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# Matrix map between complex uncertain sequences

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Abstract. In this article we define matrix maps between complex uncertain sequences. We introduce the notion of bounded sequences of complex uncertain sequence for almost sure, mean, measure and distribution. We introduce the limitation method for different notion of boundedness of sequence of complex uncertain variables and establish relation between the different notions. The necessary condition for a matrix map to be a limitation method is established.

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## 1. Introduction

When uncertainty acts neither fuzziness nor randomness, we are unable to deal with this kind of uncertainty by fuzzy set theory or probability theory. In order to handle this, an uncertainty theory was established by Liu [1]. At the same time, Liu proposed convergence concepts in different notion and established relationship between them. Up to now, uncertainty theory has successfully been applied to uncertain programming, uncertain risk analysis and uncertain reliability analysis, uncertain logic, uncertain differential equation, uncertain graphs etc. In our daily life, uncertainty became noticeable in real quantities as well as in complex quantities. The concept of complex uncertain variable was introduced by Peng [9] and after that it has been applied by Chen et al. [8], Tripathy and Dowari [4], Tripathy and Nath [7], Nath and Tripathy [3] for studying sequences of complex uncertain variables. The limitation method was founded in the book of Petersen [6], limitation method is a linear transformation which turns bounded sequences to a bounded sequences. One may refer to Petersen [6], for further details about the limitation method.

## 2. Preliminaries

In this section, we introduced some basic concepts on uncertainty theory which will be used throughout the paper.

**Definition 2.1.** ([1]) Let $\mathcal{L}$ be a $\sigma-$algebra on a nonempty set $\Gamma$. A set function $\mathcal{M}$ is called an uncertain measure if it satisfies the following axioms:
**Axiom 1.** (Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$;

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**Axiom 2.** (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any $\Lambda \in \mathcal{L}$;
**Axiom 3.** (Subadditivity Axiom) For every countable sequence of $\{\Lambda_j\} \in \mathcal{L}$, we have

$$\mathcal{M}\left\{\bigcup_{j=1}^{\infty} \bigwedge_{j}\right\} \leq \sum_{j=1}^{\infty} \mathcal{M}\{\Lambda_j\}.$$

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space, and each element $\Lambda$ in $\mathcal{L}$ is called an event.In order to obtain uncertainty measure of compound event, a product uncertain measure is defined by Liu [1] as follows:
**Axiom 4.** (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$, be uncertainty spaces for $k = 1, 2, \ldots$.. The product uncertainty measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\},$$

where $\Lambda_k$ are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

**Definition 2.2.** ([1]) An uncertain variable $\xi$ is a measurable function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers i.e., for any Borel set $B$ of real numbers, the set $\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$ is an event.

In this paper, we introduce the notion of bounded sequences of complex uncertain sequence for almost sure, mean, measure and distribution. We introduce the limitation method for different notion of boundedness of sequence of complex uncertain variables.

**Definition 2.3.** Let $(\zeta_n)$ be a sequence of complex uncertain variables in the uncertainty space $(\Gamma, L, \mathcal{M})$. Then, the sequence $(\zeta_n)$ is almost surely bounded if there exists an event $\Lambda$ with $\mathcal{M}\{\Lambda\} = 1$ and there exists $H > 0$ such that $\sup_n \|\zeta_n(\gamma)\| < H$ for every $\gamma \in \Lambda$.

**Definition 2.4.** A complex uncertain sequence $(\zeta_n)$ is bounded in measure, if there exists $H > 0$ such that $\sup_n \mathcal{M}\{\|\zeta_n\| > H\} = 0$.

**Definition 2.5.** A complex uncertain sequence $(\zeta_n)$ is bounded in mean, if there exists $H > 0$ such that $\sup_n E[\|\zeta_n\|] < H$.

**Definition 2.6.** Let, $\phi, \phi_1, \phi_2, \ldots$ be the complex uncertain distribution of complex uncertain variables $\xi_1, \xi_2, \ldots$, respectively. We say the complex uncertain sequence $(\zeta_n)$ is bounded in distribution if $\sup_n \phi_n(c) \leq \phi(c)$ for all c at which $\phi(c)$ is continuous.

**Definition 2.7.** A linear transformation $T$ from a complex uncertain sequence into complex uncertain sequence is said to be almost sure limitation method if it transforms bounded complex uncertain sequence into a bounded complex uncertain sequence i.e there exists constants $G, H > 0$ such that $\sup_n \|T(\zeta_n(\gamma))\| < G$ for every $\sup_n \|\zeta_n(\gamma)\| < H$ in $(\Gamma, L, \mathcal{M})$.

**Definition 2.8.** A linear transformation is said to be limitation method in measure if

$$\sup_n \mathcal{M}\{\|T(\zeta_n)\| > G\} = 0 \text{ for every } \sup_n \mathcal{M}\{\|\zeta_n\| > H\} = 0.$$

**Definition 2.9.** A linear transformation is said to be limitation method in mean, if $\sup_n E[\|T(\zeta_n)\|] < G$ for every $\sup_n E[\|\zeta_n\|] < H$.

**Definition 2.10.** Let, $\phi, \phi_1, \phi_2, ....$ be the complex uncertain distribution of complex uncertain variables $\zeta_1, \zeta_2, ....$ respectively. We say the linear transformation $T$ is limitation method in distribution, if $\sup_n \phi_n(T(c)) \leq \phi(T(c))$ for every $\sup_n \phi_n(c) \leq \phi(c)$ for all c at which $\phi(c)$ and $\phi(T(c))$ is continuous.

## 3. Main results

In this section, some relationships among the Limitation methods will be studied and derive some new results for a matrix map to be a limitation methods.

**Theorem 3.1.** *If the linear transformation $T$ is limitation method in mean, then $T$ is a limitation method in measure.*

*Proof.* From the definition of limitation method in mean it follows that, $\sup_n E\left(\|T(\zeta_n)\|\right) < G$ whenever $\sup_n E\left(\|\zeta_n\|\right) < H$.

For any $\frac{H}{\varepsilon} > 0$, we have from the Markov inequality,

$$\mathcal{M}\{\|\zeta_n\| \geq \frac{H}{\varepsilon}\} \leq \frac{E[\|\zeta_n\|]}{\frac{H}{\varepsilon}} < \varepsilon,$$

since $\varepsilon$ is arbitrary.

Therefore, $\sup_n \mathcal{M}\left\{\|\zeta_n\| > \frac{H}{\varepsilon}\right\} = 0.$

For any $\frac{G}{\varepsilon} > 0$, we have from the Markov inequality,

$$\mathcal{M}\{\|T(\zeta_n)\| \geq \frac{G}{\varepsilon}\} \leq \frac{E[\|T(\zeta_n)\|]}{\frac{G}{\varepsilon}} < \varepsilon,$$

since $\varepsilon$ is arbitrary.

Therefore, $\sup_n \mathcal{M}\left\{\|T(\zeta_n)\| > \frac{G}{\varepsilon}\right\} = 0.$ □

**Remark 3.1.** Bounded in measure does not imply bounded in mean in general.

**Example 3.1.** Consider an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to be $\gamma_1, \gamma_2, .....$ with

$$M\{\Lambda\} = \begin{cases} \sup \dfrac{1-n}{n^2}, & \text{if } \sup\limits_{\gamma_n \in \Lambda} \dfrac{1-n}{n^2} < \dfrac{1}{2}; \\ 1 - \sup \dfrac{1-n}{n^2}, & \text{if } \sup\limits_{\gamma_n \in \Lambda^c} \dfrac{1-n}{n^2} < \dfrac{1}{2}; \\ \dfrac{1}{2}, & \text{otherwise.} \end{cases}$$

Then, we define complex uncertain variables by,

$$\zeta_n(\gamma) = \begin{cases} in^2(n^2+n-1), & \text{if } \gamma = \gamma_n; \\ 0, & \text{otherwise,} \end{cases}$$

for $n = 1, 2....$ and $\zeta_n = 0$. For some small number $\varepsilon > 0$, we have,

$$\begin{aligned} \mathcal{M}\{\|\zeta_n\| > \varepsilon\} &= \mathcal{M}\{\gamma | \|\zeta_n(\gamma)\| > \varepsilon\} \\ &= \mathcal{M}\{\gamma_n\} = \frac{1-n}{n^2}. \end{aligned}$$

Therefore, $\sup_n \mathcal{M}\{\|\zeta_n\| > \varepsilon\} = 0$. However, for each $n \geq 2$, the complex uncertain distribution of $\zeta_n$,

$$\phi_n(c) = \phi_n(a+ib) = \begin{cases} 0, & \text{if } a < 0, b < +\infty; \\ 0, & \text{if } a \geq 0, b < 0; \\ \frac{1-n}{n^2}, & \text{if } a \geq 0, 0 \leq b < (n^2+n-1); \\ 1, & \text{if } a \geq 0, b \geq (n^2+n-1). \end{cases}$$

So for each $n \geq 2$, we have

$$E[\|\zeta_n\|] = \int_0^{(n^2+n-1)n^2} (1 - \frac{1-n}{n^2})dx \ = \ (n^2+n-1)^2.$$

Therefore, $\sup_n E[\|\zeta_n\|]$ is infinite.

**Theorem 3.2.** *Let $(\zeta_n)$ be a sequence of complex uncertain variables. Then, $(\zeta_n)$ is bounded in measure if its real and imaginary parts are bounded in measure.*

*Proof.* Let,$(\xi_n)$ and $(\eta_n)$ be real and imaginary parts of $(\zeta_n)$ respectively which are bounded in measure. From the definition of bounded in measure, there exists $H > 0$ such that

$$\sup_n \mathcal{M}\left\{\|\xi_n\| > \frac{H}{\sqrt{2}}\right\} = 0,$$

and

$$\sup_n \mathcal{M}\left\{\|\eta_n\| > \frac{H}{\sqrt{2}}\right\} = 0.$$

Also,

$$\|\zeta_n\| = \sqrt{|\xi_n|^2 + |\eta_n|^2}.$$

Thus, we have

$$\{\|\zeta_n\| > H\} \subset \left\{|\xi_n| > \frac{H}{\sqrt{2}}\right\} \bigcup \left\{|\eta_n| > \frac{H}{\sqrt{2}}\right\}.$$

Using sub additivity axiom of uncertain measure, we obtain

$$\mathcal{M}\{\|\zeta_n\| > H\} \leq \mathcal{M}\left\{|\xi_n| > \frac{H}{\sqrt{2}}\right\} + \mathcal{M}\left\{|\eta_n| > \frac{H}{\sqrt{2}}\right\}.$$

Hence, we have

$$0 \leq \sup \mathcal{M}\{\|\zeta_n\| > H\} \leq \sup \mathcal{M}\left\{|\xi_n| > \frac{H}{\sqrt{2}}\right\} + \sup \mathcal{M}\left\{|\eta_n| > \frac{H}{\sqrt{2}}\right\} = 0.$$

Therefore,

$$\sup \mathcal{M}\{\|\zeta_n\| > H\} = 0.$$

Hence, $(\zeta_n)$ is bounded in measure. □

**Theorem 3.3.** *Let, $T$ be a linear transformation. Then, $T$ is limitation method in measure if its real and imaginary parts are bounded in measure.*

*Proof.* The proof is similar to Theorem 3.2, so it is omitted. □

**Theorem 3.4.** *Let $(\zeta_n)$ be a complex uncertain sequence with real and imaginary parts $(\xi_n)$ and $(\eta_n)$, respectively, for $n = 1, 2.....$ If uncertain sequences $(\xi_n)$ and $(\eta_n)$ bounded in measure, then the complex uncertain sequence $(\zeta_n)$ bounded in distribution.*

*Proof.* Let, $c = a + ib$ be a given continuity point of the complex uncertainty distribution $\phi$.

For any $\alpha > a, \beta > b$, we have,

$$\begin{aligned}\{\xi_n \leqslant a, \eta_n \leqslant b\} = &\{\xi_n \leqslant a, \eta_n \leqslant b, \xi_k \leqslant \alpha, \eta_k \leqslant \beta\} \bigcup \{\xi_n \leqslant a, \eta_n \leqslant b, \xi_k > \alpha, \eta_k > \beta\} \\ &\bigcup \{\xi_n \leqslant a, \eta_n \leqslant b, \xi_k \leq \alpha, \eta_k > \beta\} \bigcup \{\xi_n \leqslant a, \eta_n \leqslant b, \xi_k > \alpha, \eta_k \leq \beta\} \\ &\subset \{\xi_k \leq \alpha, \eta_k \leq \beta\} \bigcup \{|\xi_n - \xi_k| \geq \alpha - a\} \bigcup \{|\xi_n - \zeta_k| \geq \beta - b\}.\end{aligned}$$

$$\begin{aligned}\phi_n(c) &= \phi_n(a + ib) \\ &\leq \phi(\alpha + i\beta) + \mathcal{M}\{|\xi_n - \xi_k| \geq \alpha - a\} + \mathcal{M}\{|\xi_n - \eta_k| \geq \beta - b\} \\ &\leq \phi(\alpha + i\beta) + \mathcal{M}\{|\xi_n| \geq \frac{\alpha - a}{2}\} + \mathcal{M}\{|\xi_k| \geq \frac{\alpha - a}{2}\} + \mathcal{M}\{|\eta_n| \geq \frac{\beta - b}{2}\} \\ &\quad + \mathcal{M}\{|\eta_k| \geq \frac{\beta - b}{2}\}.\end{aligned}$$

Since $(\xi_n)$ and $(\eta_n)$ are bounded in measure, we have

$$\sup_n \mathcal{M}\{|\xi_n| \geq \frac{\alpha - a}{2}\} = 0, \ \sup_n \mathcal{M}\{|\xi_k| \geq \frac{\alpha - a}{2}\} = 0,$$

and

$$\sup_n \mathcal{M}\{|\eta_n| \geq \frac{\beta - b}{2}\} = 0, \ \sup_n \mathcal{M}\{|\eta_k| \geq \frac{\beta - b}{2}\} = 0.$$

Thus, we obtain

$$\sup_n \phi_n(c) \leqslant \phi(\alpha + i\beta),$$

for any $\alpha > a, \ \beta > b$. Taking $\alpha + i\beta \longrightarrow a + ib$, we have

$$\sup_n \phi_n(c) \leqslant \phi(c).$$

□

**Remark 3.2.** Bounded in distribution does not imply bounded in measure in general.

**Example 3.2.** Consider an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to be $\gamma_1, \gamma_2, .....$ with

$$M\{\Lambda\} = \begin{cases} \sup \dfrac{n}{2n+1}, & \text{if } \sup\limits_{\gamma_n \in \Lambda} \dfrac{n}{2n+1} < \dfrac{1}{2}; \\ 1 - \sup \dfrac{n}{2n+1}, & \text{if } \sup\limits_{\gamma_n \in \Lambda^c} \dfrac{n}{2n+1} < \dfrac{1}{2}; \\ \dfrac{1}{2}, & \text{otherwise.} \end{cases}$$

Then, we define complex uncertain variables by,

$$\zeta_n(\gamma) = \begin{cases} in, & \text{if } \gamma = \gamma_n, \\ 0, & \text{otherwise,} \end{cases}$$

for $n = 1, 2....$ and $\zeta_n = 0$. For some small number $\varepsilon > 0$, we have,

$$\mathcal{M}\{\|\zeta_n\| \geq \varepsilon\} = \mathcal{M}\{\gamma | \|\zeta_n(\gamma)\| \geq \varepsilon\} = \mathcal{M}\{\gamma_n\} = \frac{n}{2n+1}.$$

Therefore,

$$\sup_n \mathcal{M}\{\|\zeta_n\| \geq \varepsilon\} = \frac{1}{2}.$$

That is, the sequence is not bounded in measure.

Then, $\{\zeta_n\}$ have the uncertain distribution

$$\phi_n(c) = \phi_n(a+ib) = \begin{cases} 0, & \text{if } a < 0, b < +\infty; \\ 0, & \text{if } a \geq 0, b < 0; \\ 1 - \frac{n}{2n+1}, & \text{if } a \geq 0, 0 \leq b < n; \\ 1, & \text{if } a \geq 0, b \geq n. \end{cases}$$

for $n = 1, 2, ....$, respectively, and the complex uncertain distribution of $\zeta$ is

$$\phi(c) = \begin{cases} 0, & \text{if } a < 0, b < +\infty; \\ 0, & \text{if } a \geq 0, b < 0; \\ 1, & \text{if } a \geq 0, b \geq 0. \end{cases}$$

Clearly, $\sup_n \phi_n(c) < \phi(c)$. That is, the sequence $(\zeta_n)$ bounded in distribution.

**Theorem 3.5.** *Let, $A = (a_{mn})$. Then, $A\zeta$ exists almost surely for all complex uncertain sequences which converges to $0$ if and only if $A\zeta$ exists almost surely for all bounded complex uncertain sequences. The necessary and sufficient condition for $A\zeta$ to exist for all sequences of either class is that $\sum\limits_{n=1}^{\infty} \mid a_{mn} \mid$ convergence for all $m = 1, 2, ...$*

*Proof.* Let, $\sum\limits_{n=1}^{\infty} \mid a_{mn} \mid$ converges for all $m = 1, 2, ....$ and let $\zeta = (\zeta_n)$ is a complex uncertain sequence which is bounded almost surely so there exists a constant $H > 0$ such that $\sup\limits_n \{\|\zeta_n(\gamma)\|\} < H$ for all $n \in \mathcal{N}$ and $\gamma \in \Gamma$.

Thus, we have

$$\begin{aligned} \left\| \sum_{n=1}^{\infty} a_{mn}\ \zeta_n(\gamma) \right\| &\leq \sum_{n=1}^{\infty} |a_{mn}|\, \|\zeta_n(\gamma)\| \\ &\leq \sup \{\|\zeta_n(\gamma)\|\} \sum_{n=1}^{\infty} |a_{mn}| \\ &< H \sum_{n=1}^{\infty} |a_{mn}|\,. \end{aligned}$$

Thus, $\sup \| \sum\limits_{n=1}^{\infty} a_{mn}\zeta_n(\gamma)\| < HK = G$. Therefore, $A\zeta_n$ exists almost surely.

If possible let, for some $m$, say $m = \mu$, $\sum\limits_{n=1}^{\infty} \mid a_{\mu n} \mid$ diverges.

The proof of the Theorem will be complete if we can show that, there is a complex uncertain sequence $(\zeta_n)$ which is converges almost surely to 0 and is such that $\sum\limits_{n=1}^{\infty} a_{\mu n}\zeta_n(\gamma)$ diverges.

Since, $\sum_{n=1}^{\infty} \mid a_{\mu n} \mid$ diverges, then there exists $0 = n_1 < n_2 < n_3.......$ such that

$$\sum_{n=n_k+1}^{n=n_{k+1}} a_{\mu n} > 1, \ (k = 1, 2, ....).$$

Taking an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to be $\gamma_1, \gamma_2, .....$ with

$$M\{\Lambda\} = \begin{cases} \sup \frac{n}{2n+1}, & \text{if } \sup\limits_{\gamma_n \in \Lambda} \frac{n}{2n+1} < \frac{1}{2}, \\ 1 - \sup \frac{n}{2n+1}, & \text{if } \sup\limits_{\gamma_n \in \Lambda^c} \frac{n}{2n+1} < \frac{1}{2}, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

Then, we define complex uncertain variables by,

$$\zeta_n(\gamma) = \begin{cases} in\frac{|a_{\mu n}|}{a_{\mu n}}, & \text{if } \gamma = \gamma_n \text{ and } a_{mun} \neq 0, \\ 0, & \text{otherwise,} \end{cases}$$

for $n = 1, 2....$ and $\zeta_n = 0$. Then, the sequence $\{\zeta_n\}$ converges as to 0, we have

$$\left\| \sum_{n=n_k+1}^{n_k+1} a_{\mu n} \ \zeta_n(\gamma) \right\| = \left\| \sum_{n=n_k+1}^{n_k+1} a_{\mu n} \ in\frac{|a_{\mu n}|}{a_{\mu n}} \right\| > n.$$

Hence, the theorem is proved. □

**Theorem 3.6.** *Let $A = (a_{mn})$. Then, $A\zeta_n$ exists in measure for all complex uncertain sequences which converges to 0 if and only if $A\zeta_n$ exists in measure for all bounded complex uncertain sequences. The necessary and sufficient condition for $A\zeta_n$ to exists for all sequences of either class is that $\sum_{n=1}^{\infty} \mid a_{mn} \mid$ convergence for all $m = 1, 2, ....$*

*Proof.* Proof of the theorem is similar to the previous Theorem 3.5, so it is omitted. □

**Theorem 3.7.** *Let $A = (a_{mn})$. Then, $A\zeta_n$ exists in mean for all complex uncertain sequences which converges to 0 if and only if $A\zeta_n$ exists in mean for all bounded complex uncertain sequences. The necessary and sufficient condition for $A\zeta_n$ to exists for all sequences of either class is that $\sum_{n=1}^{\infty} \mid a_{mn} \mid$ convergence for all $m = 1, 2, ....$*

*Proof.* Proof of the theorem is similar to the Theorem 3.5, so it is omitted. □

**Theorem 3.8.** *Let $A = (a_{mn})$. Then, $A\zeta_n$ exists in distribution for all complex uncertain sequences which converges to 0 if and only if $A\zeta_n$ exists in distribution for all bounded complex uncertain sequences. The necessary and sufficient condition for $A\zeta_n$ to exists for all sequences of either class is that $\sum_{n=1}^{\infty} \mid a_{mn} \mid$ convergence for all $m = 1, 2, ....$*

*Proof.* Proof of the above theorem is similar to the Theorem 3.5, so it is omitted. □

## 4. Conclusion and future scope

In this paper, we have defined different types of bounded sequences of complex uncertain variables. We have also introduced the limitation method for a different notion of boundedness of the sequence of complex uncertain variables and have established the necessary condition for a matrix map to be a limitation method. This concept can be applied for introducing some other classes of sequences of complex uncertain variables and can extend their properties. Further other matrix classes transforming between sequences of complex uncertain variables can be studied.

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