d_G(vw)$.

Therefore every pair of adjacent edges have distinct degrees, if and only if every vertex adjacent to the vertices having distinct degrees. Hence G is a highly irregular intuitionistic fuzzy graph, if and only if G is a neighbourly edge irregular intuitionistic fuzzy graph. □

Theorem 3.5. *Let $G:(\sigma,\mu)$ be a connected intuitionistic fuzzy graph on $G^*:(V,E)$ and $\mu:(\mu_1,\mu_2)$ is a constant function. Then G is highly irregular intuitionistic fuzzy graph if and only if G is neighbourly edge totally irregular intuitionistic fuzzy graph.*

Proof. Proof is similar to the above Theorem 3.4. □

Theorem 3.6. *Let $G:(\sigma,\mu)$ be an intuitionistic fuzzy graph on $G^*:(V,E)$, a star $K_{1,n}$. Then G is a totally edge regular intuitionistic fuzzy graph. Also, if the membership values and the nonmembership values of no two edges are same, then G is a neighbourly edge irregular intuitionistic fuzzy graph.*

Proof. Let $v_1,v_2,v_3,...,v_n$ be the vertices adjacent to the vertex x. Let $e_1,e_2,e_3,...,e_n$ be the edges of a star G^* in that order having membership values $p_1,p_2,p_3,...,p_n$ and nonmembership values $q_1,q_2,q_3,...,q_n$ that $0\leq p_i+q_i\leq 1$ for every $1\leq i\leq n$. Then, $td_G(e_i)=(td_{\mu_1}(e_i),td_{\mu_2}(e_i))=(d_{\mu_1}(e_i)+\mu_1(e_i,d_{\mu_2}(e_i)+\mu_2(e_i)=((p_1+p_2+p_3+...+p_n)-p_i+p_i,(q_1+q_2+q_3+...+q_n)-q_i+q_i)=(p_1+p_2+p_3+...+p_n,q_1+q_2+q_3+...+q_n)$. All edges e_i , $(1\leq i\leq n)$, having same total degrees. Hence G is a totally edge regular

intuitionistic fuzzy graph. Now, if $p_i \neq p_j$ and $q_i \neq q_j$ for every $1 \leq i, j \leq n$, then we have
$d_G(e_i) = (d_{\mu_1}(e_i), d_{\mu_2}(e_i)) = (d_{\sigma_1}(x)+d_{\sigma_1}(v_i)-2\mu_1(xv_i), d_{\sigma_2}(x)+d_{\sigma_2}(v_i)-2\mu_2(xv_i)) = ((p_1+p_2+p_3+...+p_n)+p_i-2p_i, (q_1+q_2+q_3+...+q_n)+q_i-2q_i) = ((p_1+p_2+p_3+...+p_n)-p_i, (q_1+q_2+q_3+...+q_n)-q_i)$ for every $1 \leq i \leq n$.

Therefore, all edges e_i ,$(1 \leq i \leq n)$, having distinct degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph. □

Theorem 3.7. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is a path on $2m(m > 1)$ vertices. If the membership values and the nonmembership values of the edges e_i, $i = 1, 3, 5, ..., 2m-1$, are p_1 and q_1,respectively, and the membership values and the nonmembership values of the edges e_i, $i = 2, 4, 6, ..., 2m-2$, are p_2 and q_2,respectively, such that $p_1 \neq p_2$ and $p_2 \neq 2p_1$ and $q_1 \neq q_2$ and $q_2 \neq 2q_1$, then G is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a path on $2m(m > 1)$ vertices. Let $e_1, e_2, e_3, ..., e_{2m-1}$ be the edges of path G^*. If the alternate edges have the same membership values and nonmembership values, such that

$$\mu(e_i) = (\mu_1(e_i), \mu_2(e_i)) = \begin{cases} (p_1, q_1) \ if \ i \ is \ odd \\ (p_2, q_2) \ if \ i \ is \ even. \end{cases}$$

where $0 \leq p_i + q_i \leq 1$ and $p_1 \neq p_2$ and $p_2 \neq 2p_1$ and $q_1 \neq q_2$ and $q_2 \neq 2q_1$, then
$d_G(e_1) = ((p_1) + (p_1 + p_2) - 2p_1, (q_1) + (q_1 + q_2) - 2q_1) = (p_2, q_2)$
For $i = 3, 5, 7, ..., 2m-3$;
$d_G(e_i) = ((p_1 + p_2) + (p_1 + p_2) - 2p_1, (q_1 + q_2) + (q_1 + q_2) - 2q_1) = (2p_2, 2q_2)$
For $i = 2, 4, 6, ..., 2m-2$;
$d_G(e_i) = ((p_1 + p_2) + (p_1 + p_2) - 2p_2, (q_1 + q_2) + (q_1 + q_2) - 2q_2) = (2p_1, 2q_1)$
$d_G(e_{2m-1}) = ((p_1 + p_2) + (p_1) - 2p_1, (q_1 + q_2) + (q_1) - 2q_1) = (p_2, q_2)$.

We observe that the adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graphs. Also we have
$td_G(e_1) = (p_1 + p_2, q_1 + q_2)$
$td_G(e_i) = (2p_1 + p_2, 2q_1 + q_2)$ for $i = 2, 4, 6, ..., 2m-2$
$td_G(e_i) = (p_1 + 2p_2, q_1 + 2q_2)$ for $i = 3, 5, 7, ..., 2m-3$
$td_G(e_{2m-1}) = (p_1 + p_2, q_1 + q_2)$.

Therefore the adjacent edges have distinct total degrees, hence G is a neighbourly edge totally irregular intuitionistic fuzzy graph. □

Theorem 3.8. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is an even cycle of length $2m$. If the alternate edges have the same membership values and the same nonmembership values , then G is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, an even cycle of length $2m$. Let $e_1, e_2, e_3, ..., e_{2m}$ be the edges of cycle G^*. If the alternate edges have the same membership values and the same nonmembership value, such that

$$\mu(e_i) = (\mu_1(e_i), \mu_2(e_i)) = \begin{cases} (p_1, q_1) \ if \ i \ is \ odd \\ (p_2, q_2) \ if \ i \ is \ even. \end{cases}$$

where $0 \leq p_i + q_i \leq 1$ and $p_1 \neq p_2$ and $q_1 \neq q_2$,then

for $i = 1, 3, 5, 7, ..., 2m-1$:
$d_G(e_i) = ((p_1+p_2)+(p_1+p_2)-2p_1, (q_1+q_2)+(q_1+q_2)-2q_1) = (2p_2, 2q_2)$
for $i = 2, 4, 6, ..., 2m$:
$d_G(e_i) = ((p_1+p_2)+(p_1+p_2)-2p_2, (q_1+q_2)+(q_1+q_2)-2q_2) = (2p_1, 2q_1)$

We observe that the adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graphs. Also we have
$td_G(e_i) = (p_1+2p_2, q_1+2q_2)$ for $i = 1, 3, 5, 7, ..., 2m-1$,
$td_G(e_i) = (2p_1+p_2, 2q_1+q_2)$ for $i = 2, 4, 6, ..., 2m$.

Therefore the adjacent edges have distinct total degrees, hence G is a neighbourly edge totally irregular intuitionistic fuzzy graph. □

Theorem 3.9. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a cycle on $m(m \geq 4)$ vertices. If the membership value and nonmembership value of the edges $e_1, e_2, e_3, ..., e_m$ are $p_1, p_2, p_3, ..., p_m$ such that $p_1 < p_2 < p_3 < ... < p_m$ and $q_1, q_2, q_3, ..., q_m$ such that $q_1 > q_2 > q_3 > ... > q_m$, respectively, then G is both neighbourly edge irregular intuitionistic fuzzy graph and neighbourly edge totally irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a cycle on $m(m \geq 4)$ vertices. Let $e_1, e_2, e_3, ..., e_m$ be the edges of cycle G^* in that order. Let membership values and nonmembership value of the edges $e_1, e_2, e_3, ..., e_m$ are $p_1, p_2, p_3, ..., p_m$ such that $p_1 < p_2 < p_3 < ... < p_m$ and $q_1, q_2, q_3, ..., q_m$ such that $q_1 > q_2 > q_3 > ... > q_m$, respectively, then
$d_G(v_1) = (p_1+p_m, q_1+q_m)$,
$d_G(v_i) = (p_{i-1}+p_i, q_{i-1}+q_i)$ for $i = 2, 3, 4, 5, ..., m$,
$d_G(e_1) = (p_2+p_m, q_2+q_m)$,
$d_G(e_i) = (p_{i-1}+p_{i+1}, q_{i-1}+q_{i+1})$ for $i = 2, 3, 4, 5, ..., m-1$,
$d_G(e_m) = (p_1+p_{m-1}, q_1+q_{m-1})$.

We observe that the adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph.
$td_G(e_1) = (p_1+p_2+p_m, q_1+q_2+q_m)$,
$td_G(e_i) = (p_{i-1}+p_i+p_{i+1}, q_{i-1}+q_i+q_{i+1})$ for $i = 2, 3, 4, 5, ..., m-1$,
$td_G(e_m) = (p_1+p_{m-1}+p_m, q_1+q_{m-1}+p_m)$.

We note that the adjacent edges have distinct total degrees. Hence G is a neighbourly edge totally irregular intuitionistic fuzzy graph. □

Now, we study strongly edge irregular intuitionistic fuzzy graphs and strongly edge totally irregular intuitionistic fuzzy graphs.

Definition 3.2. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. Then G is said to be:
(i)A strongly edge irregular intuitionistic fuzzy graph if every pair of edges having distinct degrees (or) no two edges have same degree. (ii) A strongly edge totally irregular intuitionistic fuzzy graph if every pair of edges having distinct total degrees (or) no two edges have same total degree.

Example 3.5. Graph which is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph.

Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ which is a cycle of length five. From Figure 5, $d_G(u) = (0.6, 0.6), d_G(v) = (0.3, 0.9), d_G(w) =$

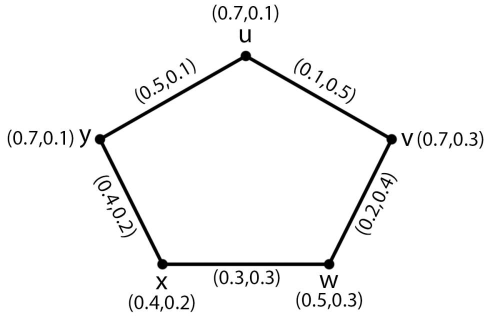

FIGURE 5. Both strongly edge irregular IFG and strongly edge totally irregular IFG.

$(0.5, 0.7), d_G(x) = (0.7, 0.5), d_G(y) = (0.9, 0.3)$. Degrees of the edges are calculated as follows: $d_G(uv) = (0.7, 0.5)$, $d_G(vw) = (0.4, 0.8)$, $d_G(wx) = (0.6, 0.6)$, $d_G(xy) = (0.8, 0.4)$, $d_G(yu) = (0.5, 0.7)$.

It is noted that every pair of edges having distinct degrees. Hence G is a strongly edge irregular intuitionistic fuzzy graph.

Total degrees of the edges are calculated as follows: $td_G(uv) = (0.8, 1.0)$, $td_G(vw) = (0.6, 1.2)$, $td_G(wx) = (0.9, 0.9)$, $td_G(xy) = (1.2, 0.6)$, $td_G(yu) = (1.0, 0.8)$.

It is observed that every pair of edges having distinct total degrees. So, G is a strongly edge totally irregular intuitionistic fuzzy graph.

Hence G is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph.

Example 3.6. Strongly edge irregular intuitionistic fuzzy graph need not be strongly edge totally irregular intuitionistic fuzzy graph.

Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$, a cycle of length three.

From Figure 6, $d_G(u) = (0.5, 1.3), d_G(v) = (0.7, 1.2), d_G(w) = (0.6, 1.1)$;

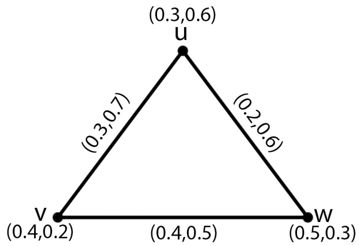

FIGURE 6. Strongly edge irregular IFG, not strongly edge totally irregular IFG.

$d_G(uv) = (0.6, 1.1), d_G(vw) = (0.5, 1.3), d_G(wu) = (0.7, 1.2)$;
$td_G(uv) = td_G(vw) = td_G(wu) = (0.9, 1.8)$.

Note that G is strongly edge irregular intuitionistic fuzzy graph, since every pair of edges having distinct degrees. Also, G is not strongly edge totally irregular intuitionistic fuzzy graph, since all the edges having same total degree. Hence strongly edge irregular intuitionistic fuzzy graph need not be strongly edge totally irregular intuitionistic fuzzy graph.

Example 3.7. Strongly edge totally irregular intuitionistic fuzzy graphs need not be strongly edge irregular intuitionistic fuzzy graphs.

Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$, a cycle of length four.

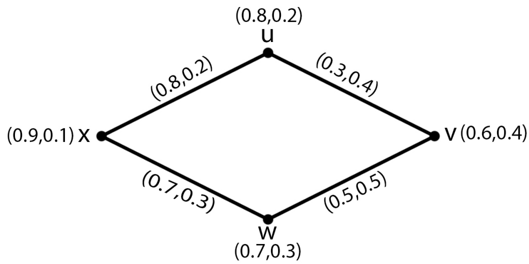

FIGURE 7. Strongly edge totally irregular IFG, not strongly edge irregular IFG.

From Figure 7, $d_G(u) = (1.1, 0.6), d_G(v) = (0.8, 0.9), d_G(w) = (1.2, 0.8), d_G(x) = (1.5, 0.5)$; $d_G(uv) = d_G(wx) = (1.3, 0.7), d_G(vw) = d_G(xu) = (1.0, 0.7)$;
$td_G(uv) = (1.6, 1.1), td_G(vw) = (1.5, 1.2), td_G(wx) = (2.0, 1.0), d_G(xu) = (1.9, 0.8)$.
It is noted that $d_G(uv) = d_G(wx)$. Hence G is not strongly edge irregular intuitionistic fuzzy graph.
But G is strongly edge totally irregular intuitionistic fuzzy graph, since $td_G(uv) \neq td_G(vw) \neq td_G(wx) \neq td_G(xu)$.
Hence strongly edge totally irregular intuitionistic fuzzy graph need not be strongly edge irregular intuitionistic fuzzy graph.

Theorem 3.10. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. Then G is a strongly edge irregular intuitionistic fuzzy graph, if and only if G is a strongly edge totally irregular intuitionistic fuzzy graph.*

Proof. Assume that $\mu : (\mu_1, \mu_2)$ is a constant function, let $\mu(uv) = c$, for all uv in E, where $c = (c_1, c_2)$ is constant.
Let uv and xy be any pair of edges in E. Then we have
$d_G(uv) \neq d_G(xy)$
$\Longleftrightarrow d_G(uv) + c \neq d_G(xy) + c$
$\Longleftrightarrow (d_{\mu_1}(uv), d_{\mu_2}(uv)) + (c_1, c_2) \neq (d_{\mu_1}(xy), d_{\mu_2}(xy)) + (c_1, c_2)$
$\Longleftrightarrow (d_{\mu_1}(uv) + c_1, d_{\mu_2}(uv) + c_2) \neq (d_{\mu_1}(xy) + c_1, d_{\mu_2}(xy) + c_2)$
$\Longleftrightarrow (d_{\mu_1}(uv) + \mu_1(uv), d_{\mu_2}(uv) + \mu_2(uv)) \neq (d_{\mu_1}(xy) + \mu_1(xy), d_{\mu_2}(xy) + \mu_2(xy))$
$\Longleftrightarrow (td_{\mu_1}(uv), td_{\mu_2}(uv)) \neq (td_{\mu_1}(xy), td_{\mu_2}(xy))$
$\Longleftrightarrow td_G(uv) \neq td_G(xy)$

Therefore every pair of edges have distinct degrees if and only if have distinct total degrees. Hence G is strongly edge irregular intuitionistic fuzzy graph if and only if G is a strongly edge totally irregular intuitionistic fuzzy graph. □

Remark 3.2. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. If G is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph, Then μ need not be a constant function.

Example 3.8. Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is a cycle of length five.

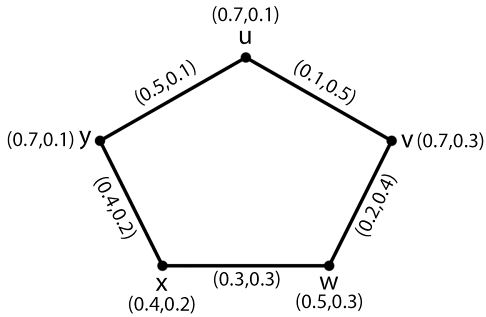

FIGURE 8. μ is not a constant function.

From Figure 8, $d_G(u) = (0.6, 0.6), d_G(v) = (0.3, 0.9), d_G(w) = (0.5, 0.7), d_G(x) = (0.7, 0.5), d_G(y) = (0.9, 0.3)$.
Also, $d_G(uv) = (0.7, 0.5), d_G(vw) = (0.4, 0.8), d_G(wx) = (0.6, 0.6), d_G(xy) = (0.8, 0.4)$, $d_G(yu) = (0.5, 0.7)$. It is noted that every pair of edges in G having distinct degrees. Hence G is a strongly edge irregular intuitionistic fuzzy graph.
Also, $td_G(uv) = (0.8, 1.0), td_G(vw) = (0.6, 1.2), td_G(wx) = (0.9, 0.9), td_G(xy) = (1.2, 0.6), td_G(yu) = (1.0, 0.8)$. Note that every pair of edges in G having distinct total degrees. Hence G is a strongly edge totally irregular intuitionistic fuzzy graph. Therefore G is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph. But μ is not a constant function.

Theorem 3.11. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$. If G is a strongly edge irregular intuitionistic fuzzy graph, then G is a neighbourly edge irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$. Let us assume that G is a strongly edge irregular intuitionistic fuzzy graph, then every pair of edges in G have distinct degrees. So every pair of adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph. □

Theorem 3.12. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$. If G is a strongly edge totally irregular intuitionistic fuzzy graph, then G is a neighbourly edge totally irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$. Let us assume that G is a strongly edge totally irregular intuitionistic fuzzy graph, then every pair of edges in G have distinct total degrees. So every pair of adjacent edges have distinct total degrees. Hence G is a neighbourly edge totally irregular intuitionistic fuzzy graph. $\square$

Remark 3.3. Converse of the above Theorems 3.11 and 3.12 need not be true.

Example 3.9. Consider $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

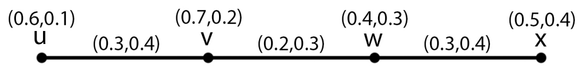

FIGURE 9. Neighbourly edge irregular IFG, not strongly edge irregular IFG; neighbourly edge totally irregular IFG, not strongly edge totally irregular IFG.

From Figure 9, $d_G(u) = d_G(x) = (0.3, 0.4), d_G(v) = d_G(w) = (0.5, 0.7)$;
$d_G(uv) = d_G(wx) = (0.2, 0.3), d_G(vw) = (0.6, 0.8)$;
$td_G(uv) = td_G(wx) = (0.5, 0.7), td_G(vw) = (0.8, 1.1)$.
Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence G is a neighbourly edge irregular intuitionistic fuzzy graph. But G is not a strongly edge irregular intuitionistic fuzzy graph, since $d_G(uv) \neq d_G(wx)$. Also, note that $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence G is a neighbourly edge totally irregular intuitionistic fuzzy graph. But G is not a strongly edge totally irregular intuitionistic fuzzy graph, since $td_G(uv) \neq td_G(wx)$.

Theorem 3.13. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. If G is a strongly edge irregular intuitionistic fuzzy graph, then G is an irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. Assume that $\mu : (\mu_1, \mu_2)$ is a constant function, let $\mu(uv) = c$, for all uv in E, where $c = (c_1, c_2)$ is constant.
Let us Suppose that G is a strongly edge irregular intuitionistic fuzzy graph. Then every pair of edges having distinct degrees. Let uv and vw be adjacent edges in G having distinct degrees, and hence $d_G(uv) \neq d_G(vw)$
$\Rightarrow (d_{\mu_1}(uv), d_{\mu_2}(uv)) \neq (d_{\mu_1}(vw), d_{\mu_2}(vw))$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2\mu_1(uv), d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2\mu_2(uv)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w) - 2\mu_1(vw), d_{\sigma_2}(v) + d_{\sigma_2}(w) - 2\mu_2(vw))$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2c_1, d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2c_2) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w) - 2c_1, d_{\sigma_2}(v) + d_{\sigma_2}(w) - 2c_2)$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v), d_{\sigma_2}(u) + d_{\sigma_2}(v)) - 2(c_1, c_2) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w), d_{\sigma_2}(v) + d_{\sigma_2}(w)) - 2(c_1, c_2)$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v), d_{\sigma_2}(u) + d_{\sigma_2}(v)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w), d_{\sigma_2}(v) + d_{\sigma_2}(w))$
$\Rightarrow (d_{\sigma_1}(u), d_{\sigma_2}(u)) + (d_{\sigma_1}(v), d_{\sigma_2}(v)) \neq (d_{\sigma_1}(v), d_{\sigma_2}(v)) + (d_{\sigma_1}(w), d_{\sigma_2}(w))$
$\Rightarrow d_G(u) + d_G(v) \neq d_G(v) + d_G(w)$

$\Rightarrow d_G(u) \neq d_G(w)$
So there exists a vertex v which is adjacent to vertices u and w having distinct degrees. Hence G is an irregular intuitionistic fuzzy graph. □

Theorem 3.14. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. If G is a strongly edge totally irregular intuitionistic fuzzy graph, then G is an irregular intuitionistic fuzzy graph.*

Proof. Proof is similar to the above Theorem 3.13. □

Remark 3.4. Converse of the above Theorems 3.13 and 3.14 need not be true.

Example 3.10. Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that G^* : (V, E) is a path on four vertices.

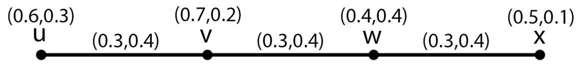

FIGURE 10. Irregular IFG, not strongly edge irregular IFG and not strongly edge totally irregular IFG

From Figure 10, $d_G(u) = d_G(x) = (0.3, 0.4), d_G(v) = d_G(w) = (0.6, 0.8)$.
Here, G is an irregular intuitionistic fuzzy graph.
Also, $d_G(uv) = d_G(wx) = (0.3, 0.4), d_G(vw) = (0.6, 0.8)$;
$td_G(uv) = td_G(wx) = (0.6, 0.8), td_G(vw) = (0.9, 1.2)$.
It is noted that $d_G(uv) = d_G(wx)$. Hence G is not a strongly edge irregular intuitionistic fuzzy graph. Also, $td_G(uv) = td_G(wx)$. Hence G is not a strongly edge totally irregular intuitionistic fuzzy graph.

Theorem 3.15. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. If G is a strongly edge irregular intuitionistic fuzzy graph, Then G is a highly irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$. Assume that $\mu : (\mu_1, \mu_2)$ is a constant function, let $\mu(uv) = c$ for all uv in E, where $c = (c_1, c_2)$ is constant.
Let v be any vertex adjacent with u, w and x. Then uv, vw and vx are adjacent edges in G. Let us suppose that G is strongly edge irregular intuitionistic fuzzy graph. Then every pair of edges in G have distinct degrees. So every pair of adjacent edges in G have distinct degrees. Hence $d_G(uv) \neq d_G(vw) \neq d_G(vx)$
$\Rightarrow (d_{\mu_1}(uv), d_{\mu_2}(uv)) \neq (d_{\mu_1}(vw), d_{\mu_2}(vw)) \neq (d_{\mu_1}(vx), d_{\mu_2}(vx))$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2\mu_1(uv), d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2\mu_2(uv)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w) - 2\mu_1(vw), d_{\sigma_2}(v) + d_{\sigma_2}(w) - 2\mu_2(vw)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(x) - 2\mu_1(vx), d_{\sigma_2}(v) + d_{\sigma_2}(x) - 2\mu_2(vx))$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v) - 2c_1, d_{\sigma_2}(u) + d_{\sigma_2}(v) - 2c_2) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w) - 2c_1, d_{\sigma_2}(v) + d_{\sigma_2}(w) - 2c_2) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(x) - 2c_1, d_{\sigma_2}(v) + d_{\sigma_2}(x) - 2c_2)$
$\Rightarrow (d_{\sigma_1}(u) + d_{\sigma_1}(v), d_{\sigma_2}(u) + d_{\sigma_2}(v)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(w), d_{\sigma_2}(v) + d_{\sigma_2}(w)) \neq (d_{\sigma_1}(v) + d_{\sigma_1}(x), d_{\sigma_2}(v) + d_{\sigma_2}(x))$
$\Rightarrow (d_{\sigma_1}(u), d_{\sigma_2}(u)) \neq (d_{\sigma_1}(w), d_{\sigma_2}(w)) \neq (d_{\sigma_1}(x), d_{\sigma_2}(x))$

$\Rightarrow d_G(u) \neq d_G(w) \neq d_G(x)$
Therefore the vertex v is adjacent to the vertices with distinct degrees.
Hence G is a highly irregular intuitionistic fuzzy graph. □

Theorem 3.16. *Let $G : (\sigma, \mu)$ be a connected intuitionistic fuzzy graph on $G^* : (V, E)$ and $\mu : (\mu_1, \mu_2)$ is a constant function. If G is a strongly edge totally irregular intuitionistic fuzzy graph, Then G is a highly irregular intuitionistic fuzzy graph.*

Proof. Proof is similar to the above Theorem 3.15. □

Remark 3.5. Converse of the above Theorems 3.15 and 3.16 need not be true.

Example 3.11. Consider $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that G^* : (V, E) is a path on four vertices. From Figure 11, $d_G(u) = d_G(x) = (0.3, 0.4), d_G(v) =$

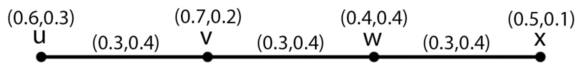

FIGURE 11. Highly irregular IFG, not strongly edge irregular IFG and not strongly edge totally irregular IFG.

$d_G(w) = (0.6, 0.8)$.
Here, G is a highly irregular intuitionistic fuzzy graph.
Note that $d_G(uv) = d_G(wx) = (0.3, 0.4), d_G(vw) = (0.6, 0.8)$.
So, G is not a strongly edge irregular intuitionistic fuzzy graph.
Also, $td_G(uv) = td_G(wx) = (0.6, 0.8), td_G(vw) = (0.9, 1.2)$.
So, G is not a strongly edge totally irregular intuitionistic fuzzy graph.

Theorem 3.17. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a star $K_{1,n}$. Then G is a totally edge regular intuitionistic fuzzy graph. Also, If the membership values and the nonmembership values of no two edges are same, then G is a strongly edge irregular intuitionistic fuzzy graph.*

Proof. Proof is similar to the Theorem 3.6. □

Theorem 3.18. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is a path on $2m(m > 1)$ vertices.If the membership values and nonmembership values of the edges $e_1, e_2, e_3, ..., e_{2m-1}$ are $p_1, p_2, p_3, ..., p_{2m-1}$ such that $p_1 < p_2 < p_3 < ... < p_{2m-1}$ and $q_1, q_2, q_3, ..., q_{2m-1}$ such that $q_1 > q_2 > q_3 > ... > q_{2m-1}$, respectively, then G is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a path on $2m(m > 1)$ vertices. Let $e_1, e_2, e_3, ..., e_{2m-1}$ be the edges of path G^* in that order. Let membership values and nonmembership values of the edges $e_1, e_2, e_3, ..., e_{2m-1}$ be $p_1, p_2, p_3, ..., p_{2m-1}$ such that $p_1 < p_2 < p_3 < ... < p_{2m-1}$ and $q_1, q_2, q_3, ..., q_{2m-1}$ such that $q_1 > q_2 > q_3 > ... > q_{2m-1}$, respectively, then
$d_G(v_1) = (p_1, q_1)$,
$d_G(v_i) = (p_{i-1} + p_i, q_{i-1} + q_i)$ for $i = 2, 3, 4, 5, ..., 2m - 1$,
$d_G(v_m) = (p_{2m-1}, q_{2m-1})$,

$d_G(e_1) = (p_2, q_2)$,
$d_G(e_i) = (p_{i-1} + p_{i+1}, q_{i-1} + q_{i+1})$ for $i = 2, 3, 4, 5, ..., 2m-2$,
$d_G(e_{2m-1}) = (p_{2m-2}, q_{2m-2})$.
We observe that any pair of edges have distinct degrees. Hence G is a strongly edge irregular inuitionistic fuzzy graph. Also we have
$td_G(e_1) = (p_1 + p_2, q_1 + q_2)$,
$td_G(e_i) = (p_{i-1} + p_i + p_{i+1}, q_{i-1} + q_i + q_{i+1})$ for $i = 2, 3, 4, 5, ..., 2m-2$,
$td_G(e_{2m-1}) = (p_{2m-2} + p_{2m-1}, q_{2m-2} + q_{2m-1})$.

Therefore any pair of edges have distinct total degrees, hence G is a strongly edge totally irregular intuitionistic fuzzy graph. □

Theorem 3.19. *Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph such that $G^* : (V, E)$ is a cycle on $m(m \geq 4)$ vertices. If the membership values and nonmembership values of the edges $e_1, e_2, e_3, ..., e_m$ are $p_1, p_2, p_3, ..., p_m$ such that $p_1 < p_2 < p_3 < ... < p_m$ and $q_1, q_2, q_3, ..., q_m$ such that $q_1 > q_2 > q_3 > ... > q_m$, respectively, then G is both strongly edge irregular intuitionistic fuzzy graph and strongly edge totally irregular intuitionistic fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$, a cycle on $m(m \geq 4)$ vertices. Let $e_1, e_2, e_3, ..., e_m$ be the edges of cycle G^* in that order. Let membership values and nonmembership values of the edges $e_1, e_2, e_3, ..., e_m$ be $p_1, p_2, p_3, ..., p_m$ such that $p_1 < p_2 < p_3 < ... < p_m$ and $q_1, q_2, q_3, ..., q_m$ such that $q_1 > q_2 > q_3 > ... > q_m$, respectively, then
$d_G(v_1) = (p_1 + p_m, q_1 + q_m)$,
$d_G(v_i) = (p_{i-1} + p_i, q_{i-1} + q_i)$ for $i = 2, 3, 4, 5, ..., m$,
$d_G(e_1) = (p_2 + p_m, q_2 + q_m)$,
$d_G(e_i) = (p_{i-1}+p_{i+1}, q_{i-1}+q_{i+1})$ for $i = 2, 3, 4, 5, ..., m-1$, $d_G(e_m) = (p_1+p_{m-1}, q_1+q_{m-1})$.

We observe that any pair of edges have distinct degrees. Hence G is a strongly edge irregular inuitionistic fuzzy graph.
$td_G(e_1) = (p_1 + p_2 + p_m, q_1 + q_2 + q_m)$,
$td_G(e_i) = (p_{i-1} + p_i + p_{i+1}, q_{i-1} + q_i + q_{i+1})$ for $i = 2, 3, 4, 5, ..., m-1$,
$td_G(e_m) = (p_1 + p_{m-1} + p_m, q_1 + q_{m-1} + q_m)$.

We note that any pair of edges have distinct total degrees. Hence G is a strongly edge totally irregular intuitionistic fuzzy graph. □

4. Application example of intuitionistic influence graph

These days, we see that graph models have many applications in different sciences such as computer science, topology, operations research, biological and social sciences. If we consider group behavior, it is observed that in a social group some people can influence thinking of others. Now with help of a directed graph which is an influence graph, we can used to model this behavior. We consider each person of a group as a vertex. There is a directed edge from vertex x to vertex y, when the person represented by vertex x influence the person represented by vertex y. This graph does not contain loops and it does not contain multiple directed edges. We now explore intuitionistic influence graph model to find out the influential person within a social group. In influence graph, the vertex (node) represents a power (authority) of a person and the edge represents the influence of a person on another person in the social group. Consider an intuitionistic influence graph of a social group. In Figure

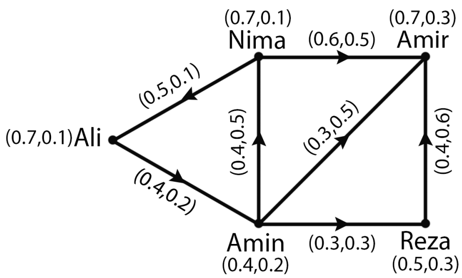

FIGURE 12. Intuitionistic influence graph.

12, the degree of power of a person is defined in terms of its trueness and falseness. The node of the intuitionistic influence graph shows the authority a person possesses in the group; for example, Amin has 40% authority in the group, but he does not have 20% power, and 20% power is not decided, whereas the edges show the influence of a person on another in a group; for example Amin can influence Nima 40%, but he can not convince him 50%, e.g., Nima follows 40% Amin's suggestions but he does not follows 50% his suggestions.

5. Conclusion

It is well known that graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological and social systems. In general graphs theory has a wide range of applications in diverse fields. In this paper, we introduced some types of edge irregular intuitionistic fuzzy graphs and properties of them. A comparative study between neighbourly edge irregular fuzzy graphs and neighbourly edge totally irregular fuzzy graph and also between strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graph did. Also some properties of neighbourly edge irregular fuzzy graphs and strongly edge irregular fuzzy graphs studied, and they examined for neighbourly edge totally irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs.

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