Annals of the University of Craiova, Mathematics and Computer Science Series
Volume 47(2), 2020, Pages 294–307
ISSN: 1223-6934

Double-framed soft filters in lattice implication algebras

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Abstract. The notion of DFS-filter (double-framed soft filter) in lattice implication algrebras is introduced and characteristics of DFS-filters are discussed and related result are investigated. We defined IDFS-filter (implicative double-framed soft filter) and extension property for IDFS-filters is established. We maked PIDFS-filter (positive implicative double-framed soft filter) and investigated its properties. Finally, relation between DFS-filter, IDFS-filter and PIDFS-filter discussed are.

2010 Mathematics Subject Classification. 06F35, 03G25, 06D72.
Key words and phrases. Double-framed soft set(DFSs), DFS-filter, IDFS-filter, PIDFS-filter, γ-inclusive set, δ-exclusive set.

1. Introduction

In the field of many-valued logic, lattice-valued logic plays an important role for two aspects: First, it extends the chain-type truth-value field of some well-known presented logic to some relatively general lattices. Second, the incompletely comparable property of truth value characterized by general lattice can more efficiently reflect the uncertainty of people thinking, judging and decision. Hence, lattice-valued logic is becoming a research field which strongly influences the development of Algebraic Logic, Computer Science and Artificial intelligence Technology.
In 1993, Xu [9] proposed the concept of lattice implication algebras, which combines lattice with implication algebra and he discussed their some properties. For the general development of lattice implication algebras, filter theory and its fuzzification play an important role. Xu and Qin[11] introduced the notion of (implicative) filters in a lattice implication algebra, and investigated their properties. In [3] and [5] Jun (together with Xu and Qin) discussed positive implicative and associative filters of a lattice implication algebra, and Jun[4] considered the fuzzification of positive implicative and associative filters of a lattice implication algebra. In [10], Xu and Qin considered the fuzzification of (implicative) filters.

Molodtsov [8] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets.

Worldwide, there has been a rapid growth in interest in soft set theory and its applications in recent years. Evidence of this can be found in the increasing number

Received March 13, 2019. Accepted June 6, 2019.
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of high-quality articles on soft sets and related topics that have been published in a variety of international journals, symposia, workshops, and international conferences in recent years. Maji et al. [7] described the application of soft set theory to a decision making problem. Maji et al. [6] also studied several operations on the theory of soft sets.

In this paper, we introduce the notion of double-framed soft filter in lattice implication algebras, and investigated related results. We discuss characterizations of double-framed soft filters. We defined implicative double-framed soft filter and extension property for double-framed soft filters is established. Also we maked positive implicative double-framed soft filter and their properties. Finally we consider relation between double-framed soft filters, implicative double-framed soft filters and positive implicative double-framed soft filters.

2. Preliminaries

Molodtsov [8] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A, B, C, \cdots \subseteq E$.

Definition 2.1 ([8])**.** A pair (f, A) is called a *soft set* over U, where f is a mapping given by

$$f : A \to \mathscr{P}(U).$$

In the other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $f(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (f, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [8].

Definition 2.2 ([2])**.** A double-framed soft pair $\langle(\alpha, \beta); A\rangle$ is called a *double-framed soft set* (briefly, DFS-set) over U, where α and β are mappings from A to $\mathscr{P}(U)$.

For a DFS-set $\langle(\alpha, \beta); A\rangle$ over U and two subsets γ and δ of U, the *γ-inclusive set* and the *δ-exclusive set* of $\langle(\alpha, \beta); A\rangle$, denoted by $i_A(\alpha; \gamma)$ and $e_A(\beta; \delta)$, respectively, are defined as follows:

$$i_A(\alpha; \gamma) := \{x \in A \mid \gamma \subseteq \alpha(x)\}$$

and

$$e_A(\beta; \delta) := \{x \in A \mid \delta \supseteq \beta(x)\},$$

respectively. The set

$$DF_A\,(\alpha, \beta)_{(\gamma,\delta)} := \{x \in A \mid \gamma \subseteq \alpha(x),\ \delta \supseteq \beta(x)\}$$

is called a *double-framed including set* of $\langle(\alpha, \beta); A\rangle$. It is clear that

$$DF_A\,(\alpha, \beta)_{(\gamma,\delta)} = i_A(\alpha; \gamma) \cap e_A(\beta; \delta).$$

Definition 2.3. ([12]). Let $(L, \vee, \wedge, 0, 1)$ be a bounded lattice with an order-reversing involution $'$,1 and 0 the greatest and be smallest element of L respectively, and $\to$: $L \times L \longrightarrow L$ be a mapping. Then $(L, \vee, \wedge,', \to, 0, 1)$ is called a *quasi-lattice implication algebra* if the following conditions hold for any $x, y, z \in L$:
(I_1) $x \to (y \to z) = y \to (x \to z)$,

(I_2) $x \to x = 1$,
(I_3) $x \to y = y' \to x'$,
(I_4) $x \to y = y \to x = 1$implies $x = y$,
(I_5) $(x \to y) \to y = (y \to x) \to x$.

Definition 2.4. A quasi-lattice implication algebra is called a *lattice implication algebra*, if it satisfies in the following conditions:
(l_1) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$
(l_2) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$

In a lattice implication algebra L, the following hold:

$$0 \to x = 1, 1 \to x = x \text{ and } x \to 1 = 1, \tag{1}$$
$$x' = x \to 0, \tag{2}$$
$$x \vee y = (x \to y) \to y, \tag{3}$$
$$x \leqslant y \text{ implies } y \to z \leqslant x \to z \text{ and } z \to x \leqslant z \to y, \tag{4}$$
$$x \to y \leqslant (y \to z) \to (x \to z), \tag{5}$$
$$((y \to x) \to y')' = x \wedge y = ((x \to y) \to x')', \tag{6}$$
$$x \leqslant (x \to y) \to y, \tag{7}$$
$$x \leqslant y \text{ if and only if } x \to y = 1. \tag{8}$$

Let L be a lattice implication algebra and J is a subset of L. Then J is called a *filter* of L, if it satisfies the following conditions:
(F_1) $1 \in J$,
(F_2) for any $x, y \in L$, if $x \in J$ and $x \to y \in J$, then $y \in J$.

A subset J of a lattice implication algebra L is said to be an *implicative filter* of L, if it satisfies the following conditions:
(IF_1) $1 \in J$,
(IF_2) for any $x, y, z \in L$, if $x \to (y \to z) \in J$ and $x \to y \in J$, then $x \to z \in J$.

Note that every implicative filter is a filter, but the converse is not true in general.

A subset J of a lattice implication algebra L is said to be a *positive implicative filter* of L, if it satisfies the following conditions:
(PIF_1) $1 \in J$,
(PIF_2) for any $x, y, z \in L$, if $x \to ((y \to z) \to y) \in J$ and $x \in J$, then $y \in J$.

Note that every positive implicative filter is a filter, but the converse is not true in general.

Notation: From now one, we let $(L, \vee, \wedge, ', \to, 0, 1)$ or L is a lattice implication algebra and U is a universal set.

3. Double-framed soft filters

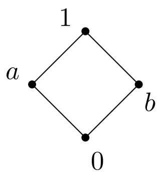
In this section, we define double-framed soft filter (briefly, DFS-filter) of lattice implication algebra and find Characteristic of it and consider many property of it.

Definition 3.1. A DFS-set $\langle(\alpha,\beta);L\rangle$ over U is called a *double-framed soft filter* (briefly, *DFS-filter*) over U if the following assertions are valid:

$$(\forall x \in L)\,(\alpha(x) \subseteq \alpha(1),\ \beta(x) \supseteq \beta(1))\,, \tag{9}$$

$$(\forall x, y \in L)\left(\begin{array}{c} \alpha(x \to y) \cap \alpha(x) \subseteq \alpha(y) \\ \beta(x \to y) \cup \beta(x) \supseteq \beta(y) \end{array}\right). \tag{10}$$

Example 3.1. Let $L = \{0, a, b, 1\}$ be a set with the following Hasse diagram and Cayley tables:



x	x'
0	1
a	b
b	a
1	0

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	a	a	1	1
1	0	a	b	1

Then L is a lattice implication algebra (See [12]). Let $\langle(\alpha,\beta);L\rangle$ be a DFS-set over $U = \mathbb{Z}$ defined by

$$\alpha : L \longrightarrow 2^{\mathbb{Z}},\ x \mapsto \begin{cases} 8\mathbb{Z} & \text{if } x \in \{0, b\}, \\ 4\mathbb{Z} & \text{if } x = a, \\ 2\mathbb{Z} & \text{if } x = 1. \end{cases}$$

and

$$\beta : L \longrightarrow 2^{\mathbb{Z}},\ x \mapsto \begin{cases} 2\mathbb{Z} & \text{if } x \in \{a, 0\}, \\ 4\mathbb{Z} & \text{if } x = b, \\ 8\mathbb{Z} & \text{if } x = 1. \end{cases}$$

Then $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over $\mathbb{Z}$.

Proposition 3.1. *Every DFS-filter $\langle(\alpha,\beta);L\rangle$ over U satisfies the following assertion.*

$$(\forall x, y \in L)\,(x \to y = 1 \ \Rightarrow\ \alpha(x) \subseteq \alpha(y),\ \beta(x) \supseteq \beta(y))\,. \tag{11}$$

Proof. Let $x, y \in L$ be such that $x \to y = 1$. By using (10), we have $\alpha(1) \cap \alpha(x) \subseteq \alpha(y)$ and $\beta(1) \cup \beta(x) \supseteq \beta(y)$. It follows from (9) that $\alpha(x) \subseteq \alpha(y)$ and $\beta(x) \supseteq \beta(y)$. $\square$

Theorem 3.2. *A DFS-set $\langle(\alpha,\beta);L\rangle$ over U is a DFS-filter over U if and only if it satisfies* (9) *and*

$$(\forall x, y, z \in L)\left(\begin{array}{c} \alpha(x \to y) \cap \alpha(y \to z) \subseteq \alpha(x \to z) \\ \beta(x \to y) \cup \beta(y \to z) \supseteq \beta(x \to z) \end{array}\right). \tag{12}$$

Proof. Assume that $\langle(\alpha,\beta);L\rangle$ be a DFS-filter over U. In any lattice implication algebra, for all $x, y, z \in L$ we have

$$(x \to y) \to ((y \to z) \to (x \to z)) = 1.$$

Then (11) imply that $\alpha(x \to y) \subseteq \alpha((y \to z) \to (x \to z))$ and $\beta(x \to y) \supseteq \beta((y \to z) \to (x \to z))$ and so (10) imply that

$$\alpha(x \to y) \cap \alpha(y \to z) \subseteq \alpha((y \to z) \to (x \to z)) \cap \alpha(y \to z) \subseteq \alpha(x \to z),$$

$$\beta(x \to y) \cup \beta(y \to z) \supseteq \beta((y \to z) \to (x \to z)) \cup \beta(y \to z) \supseteq \beta(x \to z).$$

Conversely, suppose that (9) and (12) are valid. If we take $x = 1$ in (12), then

$$\alpha(1 \to y) \cap \alpha(y \to z) \subseteq \alpha(1 \to z),$$

$$\beta(1 \to y) \cup \beta(y \to z) \supseteq \beta(1 \to z).$$

Since in any lattice implication algebra $1 \to x = x$, for all $x \in L$, hence

$$\alpha(y) \cap \alpha(y \to z) \subseteq \alpha(z),$$

$$\beta(y) \cup \beta(y \to z) \supseteq \beta(z),$$

for all $y, z \in L$. Therefore, $\langle(\alpha, \beta); L\rangle$ is a DFS-filter over U and this completes the proof. □

Theorem 3.3. *For any DFS-set $\langle(\alpha, \beta); L\rangle$ over U, the following assertions are equivalent.*

(1) *$\langle(\alpha, \beta); L\rangle$ is a DFS-filter over U.*

(2) $(\forall x, y, z \in L) \left(x \to (y \to z) = 1 \;\Rightarrow\; \begin{cases} \alpha(z) \supseteq \alpha(x) \cap \alpha(y) \\ \beta(z) \subseteq \beta(x) \cup \beta(y) \end{cases} \right).$

Proof. (1 ⇒ 2) Assume that $\langle(\alpha, \beta); L\rangle$ be a DFS-filter over U and for arbitrary $x, y, z \in L$, $x \to (y \to z) = 1$. By using (11) we have $\alpha(x) \subseteq \alpha(y \to z)$ and $\beta(x) \supseteq \beta(y \to z)$. Also, it follows from (10) that $\alpha(y \to z) \cap \alpha(y) \subseteq \alpha(z)$ and $\beta(y \to z) \cup \beta(y) \supseteq \beta(z)$. Thus

$$\alpha(x) \cap \alpha(y) \subseteq \alpha(y \to z) \cap \alpha(y) \subseteq \alpha(z),$$

$$\beta(x) \cup \beta(y) \supseteq \beta(y \to z) \cup \beta(y) \supseteq \beta(z).$$

(2 ⇒ 1) Taking $z = 1$ and $x = y$ implies that $x \to (x \to 1) = x \to 1 = 1$ for all $x \in L$. Hence $\alpha(x) = \alpha(x) \cap \alpha(x) \subseteq \alpha(1)$ and $\beta(x) = \beta(x) \cup \beta(x) \supseteq \beta(1)$ for all $x \in L$. Therefore (9) exist. In any lattice implication algebra $x \to x = 1$ for all $x \in L$, so taking $t = x \to y$ in $(x \to y) \to (x \to y) = 1$ imply that $t \to (x \to y) = 1$. Then

$$\alpha(y) \supseteq \alpha(t) \cap \alpha(x) = \alpha(x \to y) \cap \alpha(x),$$

$$\beta(y) \subseteq \beta(t) \cup \beta(x) = \beta(x \to y) \cup \beta(x).$$

Therefore, (10) exist and $\langle(\alpha, \beta); L\rangle$ is a DFS-filter over U. □

Theorem 3.4. *A DFS-set $\langle(\alpha, \beta); L\rangle$ over U is a DFS-filter over U if and only if it satisfies* (9), (11) *and*

$$(\forall x, y \in L) \left(\begin{array}{l} \alpha((x \to y')') \supseteq \alpha(x) \cap \alpha(y) \\ \beta((x \to y')') \subseteq \beta(x) \cup \beta(y) \end{array} \right). \tag{13}$$

Proof. Assume that $\langle(\alpha, \beta); L\rangle$ be a DFS-filter over U. Then (11) is clear by Proposition 3.1. Using the conditions (I_3) and (I_2) imply that

$$x \to (y \to (x \to y')') = x \to ((x \to y') \to y') = (x \to y') \to (x \to y') = 1,$$

Hence for all $x, y \in L$

$$x \to (y \to (x \to y')') = 1. \tag{14}$$

Taking $z = (x \to y')'$ in (14) and using Theorem 3.3(2), we have $\alpha((x \to y')') \supseteq \alpha(x) \cap \alpha(y)$ and $\beta((x \to y')') \subseteq \beta(x) \cup \beta(y)$ for all $x, y \in L$.

Conversely suppose that a DFS-set $\langle(\alpha, \beta); L\rangle$ over U satisfies conditions (9),(11) and

(13). Since $(y \to (x \to y')')' \to x' = 1$ for all $x, y \in L$ by (14) and (I_3), then using (11) and (I_3) imply that

$$\alpha(((x \to y') \to y')') = \alpha((y \to (x \to y')')') \subseteq \alpha(x'),$$

$$\beta(((x \to y') \to y')') = \beta((y \to (x \to y')')') \supseteq \beta(x').$$

It follows from Theorem 3.3, we have

$$\alpha(x \to y') \cap \alpha(y) \subseteq \alpha(((x \to y') \to y')') \subseteq \alpha(x'),$$

$$\beta(x \to y') \cup \beta(y) \supseteq \beta(((x \to y') \to y')') \supseteq \beta(x').$$

Finally for all $x, y \in L$ we have

$$\alpha(y \to x') \cap \alpha(y) \subseteq \alpha(x'),$$

$$\beta(y \to x') \cup \beta(y) \supseteq \beta(x').$$

Therefore $\langle(\alpha, \beta); L\rangle$ is a DFS-filter over U and this completes the proof. □

Theorem 3.5. *A DFS-set $\langle(\alpha, \beta); L\rangle$ over U is a DFS-filter over U if and only if it satisfies the condition* (9) *and*

$$(\forall x, y, z \in L) \begin{pmatrix} \alpha(z \to x) \supseteq \alpha((z \to y) \to x) \cap \alpha(y) \\ \beta(z \to x) \subseteq \beta((z \to y) \to x) \cup \beta(y) \end{pmatrix}. \tag{15}$$

Proof. Suppose that $\langle(\alpha, \beta); L\rangle$ is a DFS-filter over U. Then the condition (9) is valid. Let $x, y, z \in L$, then $((z \to y) \to x) \to (y \to (z \to x)) = 1$, since $x \to (z \to x) = 1$ and $y \to (z \to y) = 1$. Hence

$$\alpha(y \to (z \to x)) \supseteq \alpha((z \to y) \to x),$$

$$\beta(y \to (z \to x)) \subseteq \beta((z \to y) \to x),$$

by (11). It follows from (10) that

$$\alpha(z \to x) \supseteq \alpha(y \to (z \to x)) \cap \alpha(y) \supseteq \alpha((z \to y) \to x) \cap \alpha(y),$$

$$\beta(z \to x) \subseteq \beta(y \to (z \to x)) \cup \beta(y) \subseteq \beta((z \to y) \to x) \cup \beta(y).$$

Hence (15) is valid.
Conversly, Assume that $\langle(\alpha, \beta); L\rangle$ be a DFS-set over U that satisfies conditions (9) and (15). Taking $z = 1$ in (15) and using (1), we have for all $x, y \in L$

$$\alpha(x) = \alpha(1 \to x) \supseteq \alpha((1 \to y) \to x) \cap \alpha(y) = \alpha(y \to x) \cap \alpha(y)$$

$$\beta(x) = \beta(1 \to x) \subseteq \beta((1 \to y) \to x) \cup \beta(y) = \beta(y \to x) \cup \beta(y)$$

Therefore the condition (10) is valid for all $x, y \in L$ and $\langle(\alpha, \beta); L\rangle$ is a DFS-filter over U. □

Theorem 3.6. *A DFS-set $\langle(\alpha, \beta); L\rangle$ over U is a DFS-filter over U if and only if for every subsets γ and δ of U such that $\gamma \in Im(\alpha)$ and $\delta \in Im(\beta)$, the γ-inclusive set and the δ-exclusive set of $\langle(\alpha, \beta); L\rangle$ are filters of L.*

Proof. Assume that $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over U. Let $\gamma,\delta\in\mathscr{P}(U)$ be such that $i_L(\alpha;\gamma)\neq\emptyset$ and $e_L(\beta;\delta)\neq\emptyset$. Then there exists $a\in i_L(\alpha;\gamma)$ and $b\in e_L(\beta;\delta)$, and so $\gamma\subseteq\alpha(a)$ and $\delta\supseteq\beta(b)$. It follows from (9) that $\gamma\subseteq\alpha(a)\subseteq\alpha(1)$ and $\delta\supseteq\beta(b)\supseteq\beta(1)$. Thus $1\in i_L(\alpha;\gamma)$ and $1\in e_L(\beta;\delta)$. Let $x,y\in L$ be such that $x\to y\in i_L(\alpha;\gamma)(x\to y\in e_L(\beta;\delta))$ and $x\in i_L(\alpha;\gamma)(x\in e_L(\beta;\delta))$. Then $\gamma\subseteq\alpha(x\to y)(\delta\supseteq\beta(x\to y))$ and $\gamma\subseteq\alpha(x)(\delta\supseteq\beta(x))$. It follows from (10) that $\gamma\subseteq\alpha(x\to y)\cap\alpha(x)\subseteq\alpha(y)(\delta\supseteq\beta(x\to y)\cup\beta(x)\supseteq\beta(y))$ that is, $y\in i_L(\alpha;\gamma)(y\in e_L(\beta;\delta))$. Thus $i_L(\alpha;\gamma)(\neq\emptyset)$ (and $e_L(\beta;\delta)(\neq\emptyset)$) are filters of L.

Conversely, suppose that for any $x\in L$, $\alpha(x)=\gamma_x(\beta(x)=\delta_x)$. Then $x\in i_L(\alpha;\gamma_x)\neq\emptyset$ $(x\in e_L(\beta;\delta_x)\neq\emptyset)$. Since $i_L(\alpha;\gamma_x)(e_L(\beta;\delta_x))$ is a filter of L, we let $1\in i_L(\alpha;\gamma_x)(1\in e_L(\beta;\delta_x))$. Hence $\alpha(1)\supseteq\gamma_x=\alpha(x)(\ \beta(1)\subseteq\delta_x=\beta(x))$. Let $x,y\in L$ be such that $\alpha(x)=\gamma_x$, and $\alpha(x\to y)=\gamma_{x\to y}(\ \beta(x)=\delta_x$ and $\beta(x\to y)=\delta_{x\to y})$. Then $x\in i_L(\alpha;\gamma_x), x\to y\in i_L(\alpha;\gamma_{x\to y})(\ x\in e_L(\beta;\delta_x), x\to y\in e_L(\beta;\delta_{x\to y}))$. Taking $\gamma=\gamma_x\cap\gamma_{x\to y}$ $(\delta=\delta_x\cup\delta_{x\to y})$ implies that $x\in i_L(\alpha;\gamma_x)\subseteq i_L(\alpha;\gamma)\neq\emptyset$ and $x\to y\in i_L(\alpha;\gamma_{x\to y})\subseteq i_L(\alpha;\gamma)\neq\emptyset(\ x\in e_L(\beta;\delta_x)\supseteq e_L(\beta;\delta)\neq\emptyset$ and $x\to y\in e_L(\beta;\delta_{x\to y})\supseteq e_L(\beta;\delta)\neq\emptyset)$. Since $i_L(\alpha;\gamma)$ $(\ e_L(\beta;\delta))$ is a filter of L, thus $y\in i_L(\alpha;\gamma)$ $(\ y\in e_L(\beta;\delta))$ Hence,

$$\alpha(y)\supseteq\gamma=\gamma_x\cap\gamma_{x\to y}=\alpha(x)\cap\alpha(x\to y)$$

$$(\beta(y)\subseteq\delta=\delta_x\cup\delta_{x\to y}=\beta(x)\cup\beta(x\to y))$$

Therefore $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over U and this completes the proof. □

Corollary 3.7. *Let $\langle(\alpha,\beta);L\rangle$ be a DFS-set over U. If $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over U, then for any $\gamma,\delta\in\mathscr{P}(U), i_L(\alpha;\gamma)\cap e_L(\beta;\delta)$ is a filter of L.*

The following example shows that the converse of Corollary 3.7 is not true in general.

Example 3.2. Let $L=\{0,a,b,c,1\}$ be a set whit the following Cayley Tables:

x	x'
0	1
a	c
b	b
c	a
1	0

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	b	1	1	1
c	a	b	b	1	1
1	0	a	b	c	1

Then L is a lattice implication algebra. Let $\langle(\alpha,\beta);L\rangle$ be a DFS-set over $U=\mathbb{Z}$ defined by

$$\alpha:L\longrightarrow 2^{\mathbb{Z}},\ x\mapsto\begin{cases}8\mathbb{Z} & \text{if } x\in\{a,b,c\},\\ 4\mathbb{Z} & \text{if } x=0,\\ 2\mathbb{Z} & \text{if } x=1.\end{cases}$$

and

$$\beta:L\longrightarrow 2^{\mathbb{Z}},\ x\mapsto\begin{cases}2\mathbb{Z} & \text{if } x\in\{a,b,c\},\\ 4\mathbb{Z} & \text{if } x=0,\\ 16\mathbb{Z} & \text{if } x=1.\end{cases}$$

For two subsets $\gamma=2\mathbb{Z}$ and $\delta=8\mathbb{Z}$ of $\mathbb{Z}$ we have $i_L(\alpha;\gamma)=e_L(\beta;\delta)=\{1\}$. Then $i_L(\alpha;\gamma)\cap e_L(\beta;\delta)=\{1\}$ that is a filter of L. But $\alpha(0\to c)\cap\alpha(0)=4\mathbb{Z}\supset 8\mathbb{Z}=\alpha(c)$. Thus $\langle(\alpha,\beta);L\rangle$ is not a DFS-filter over $\mathbb{Z}$.

Given a DFS-set $\langle(\alpha,\beta);L\rangle$ over U and $a\in L$, by $[\alpha(a))$ and $(\beta(a)]$ we mean

$$\emptyset \neq [\alpha(a)) := \{x\in L|\alpha(a)\subseteq\alpha(x)\},$$
$$\emptyset \neq (\beta(a)] := \{x\in L|\beta(a)\supseteq\beta(x)\}.$$

Theorem 3.8. *Every $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over U if and only if for any $a\in L$, $[\alpha(a))$ and $(\beta(a)]$ are filters of L.*

Proof. Suppose that $\langle(\alpha,\beta);L\rangle$ be a DFS-filter over U and $a\in L$. Obviously $1\in[\alpha(a))$ and $1\in(\beta(a)]$ by (9). Let $x,y\in L$ be such that $x\to y\in[\alpha(a))$ and $x\in[\alpha(a))$($x\to y\in(\beta(a)]$ and $x\in(\beta(a)]$). Then $\alpha(x\to y)\supseteq\alpha(a)$ and $\alpha(x)\supseteq\alpha(a)$ ($\beta(x\to y)\subseteq\beta(a)$ and $\beta(x)\subseteq\beta(a)$). It follows from (10) that

$$\alpha(y)\supseteq\alpha(x)\cap\alpha(x\to y)\supseteq\alpha(a),\quad(\beta(y)\subseteq\beta(x)\cup\beta(x\to y)\subseteq\beta(a))$$

Thus $y\in[\alpha(a))$ ($y\in(\beta(a)]$) and $[\alpha(a))$ ($(\beta(a)]$) is a filter of L for any $a\in L$. Conversely, assume that for any $a\in L$, $[\alpha(a))$ and $(\beta(a)]$ are filters of L. Since $1\in[\alpha(a))$ ($1\in(\beta(a)]$), then (9) is valid. Note that $x\to y\in[\alpha(x\to y))$ and $x\in[\alpha(x))$ ($x\to y\in(\beta(x\to y)]$ and $x\in(\beta(x)]$), put $\alpha(x\to y)=A$ and $\alpha(x)=B$ ($\beta(x\to y)=A$ and $\beta(x)=B$), take $C=A\cap B$ ($C=A\cup B$) and let $x\to y\in[C), x\in[C)$ ($x\to y\in(C], x\in(C]$) then $y\in[C)$ ($y\in(C]$). Thus $\alpha(y)\supseteq C=A\cap B=\alpha(x\to y)\cap\alpha(x)$ ($\beta(y)\subseteq C=A\cup B=\beta(x\to y)\cup\beta(x)$). Thus $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over U and this completes the proof. □

Corollary 3.9. *In any DFS-filter $\langle(\alpha,\beta);L\rangle$ over U, $[\alpha(a))\cap(\beta(a)]$ is a filter of L, for all $a\in L$.*
Converse of Corollary 3.9 is not true as seen in the following example.

Example 3.3. Consider the lattice implication algebra $L=\{0,a,b,c,1\}$ and DFS-set $\langle(\alpha,\beta);L\rangle$ over $\mathbb{Z}$ in Example 3.2. By taking $a=0$ we have

$$[\alpha(0))=\{x\in L|4\mathbb{Z}\subseteq\alpha(x)\}=\{1\}$$
$$(\beta(0)]=\{x\in L|4\mathbb{Z}\supseteq\beta(x)\}=\{1\}$$

Therefore $[\alpha(0))\cap(\beta(0)]=\{1\}$ is a filter of L but we know that $\langle(\alpha,\beta);L\rangle$ is not a DFS-filter over $\mathbb{Z}$.

Theorem 3.10. *Let $\langle(\alpha,\beta);L\rangle$ is a DFS-set over U and $a\in L$. Then the following assertions are valid.*

(1) *If $[\alpha(a))$ and $(\beta(a)]$ are filters of L, then $\langle(\alpha,\beta);L\rangle$ satisfies the following implication.*

$$(\forall x,y\in L)\left(\begin{array}{c}\alpha(a)\subseteq\alpha(x\to y)\cap\alpha(x)\Rightarrow\alpha(a)\subseteq\alpha(y)\\ \beta(a)\supseteq\beta(x\to y)\cup\beta(x)\Rightarrow\beta(a)\supseteq\beta(y)\end{array}\right).\tag{16}$$

(2) *If $\langle(\alpha,\beta);L\rangle$ satisfies* (9) *and* (16), *then $[\alpha(a))$ and $(\beta(a)]$ are filters of L.*

Proof. (1)Proof is clear by Theorem 3.8.
(2) Suppose that $\langle(\alpha,\beta);L\rangle$ satisfies (9) and (16). Since $\langle(\alpha,\beta);L\rangle$ satisfies (9), we have $1\in[\alpha(a))$ and $1\in(\beta(a)]$. Let $x,y\in L$ be such that $x\to y\in[\alpha(a))$ and $x\in[\alpha(a))$ ($x\to y\in(\beta(a)]$ and $x\in(\beta(a)]$). Then $\alpha(a)\subseteq\alpha(x\to y)$ and $\alpha(a)\subseteq\alpha(x)$ ($\beta(a)\supseteq\beta(x\to y)$ and $\beta(a)\supseteq\beta(x)$), which implies that $\alpha(a)\subseteq\alpha(x)\cap\alpha(x\to y)$ ($\beta(a)\supseteq\beta(x)\cup\beta(x\to y)$). Finally, it follows from (16) that $\alpha(a)\subseteq\alpha(y)$ ($\beta(a)\supseteq\beta(y)$), i.e., $y\in[\alpha(a))$ ($y\in(\beta(a)]$). Therefore $[\alpha(a))$ and $(\beta(a)]$ are filters of L. □

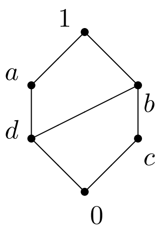
4. Implicative double-framed soft filters

In this section, we define implicative double-framed soft filter (briefly, IDFS-filter) of lattice implication algebra and find Characteristic of it and consider many property of it. Finally we finding relation between DFS-filters and IDFS-filters.

Definition 4.1. A DFS-set $\langle(\alpha, \beta); L\rangle$ over U is called an *implicative double-framed soft filter* (briefly, *IDFS-filter*) over U if it satisfies (9) and the following assertion is valid:

$$(\forall x, y \in L) \left(\begin{array}{l} \alpha(x \to (y \to z)) \cap \alpha(x \to y) \subseteq \alpha(x \to z) \\ \beta(x \to (y \to z)) \cup \beta(x \to y) \supseteq \beta(x \to z) \end{array} \right). \tag{17}$$

Example 4.1. Let $L = \{0, a, b, c, d, 1\}$ be a set with the following Hasse diagram and Cayley tables:



x	x'
0	1
a	c
b	d
c	a
d	b
1	0

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

Then L is a lattice implication algebra (See [12]). Let $\langle(\alpha, \beta); L\rangle$ be a DFS-set over $U = \mathbb{Z}$ defined by

$$\alpha : L \longrightarrow 2^{\mathbb{Z}}, \; x \mapsto \begin{cases} 4\mathbb{Z} & \text{if } x \in \{0, a, d\}, \\ 2\mathbb{Z} & \text{if } x \in \{1, b, c\}. \end{cases}$$

and

$$\beta : L \longrightarrow 2^{\mathbb{Z}}, \; x \mapsto \begin{cases} 2\mathbb{Z} & \text{if } x \in \{0, a, d\}, \\ 4\mathbb{Z} & \text{if } x \in \{1, b, c\}. \end{cases}$$

Then $\langle(\alpha, \beta); L\rangle$ is a IDFS-filter over $\mathbb{Z}$.

Theorem 4.1. *Every IDFS-filter is a DFS-filter.*

Proof. Let $\langle(\alpha, \beta); L\rangle$ be an IDFS-filter of L over U. Then (9) is valid. If we take $x = 1$ in (17), then we have

$$(\forall y, z \in L) \left(\begin{array}{l} \alpha(y \to z) \cap \alpha(y) \subseteq \alpha(z) \\ \beta(y \to z) \cup \beta(y) \supseteq \beta(z) \end{array} \right).$$

Therefore $\langle(\alpha, \beta); L\rangle$ is a DFS-filter of L over U. □

As seen in the following example, the converse of Theorem 4.1 is not true in general.

Example 4.2. Consider the lattice implication algebra $L = \{0, a, b, c, d, 1\}$ in Example 4.1. Let $\langle(\alpha, \beta); L\rangle$ be a DFS-set over $U = \mathbb{Z}$ defined by

$$\alpha : L \longrightarrow 2^{\mathbb{Z}}, \; x \mapsto \begin{cases} 2\mathbb{Z} & \text{if } x = 1, \\ 4\mathbb{Z} & \text{if } x \in \{0, a, b, c\}. \end{cases}$$

and

$$\beta : L \longrightarrow 2^{\mathbb{Z}}, \; x \mapsto \begin{cases} 2\mathbb{Z} & \text{if } x \in \{0, a, d\}, \\ 4\mathbb{Z} & \text{if } x \in \{1, b, c\}. \end{cases}$$

Then $\langle(\alpha,\beta);L\rangle$ is a DFS-filter over $\mathbb{Z}$ but it is not an IDFS-filter over $\mathbb{Z}$, since

$$\alpha(d\to(b\to 0))\cap\alpha(d\to b)=2\mathbb{Z}\supset 4\mathbb{Z}=\alpha(d\to 0).$$

Theorem 4.2. *Let $\langle(\alpha,\beta);L\rangle$ be a DFS-set over U and $a\in L$. If $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter of L over U, then $[\alpha(a))$ and $(\beta(a)]$ are implicative filters of L.*

Proof. Since $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter, so we have (9). Thus $1\in[\alpha(a))$ $(1\in(\beta(a)])$. Let $x,y,z\in L$ such that $x\to y\in[\alpha(a))$ and $x\to(y\to z)\in[\alpha(a))$ $(x\to y\in(\beta(a)]$ and $x\to(y\to z)\in(\beta(a)])$. Then $\alpha(x\to y)\supseteq\alpha(a)$ and $\alpha(x\to(y\to z))\supseteq\alpha(a)$ $(\beta(x\to y)\subseteq\beta(a)$ and $\beta(x\to(y\to z))\subseteq\beta(a)$). It follows from (17) that

$$(\forall x,y,z\in L)\left(\begin{array}{c}\alpha(x\to z)\supseteq\alpha(x\to(y\to z))\cap\alpha(x\to y)\supseteq\alpha(a)\\ (\beta(x\to z)\subseteq\beta(x\to(y\to z))\cup\alpha(x\to y)\subseteq\beta(a))\end{array}\right).$$

Thus $x\to z\in[\alpha(a))$ $(x\to z\in(\beta(a)])$ and $[\alpha(a))$ $((\beta(a)])$ is an implicative filter of L. □

Theorem 4.3. *Let $\langle(\alpha,\beta);L\rangle$ be a DFS-set over U. If $\langle(\alpha,\beta);L\rangle$ is an DFS-filter over U and satisfying the following condition,*

$$(\forall x,y,z\in L)\left(\begin{array}{c}\alpha(y\to z)\supseteq\alpha(x\to(y\to(y\to z)))\cap\alpha(x)\\ \beta(y\to z)\subseteq\beta(x\to(y\to(y\to z)))\cup\beta(x)\end{array}\right).\tag{18}$$

Then $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter over U.

Proof. Note that by using $(x\to(y\to z))\to((x\to y)\to(x\to(x\to z)))=1$, for all $x,y,z\in L$. It follows from (9) and (17) that for all $x,y,z\in L$

$$\alpha(x\to z)\supseteq\alpha((x\to y)\to(x\to(x\to z)))\cap\alpha(x\to y)\supseteq\alpha(x\to(y\to z))\cap\alpha(x\to y)$$

$$\beta(x\to z)\subseteq\beta((x\to y)\to(x\to(x\to z)))\cup\beta(x\to y)\subseteq\beta(x\to(y\to z))\cup\beta(x\to y).$$

Therefore $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter of L over U. □

Theorem 4.4. *Let $\langle(\alpha,\beta);L\rangle$ be a DFS-set over U and $a\in L$. The following statements are equivalent:*

(1) *$\langle(\alpha,\beta);L\rangle$ is an IDFS-filter of L over U.*

(2) *$\langle(\alpha,\beta);L\rangle$ is a DFS-filter of L over U that satisfies the condition.*

$$(\forall x,y\in L)\left(\begin{array}{c}\alpha(x\to y)\supseteq\alpha(x\to(x\to y))\\ \beta(x\to y)\subseteq\beta(x\to(x\to y))\end{array}\right).\tag{19}$$

(3) *$\langle(\alpha,\beta);L\rangle$ is a DFS-filter of L over U that satisfies the condition.*

$$(\forall x,y,z\in L)\left(\begin{array}{c}\alpha((x\to y)\to(x\to z))\supseteq\alpha(x\to(y\to z))\\ \beta((x\to y)\to(x\to z))\subseteq\beta(x\to(y\to z))\end{array}\right).\tag{20}$$

(4) (9) *is valid and satisfies the condition.*

$$(\forall x,y,z\in L)\left(\begin{array}{c}\alpha(x\to y)\supseteq\alpha(z\to(x\to(x\to y)))\cap\alpha(z)\\ \beta(x\to y)\subseteq\beta(z\to(x\to(x\to y)))\cup\alpha(z)\end{array}\right).\tag{21}$$

Proof. (1⇒ 2) Suppose that $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter of L over U. Then $\langle(\alpha,\beta);L\rangle$ is a DFS-filter by Theorem 4.1. Taking $x=y$ and $y=z$ in (17) we have for all $x,y\in L$

$$\alpha(x\to(x\to y))\cap\alpha(x\to x)\subseteq\alpha(x\to y)$$
$$\beta(x\to(x\to y))\cup\beta(x\to x)\supseteq\beta(x\to y)$$

Hence (19) is valid.

(2⇒ 3) Suppose that $\langle(\alpha,\beta);L\rangle$ is a DFS-filter of L over U that satisfies the condition (19). We know for any $x,y,z\in L$,

$$\begin{aligned} x\to(x\to((x\to y)\to z)) &= x\to((x\to y)\to(x\to z)) \\ &= x\to((x\wedge y)\to z) \\ &\geqslant x\to(y\to z) \end{aligned}$$

Since by using $(x\to((x\to y)\to(x\to z)))\to(x\to(y\to z))=1$, for all $x,y,z\in L$, it follows from (11) and (19) that for all $x,y,z\in L$

$$\begin{aligned} \alpha((x\to y)\to(x\to z)) &= \alpha(x\to((x\to y)\to z)) \\ &\supseteq \alpha(x\to(x\to((x\to y)\to z))) \\ &= \alpha(x\to((x\to y)\to(x\to z))) \\ &\supseteq \alpha(x\to(y\to z)) \end{aligned}$$

Also, for all $x,y,z\in L$ we have

$$\begin{aligned} \beta((x\to y)\to(x\to z)) &= \beta(x\to((x\to y)\to z)) \\ &\subseteq \beta(x\to(x\to((x\to y)\to z))) \\ &= \beta(x\to((x\to y)\to(x\to z))) \\ &\subseteq \beta(x\to(y\to z)). \end{aligned}$$

Therefore (3) is valid.

(3 ⇒ 4) Assume that (3) holds. By using (10) and (20) we have for any $x,y,z\in L$

$$\begin{aligned} \alpha(z\to(x\to(x\to y)))\cap\alpha(z) &\subseteq \alpha(x\to(x\to y)) \\ &\subseteq \alpha((x\to x)\to(x\to y)) \\ &= \alpha(x\to y) \end{aligned}$$

Therefore (21) is valid.

(4 ⇒ 1) Suppose (4) holds. Taking $z=1$ in (21) imply that

$$\begin{aligned} \alpha(x\to y) &\supseteq \alpha(1\to(x\to(x\to y)))\cap\alpha(1)=\alpha(x\to(x\to y)) \\ \beta(x\to y) &\subseteq \beta(1\to(x\to(x\to y)))\cup\beta(1)=\beta(x\to(x\to y)) \end{aligned}$$

Taking $y=x, z=y$ in $\alpha(x\to(y\to z))\cap\alpha(x\to y), \beta(x\to(y\to z))\cup\beta(x\to y)$ imply that

$$\begin{aligned} \alpha(x\to(x\to y))\cap\alpha(x\to x) = \alpha(x\to(x\to y))\cap\alpha(1)\subseteq\alpha(x\to y), \\ \beta(x\to(x\to y))\cup\beta(x\to x) = \beta(x\to(x\to y))\cup\beta(1)\supseteq\beta(x\to y), \end{aligned}$$

for any $x,y\in L$. Therefore (1) is valid. □

Theorem 4.5. *Let $\langle(\alpha,\beta);L\rangle$ be an IDFS-filter over U. Then it satifies the following condition:*

$$(\forall x,y,z\in L)\begin{pmatrix} \alpha((x\to y)\to x)\subseteq\alpha(x) \\ \beta((x\to y)\to x)\supseteq\beta(x) \end{pmatrix} \tag{22}$$

Proof. Note that for all $x,y\in L$ we have

$$(x\to y)\to x \leq (x\to y)\to((x\to y)\to y)$$

$$(x\to y)\to y \leq (y\to(x\to y))\to(y\to y)=x$$

Then (11) and (19) imply that

$$\begin{aligned}\alpha((x \to y) \to x) &\subseteq \alpha((x \to y) \to ((x \to y) \to y)) \\ &\subseteq \alpha((x \to y) \to y) \\ &\subseteq \alpha(x)\end{aligned}$$

and

$$\begin{aligned}\beta((x \to y) \to x) &\supseteq \beta((x \to y) \to ((x \to y) \to y)) \\ &\supseteq \beta((x \to y) \to y) \\ &\supseteq \beta(x)\end{aligned}$$

□

5. Positive implicative double-framed soft filters

In this section, we define positive implicative double-framed soft filter (briefly, PIDFS-filter) of lattice implication algebra and find Characteristic of it and finally we finding relation between DFS-filters, IDFS-filters and PIDFS-filters.

Definition 5.1. A DFS-set $\langle(\alpha, \beta); L\rangle$ over U is called a *positive implicative double-framed soft filter* (briefly, *PIDFS-filter*) over U if it satisfies (9) and the following condition.

$$(\forall x, y, z \in L)\begin{pmatrix} \alpha(x \to ((y \to z) \to y)) \cap \alpha(x) \subseteq \alpha(y) \\ \beta(x \to ((y \to z) \to y)) \cup \beta(x) \supseteq \beta(y)\end{pmatrix} \quad (23)$$

Example 5.1. Consider the lattice implication algebra $L = \{0, a, b, c, d, 1\}$ and DFS-set $\langle(\alpha, \beta); L\rangle$ over $\mathbb{Z}$ in Example 4.1. It is routine to verify that $\langle(\alpha, \beta); L\rangle$ is a PIDFS-filter over $\mathbb{Z}$.

Theorem 5.1. *Let $\langle(\alpha, \beta); L\rangle$ be a DFS-filter of L over U. Then $\langle(\alpha, \beta); L\rangle$ is a PIDFS-filter over U if and only if it satisfies the following condition.*

$$(\forall x, y, z \in L)\begin{pmatrix} \alpha((x \to y) \to x) \subseteq \alpha(x) \\ \beta((x \to y) \to x) \supseteq \beta(x)\end{pmatrix} \quad (24)$$

Proof. Suppose that $\langle(\alpha, \beta); L\rangle$ be a PIDFS-filter of L over U. Take $x = 1$ in (23), then we have

$$\alpha(1 \to ((y \to z) \to y)) \cap \alpha(1) = \alpha((y \to z) \to y) \subseteq \alpha(y)$$
$$\beta(1 \to ((y \to z) \to y)) \cup \beta(1) = \alpha((y \to z) \to y) \supseteq \alpha(y)$$

Conversely, assume that we have (24). Then by using (10)

$$\alpha(x \to ((y \to z) \to y)) \cap \alpha(x) \subseteq \alpha((y \to z) \to y) \subseteq \alpha(y)$$
$$\beta(x \to ((y \to z) \to y)) \cup \beta(x) \supseteq \beta((y \to z) \to y) \supseteq \beta(y)$$

Therefore $\langle(\alpha, \beta); L\rangle$ is a $PIDFS$-filter over U. □

Proof. Taking $x = 1$ in (24) imply that

$$\alpha(1 \to ((y \to z) \to y)) \cap \alpha(1) = \alpha((y \to z) \to y) \cap \alpha(1) = \alpha((y \to z) \to y) \subseteq \alpha(y)$$
$$\beta(1 \to ((y \to z) \to y)) \cup \beta(1) = \beta((y \to z) \to y) \cup \beta(1) = \beta((y \to z) \to y) \supseteq \beta(y)$$

Thus in any PIDFS-filter $\langle(\alpha, \beta); L\rangle$ the given assertions are valid. □

Theorem 5.2. *Every PIDFS-filter is a DFS-filter.*

Proof. Let $\langle(\alpha,\beta);L\rangle$ be a PIDFS-filter over U. Taking $z=y$ in (23) and using (1) imply that

$$(\alpha(x\to((y\to y)\to y))\cap\alpha(x)=\alpha(x\to y)\cap\alpha(x)\subseteq\alpha(y)$$
$$(\beta(x\to((y\to y)\to y))\cup\beta(x)=\beta(x\to y)\cup\beta(x)\supseteq\beta(y)$$

Then $\langle(\alpha,\beta);L\rangle$ is a DFS-filter. □

Theorem 5.3. *Every PIDFS-filter $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter.*

Proof. Let $\langle(\alpha,\beta);L\rangle$ is a PIDFS-filter in any lattice implication algebra for all $x,y\in L$ we have

$$x\to(x\to z)=((x\to z)\to z)\to(x\to z).$$

Then

$$\begin{aligned}x\to(y\to z)&=y\to(x\to z)\\&\leq(x\to y)\to(x\to(x\to z))\\&=(x\to y)\to((x\to z)\to z)\to(x\to z).\end{aligned}$$

It follows from (11) and (23) that

$$\begin{aligned}\alpha(x\to(y\to z))\cap\alpha(x\to y)&\subseteq\alpha((x\to y)\to((x\to z)\to z)\to(x\to z))\cap\alpha(x\to y)\\&\subseteq\alpha(x\to z).\end{aligned}$$

And

$$\begin{aligned}\beta(x\to(y\to z))\cup\beta(x\to y)&\supseteq\beta((x\to y)\to((x\to z)\to z)\to(x\to z))\cup\beta(x\to y)\\&\supseteq\beta(x\to z).\end{aligned}$$

Hence $\langle(\alpha,\beta);L\rangle$ is an IDFS-filter. □

Corollary 5.4. *In any DFS-set, the concepts of IDFS-filter and PIDFS-filter are coincide.*

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