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Antiplane contact problems for viscoelastic materials

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Abstract. We survey our recent results in the study of two antiplane problems modelling contact between a viscoelastic body and a rigid foundation. The first problem involves frictionless adhesive contact while the second one is frictional. For each problem we present the mathematical model, its variational formulation, and state an existence and uniqueness result.

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1. Introduction

In this paper we make a survey on our recent results obtained in the study of two antiplane contact problems. We consider quasistatic processes for homogeneous isotropic viscoelastic materials. The first problem involves frictionless adhesive contact and, following [1], the adhesion process is modeled by a surface internal variable, the bonding field; the tangential shear due to the bounding field is included in the model. For the second problem, where the contact is frictional, the friction is modeled with a slip dependent version of Tresca's law. Our aim is to present results concerning the well posedness of these problems.

The paper is structured as follows. In Section 2 we present the physical setting and we state the corresponding mathematical models. In Section 3 we list the assumptions on the data, derive the variational formulations of the mechanical problems, and state existence and uniqueness results of the weak solution for the models. The proofs of our results can be found in [2, 4]. For the adhesive problem studied in [4] we use a version of the Cauchy-Lipschitz theorem and for the frictional problem studied in [2] we use the Banach's fixed point theorem and results for elliptic variational inequalities.

2. Mathematical modelling

We consider a body $\mathcal{B}$ identified with a region in $\mathbb{R}^3$ it occupies in a fixed and undistorted reference configuration. We assume that $\mathcal{B}$ is a cylinder with generators parallel to the x_3-axes with a cross-section which is a regular region Ω in the x_1, x_2-plane, $Ox_1x_2x_3$ being a Cartesian coordinate system. The cylinder is assumed to be sufficiently long so that end effects in the axial direction are negligible. Thus, $\mathcal{B} = \Omega \times (-\infty, +\infty)$. Let $\partial\Omega = \Gamma$. We assume that Γ is divided into three disjoint measurable parts Γ_1, Γ_2 and Γ_3 such that the one-dimensional measure of Γ_1, denoted $meas\,\Gamma_1$, is strictly positive. Let $T > 0$ and let $[0, T]$ denote the time interval of interest. The cylinder is clamped on $\Gamma_1 \times (-\infty, +\infty)$ and is in contact with a rigid foundation on $\Gamma_3 \times (-\infty, +\infty)$. Moreover, the cylinder is subjected to time dependent

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volume forces of density $\boldsymbol{f}_0$ on $\mathcal{B}$ and to time dependent surface tractions of density $\boldsymbol{f}_2$ on $\Gamma_2 \times (-\infty, +\infty)$. We assume that

$$\boldsymbol{f}_0 = (0, 0, f_0) \quad \text{with} \quad f_0 = f_0(x_1, x_2, t) : \Omega \times [0, T] \to \mathbb{R}, \tag{1}$$

$$\boldsymbol{f}_2 = (0, 0, f_2) \quad \text{with} \quad f_2 = f_2(x_1, x_2, t) : \Gamma_2 \times [0, T] \to \mathbb{R}. \tag{2}$$

The body forces (1) and the surface tractions (2) would be expected to give rise to a deformation of the cylinder whose displacement denoted by $\boldsymbol{u}$ is of the form

$$\boldsymbol{u} = (0, 0, u) \quad \text{with} \quad u = u(x_1, x_2, t) : \Omega \times [0, T] \to \mathbb{R}. \tag{3}$$

Such kind of deformation is called an *antiplane shear*. From (3) it follows that, in the case of the antiplane problem, the infinitesimal strain tensor becomes

$$\boldsymbol{\varepsilon}(\boldsymbol{u}) = \begin{pmatrix} 0 & 0 & \frac{1}{2}u_{,1} \\ 0 & 0 & \frac{1}{2}u_{,2} \\ \frac{1}{2}u_{,1} & \frac{1}{2}u_{,2} & 0 \end{pmatrix}, \tag{4}$$

where the index that follows a comma indicates a partial derivative with respect to the corresponding component of the spatial variable.

Let $\boldsymbol{\sigma} = (\boldsymbol{\sigma}_{ij})$ denote the stress field. We consider the linear constitutive law for a viscoelastic material

$$\boldsymbol{\sigma} = 2\theta\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}) + \lambda(\text{tr } \boldsymbol{\varepsilon}(\boldsymbol{u}))\boldsymbol{I} + 2\mu\boldsymbol{\varepsilon}(\boldsymbol{u}), \tag{5}$$

where $\lambda > 0$ and $\mu > 0$ are the Lamé coefficients, $\theta > 0$ is the coefficient of viscosity, tr $\boldsymbol{\varepsilon}(\boldsymbol{u}) = \varepsilon_{ii}(\boldsymbol{u})$ and $\boldsymbol{I}$ is the unit tensor in $\mathbb{R}^3$. Here and below the dot above represents the derivative with respect to the time variable and the convention summation upon repeated index is used.

We neglect the inertial term in the equation of motion and obtain the quasistatic approximation for the process. Thus, keeping in mind (1), (3) and (5) we deduce that the equation of equilibrium reduces to the following scalar equation

$$\theta\Delta\dot{u} + \mu\Delta u + f_0 = 0 \quad \text{on} \quad \Omega \times (0, T).$$

Recall that, since the cylinder is clamped on $\Gamma_1 \times (-\infty, +\infty) \times (0, T)$, the displacement field vanishes there. Thus, (3) implies that

$$u = 0 \quad \text{on} \quad \Gamma_1 \times (0, T).$$

Let $\boldsymbol{\nu}$ denote the unit normal on $\Gamma \times (-\infty, +\infty)$. We have $\boldsymbol{\nu} = (\nu_1, \nu_2, 0)$ where $\nu_i = \nu_i(x_1, x_2) : \Gamma \to \mathbb{R},\ i = 1, 2$. From (5) we deduce that the Cauchy stress vector is given by

$$\boldsymbol{\sigma\nu} = (0, 0, \theta\, \partial_\nu \dot{u} + \mu \partial_\nu u). \tag{6}$$

Here and below we use the notation $\partial_\nu u = u_{,1}\nu_1 + u_{,2}\nu_2$. Keeping in mind the traction boundary condition $\boldsymbol{\sigma\nu} = \boldsymbol{f}_2$ on $\Gamma_2 \times (0, T)$, it follows from (2) and (6) that

$$\theta\, \partial_\nu \dot{u} + \mu\, \partial_\nu u = f_2 \quad \text{on} \quad \Gamma_2 \times (0, T).$$

For the displacement field vector $\boldsymbol{u}$ we denote by u_ν and $\boldsymbol{u}_\tau$ its *normal* and *tangential* components on the boundary given by $u_\nu = \boldsymbol{v} \cdot \boldsymbol{\nu}$, $\boldsymbol{u}_\tau = \boldsymbol{u} - u_\nu\boldsymbol{\nu}$, and for the stress field $\boldsymbol{\sigma}$ we denote by σ_ν and $\boldsymbol{\sigma}_\tau$ the *normal* and the *tangential* components on the boundary, that is $\sigma_\nu = (\boldsymbol{\sigma\nu}) \cdot \boldsymbol{\nu}$, $\boldsymbol{\sigma}_\tau = \boldsymbol{\sigma\nu} - \sigma_\nu\boldsymbol{\nu}$. Using again (3) we find

$$u_\nu = 0, \quad \boldsymbol{u}_\tau = (0, 0, u) \tag{7}$$

and, similarly, from (5) and we obtain

$$\sigma_\nu = 0, \quad \boldsymbol{\sigma}_\tau = (0, 0, \sigma_\tau), \quad \text{where} \quad \sigma_\tau = \theta\, \partial_\nu \dot{u} + \mu\, \partial_\nu u. \tag{8}$$

We now describe the frictionless adhesive contact condition on $\Gamma_3 \times (-\infty, +\infty)$. Following [1], we introduce the surface state variable β, the bonding field, which is a measure of the fractional intensity of adhesion between the surface and the foundation. This variable is restricted to values $0 \leq \beta \leq 1$; when $\beta = 0$ all the bonds are severed and there are no active bonds; when $\beta = 1$ all the bonds are active. When $0 < \beta < 1$ it measures the fraction of active bonds and partial adhesion takes place.

We assume that the resistance to tangential motion is generated by the glue, in comparison to which the frictional traction can be neglected. Thus, the tangential traction depends only on the intensity of adhesion and the tangential displacement.

$$-\sigma_\tau = p(\beta, u).$$

Using now (8), it is straightforward to see that

$$\theta\, \partial_\nu \dot{u} + \mu\, \partial_\nu u = p(\beta, u) \quad \text{on } \Gamma_3 \times (0, T).$$

In particular, we may consider the case

$$p(\beta, r) = \begin{cases} -q(\beta)\, L & \text{if} \quad r < -L \\ q(\beta)\, r & \text{if} \quad |r| \leq L \\ q(\beta)\, L & \text{if} \quad r > L, \end{cases} \tag{9}$$

where $L > 0$ is the limit bound constant and q is a nonnegative tangential stiffness function, see [3].

The evolution of the bonding field is assumed to depend generally on β and $\boldsymbol{u}_\tau$. Keeping in mind (7), we can describe it by the equation

$$\dot{\beta} = H_{ad}(\beta, R(|u|)) \qquad \text{on } \Gamma_3 \times (0, T),$$

where H_{ad} is a general function discussed below. The function $R : \mathbb{R} \to \mathbb{R}$ is a truncation and is defined as

$$R(s) = \begin{cases} L & \text{if } s \geq L \\ s & \text{if } |s| \leq L \\ -L & \text{if } s \leq -L, \end{cases} \tag{10}$$

where $L > 0$ is a characteristic length of the bonds. We use it as an argument in H_{ad} since usually, when the glue is streched beyond the limit L it does not contribute more to the bond strenght. An example of such a function H_{ad} is given by

$$H_{ad}(\beta, r) = -\gamma_\nu \frac{\beta_+}{1 + \beta_+} r^2 \tag{11}$$

where γ_ν is the bonding energy constant and $\beta_+ = \max\{0, \beta\}$.

Let $\boldsymbol{u}_0 = (0, 0, u_0)$ be the initial displacement and β_0 the initial bonding field. The mechanical model of the *antiplane frictionless contact problem with adhesion* is complete and can be stated as follows.

Problem P_1**.** *Find an displacement field* $u : \Omega \times [0,T] \to \mathbb{R}$ *and an adhesion field* $\beta : \Gamma_3 \times [0,T] \to [0,1]$ *such that*

$$\theta\Delta\dot{u} + \mu\Delta u + f_0 = 0 \quad \text{in} \quad \Omega \times (0,T), \tag{12}$$

$$u = 0 \quad \text{on} \quad \Gamma_1 \times (0,T), \tag{13}$$

$$\theta\,\partial_\nu \dot{u} + \mu\,\partial_\nu u = f_2 \quad \text{on} \quad \Gamma_2 \times (0,T), \tag{14}$$

$$-\theta\,\partial_\nu \dot{u} - \mu\,\partial_\nu u = p(\beta, u) \quad \text{on} \quad \Gamma_3 \times (0,T), \tag{15}$$

$$\dot{\beta} = H_{ad}(\beta, R(|u|)) \quad \text{on} \quad \Gamma_3 \times (0,T), \tag{16}$$

$$u(0) = u_0 \quad \text{in} \quad \Omega, \tag{17}$$

$$\beta(0) = \beta_0 \quad \text{on} \quad \Gamma_3. \tag{18}$$

Note that when the displacement field u which solves Problem P_1 is known, then the stress tensor can be calculated using the formula (5).

We now describe the frictional contact condition on $\Gamma_3 \times (-\infty, +\infty)$. We assume that the friction is invariant with respect with the x_3 axis and is modelled with the following conditions on Γ_3, for all $t \in [0,T]$:

$$\begin{cases} |\boldsymbol{\sigma}_\tau(t)| \le g(\int_0^t |\dot{\boldsymbol{u}}_\tau(s)|ds), \\ |\boldsymbol{\sigma}_\tau(t)| < g(\int_0^t |\dot{\boldsymbol{u}}_\tau(s)|ds) \Rightarrow \dot{\boldsymbol{u}}_\tau(t) = 0, \\ |\boldsymbol{\sigma}_\tau(t)| = g(\int_0^t |\dot{\boldsymbol{u}}_\tau(s)|ds) \Rightarrow \exists\, \gamma \ge 0 \text{ such that } \boldsymbol{\sigma}_\tau = -\gamma \dot{\boldsymbol{u}}_\tau. \end{cases} \tag{19}$$

Here $g : \Gamma_3 \times \mathbb{R} \to \mathbb{R}_+$ is a given function, $\dot{\boldsymbol{u}}_\tau$ denotes the tangential velocity on the contact boundary and $|\cdot|$ denotes the Euclidian norm on $\mathbb{R}^d$ ($d = 1$ or 3). This is Tresca's friction law where the friction bound g is assumed to depend on the accumulated slip of the surface. From a mechanical point of view this accumulated slip represents the changes in the contact surface structure that resulted from sliding. In the previous formula, when the friction bound g is a given function such that $g : \Gamma_3 \to \mathbb{R}_+$, we obtained the simple Tresca's friction law. The strict inequality holds in the *stick* zone and the equality in the *slip* zone.

Using now (7) and (8), it is straightforward to see that on Γ_3, for all $t \in [0,T]$ the conditions (19) imply

$$\begin{cases} |\theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t)| \le g(\int_0^t |\dot{u}(s)|ds), \\ |\theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t)| < g(\int_0^t |\dot{u}(s)|ds) \Rightarrow \dot{u}(t) = 0, \\ |\theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t)| = g(\int_0^t |\dot{u}(s)|ds) \\ \quad \Rightarrow \exists\, \gamma \ge 0 \text{ such that } \theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t) = -\gamma \dot{u}(t). \end{cases}$$

Finally, we prescribe the initial displacement, $u(0) = u_0$ in Ω.

Now, the mechanical model of the *antiplane frictional contact problem* is complete and it can be stated as follows.

Problem P_2**.** *Find the displacement field* $u : \Omega \times [0,T] \to \mathbb{R}$ *such that:*

$$\theta\Delta\dot{u} + \mu\Delta u + f_0 = 0 \quad \text{in } \Omega \times (0,T), \tag{20}$$

$$u = 0 \quad \text{on } \Gamma_1 \times (0,T), \tag{21}$$

$$\theta\,\partial_\nu \dot{u} + \mu\,\partial_\nu u = f_2 \quad \text{on } \Gamma_2 \times (0,T), \tag{22}$$

$$\left\{\begin{array}{l} |\theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t)| \le g(\int_0^t |\dot{u}(s)|ds), \\ |\theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t)| < g(\int_0^t |\dot{u}(s)|ds) \Rightarrow \dot{u}(t) = 0, \\ |\theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t)| = g(\int_0^t |\dot{u}(s)|ds) \\ \quad \Rightarrow \exists\,\gamma \ge 0 \text{ such that } \theta\,\partial_\nu \dot{u}(t) + \mu\,\partial_\nu u(t) = -\gamma \dot{u}(t) \end{array}\right. \qquad \text{on } \Gamma_3 \times (0,T), \tag{23}$$

$$u(0) = u_0 \qquad \text{in } \ \Omega. \tag{24}$$

Note again that when the displacement field u which solves Problem P_2 is known, then the stress tensor can be calculated using the formula (5).

3. Variational formulations and main results

In this section we derive the variational formulations of the problems P_1, P_2 and state two existence and uniqueness results, Theorem 3.1 and Theorem 3.2. To this end we introduce the closed subspace of $H^1(\Omega)$ defined by

$$V = \{v \in H^1(\Omega)\,|\,v = 0 \ \text{ on } \ \Gamma_1\}.$$

Since $meas\,\Gamma_1 > 0$, it follows that V is a real Hilbert space endowed with the inner product

$$(u,v)_V = \int_\Omega \nabla u \cdot \nabla v\,dx \qquad \forall u, v \in V,$$

and the associated norm $\|\cdot\|_V$.

The adhesive problem. In the study of the mechanical Problem P_1, we assume that the tangential contact function $p : \Gamma_3 \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ satisfies

$$\left\{\begin{array}{l} \text{(a) There exists } \ L_p > 0 \text{ such that} \\ \qquad |p(\boldsymbol{x},\beta_1,r_1) - p(\boldsymbol{x},\beta_2,r_2)| \le L_p\,(|\beta_1 - \beta_2| + |r_1 - r_2|) \\ \qquad \forall\,\beta_1,\ \beta_2 \in \mathbb{R},\ r_1,\ r_2 \in \mathbb{R}, \text{ a.e. } \boldsymbol{x} \in \Gamma_3; \\ \text{(b) The map } \boldsymbol{x} \mapsto p(\boldsymbol{x},\beta,r) \text{ is Lebesgue measurable} \\ \qquad \text{on } \Gamma_3,\ \forall\,\beta \in \mathbb{R},\ r \in \mathbb{R}; \\ \text{(c) The map } \boldsymbol{x} \mapsto p(\boldsymbol{x},0,0) \in L^\infty(\Gamma_3). \end{array}\right. \tag{25}$$

Clearly, if $q : \mathbb{R} \to \mathbb{R}$ is a bounded Lipschitz continuous function, then the tangential contact function (9) satisfies condition (25). We conclude that our results below are valid for the corresponding contact problems.

Next, the adhesion rate function $H_{ad} : \Gamma_3 \times \mathbb{R} \times [-L, L] \to \mathbb{R}$ satisfies

$$\left\{\begin{array}{l} \text{(a) There exists } L_{H_{ad}} > 0 \text{ such that} \\ \qquad |H_{ad}(\boldsymbol{x}, b_1, r_1) - H_{ad}(\boldsymbol{x}, b_2, r_2)| \\ \qquad \le L_{H_{ad}} (|b_1 - b_2| + |r_1 - r_2|) \\ \qquad \forall\, b_1,\, b_2 \in \mathbb{R},\, r_1,\, r_2 \in [-L, L], \text{ a.e. } \boldsymbol{x} \in \Gamma_3; \\ \text{(b) The map } \boldsymbol{x} \mapsto H_{ad}(\boldsymbol{x}, b, r) \text{ is Lebesgue measurable} \\ \qquad \text{on } \Gamma_3,\ \forall\, b \in \mathbb{R},\, r \in [-L, L]; \\ \text{(c) The map } (b, r) \mapsto H_{ad}(\boldsymbol{x}, b, r) \text{ is continuous on} \\ \qquad \mathbb{R} \times [-L, L], \text{ a.e. } \boldsymbol{x} \in \Gamma_3; \\ \text{(d) } H_{ad}(\boldsymbol{x}, 0, r) = 0 \quad \forall\, r \in [-L, L], \text{ a.e. } \boldsymbol{x} \in \Gamma_3; \\ \text{(e) } H_{ad}(\boldsymbol{x}, b, r) \ge 0 \quad \forall\, b \le 0,\, r \in [-L, L], \text{ a.e. } \boldsymbol{x} \in \Gamma_3 \quad \text{and} \\ \qquad H_{ad}(\boldsymbol{x}, b, r) \le 0 \quad \forall\, b \ge 1,\, r \in [-L, L], \text{ a.e. } \boldsymbol{x} \in \Gamma_3. \end{array}\right. \tag{26}$$

It is straightforward to see that if the adhesion coefficient $\gamma_\nu \in L^\infty(\Gamma_3)$ satisfies $\gamma_\nu \ge 0$ a.e. on Γ_3 then the function H_{ad} in example (11) satisfies (26). We conclude that all the results below are valid for this choice of H_{ad}.

We assume that the body forces and surface tractions have the regularity

$$f_0 \in W^{1,\infty}(0, T; L^2(\Omega)), \qquad f_2 \in W^{1,\infty}(0, T; L^2(\Gamma_2)). \tag{27}$$

Finally, the initial data satisfy

$$u_0 \ \in V, \quad \beta_0 \in L^\infty(\Gamma_3) \ \text{ and } \ 0 \le \beta_0 \le 1 \text{ a.e. } \boldsymbol{x} \in \Gamma_3. \tag{28}$$

We note that conditions (28) and (26) ensure that the adhesion field is restricted to values between 0 and 1. Indeed, if $\{u,\, \beta\}$ are regular functions which satisfy (16), (18), using the arguments in [3] and conditions (28), (26), it can be shown that $0 \le \beta(x, t) \le 1$ for all $x \in \Gamma_3$, $t \in [0, T]$.

Next, we define the function $f : [0, T] \to V$ and the functional $j : L^\infty(\Gamma_3) \times V \times V \to \mathbb{R}$ by equalities

$$(f(t), v)_V = \int_\Omega f_0(t) v\, dx + \int_{\Gamma_2} f_2(t) v\, da \quad \forall v \in V,\ t \in [0, T],$$

$$j(\beta, u, v) = \int_{\Gamma_3} p(\beta, u) v\, da \qquad \forall \beta \in L^\infty(\Gamma_3),\ \forall\, u,\, v\, \in V.$$

Performing integrates by part we obtain the following variational formulation of the Problem P_1.

Problem P_V^1. *Find a displacement field $u : [0, T] \to V$ and an adhesion field $\beta : [0, T] \to L^\infty(\Gamma_3)$ such that $u(0) = u_0$, $\beta(0) = \beta_0$ and*

$$\begin{gathered} \theta(\dot{u}(t), v)_V + \mu(u(t), v)_V + j(\beta(t), u(t), v) = (f(t), v)_V \\ \forall\, v \in V,\ \forall\, t \in [0, T], \end{gathered} \tag{29}$$

$$\dot{\beta}(t) = H_{ad}(\beta(t), R(|u(t)|)), \quad \forall t \in [0, T]. \tag{30}$$

We have the following existence and uniqueness result.

Theorem 3.1. *Assume that (25)–(28) hold. Then, for each $\theta > 0$ there exists a unique solution $\{u,\, \beta\}$ of Problem P_V^1. Moreover, the solution satisfies*

$$u \in W^{2,\infty}(0, T; V), \quad \beta \in W^{2,\infty}(0, T; L^\infty(\Gamma_3)). \tag{31}$$

The proof of Theorem 3.1 was obtained in [4] and it is based on a version of Cauchy-Lipschitz theorem. We conclude by this theorem that the mechanical Problem P_1 has a unique weak solution with regularity (31).

The frictional problem. In the study of the mechanical Problem P_2, we assume that the *friction bound* function g, defined on $\Gamma_3 \times \mathbb{R}$ with values in $\mathbb{R}_+$, satisfy the following properties:

$$\left\{\begin{array}{l} (a)\ \exists\, L_g > 0 \text{ such that } |g(x,r_1) - g(x,r_2)| \le L_g |r_1 - r_2| \\ \qquad \forall\, r_1, r_2 \in \mathbb{R}, \quad \text{a.e. } x \in \Gamma_3; \\ (b)\ \forall\, r \in \mathbb{R},\ g(\cdot, r) \text{ is Lebesgue measurable on } \Gamma_3; \\ (c)\ g(\cdot, 0) \in L^2(\Gamma_3). \end{array}\right. \tag{32}$$

The forces and tractions are assumed to have the regularity

$$f_0 \in L^\infty(0,T; L^2(\Omega)), \qquad f_2 \in L^\infty(0,T; L^2(\Gamma_2)) \tag{33}$$

and the initial data is chosen such that

$$u_0 \in V. \tag{34}$$

For almost any $t \in (0,T)$ we define the operator

$$S_t : V \to L^2(\Gamma), \qquad S_t(v) = \int_0^t |v(s)|\, ds,$$

and we introduce the function $f : [0,T] \to V$ and the functional $j : L^2(\Gamma) \times V \to \mathbb{R}_+$ using the equalities

$$(f(t), v)_V = \int_\Omega f_0(t) v\, dx + \int_{\Gamma_2} f_2(t) v\, da \quad \forall v \in V \quad \text{ a.e. } t \in (0,T), \tag{35}$$

$$j(u,v) = \int_{\Gamma_3} g(u)\, |v| da \qquad \forall\, u \in L^2(\Gamma), \quad \forall v \in V. \tag{36}$$

In [2] the following variational formulation of Problem P_2 has been derived.

Problem P_V^2. *Find a displacement field $u : [0,T] \to V$ such that $u(0) = u_0$ and*

$$\begin{aligned} \theta\, (\dot u(t), v - \dot u(t))_V + \mu\, (u(t), v - \dot u(t))_V + j(S_t(\dot u), v) - j(S_t(\dot u), \dot u(t)) \\ \ge (f(t), v - \dot u(t))_V \quad \forall v \in V, \quad a.e.\ t \in (0,T). \end{aligned} \tag{37}$$

The well-posedness of problem P_V^2 is given by the following result.

Theorem 3.2. *Assume that* (32), (33) *and* (34) *hold. Then the variational Problem P_V^2 has a unique solution $u \in W^{1,\infty}(0,T;V)$.*

The proof of Theorem 3.2 was be carried out in several steps and it is based on fixed point arguments. Details can be found in [2]. We conclude by this theorem that the mechanical Problem P_2 has a unique weak solution with regularity $W^{1,\infty}(0,T;V)$.

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