Combination and fusion of some 2D invariant moments with generative and discriminative classifiers for recognition of isolated handwritten Tifinagh characters

M. Oujaoura, I. Rahil, W. Bouarifi, and A. Atlas

Abstract. In order to improve the recognition rate and accuracy of the Tifinagh OCR, this document proposes an approach to build a powerful automatic recognition system of isolated handwritten Tifinagh characters by using a combination and fusion of some features extraction methods. The Krawtchouk, Chebyshev and Gaussian Hermite moments are used as descriptors in the features extraction phase due to their invariance to translation, rotation and scaling changes. In the classification phase, the discriminative power of the neural networks, the multiclass SVM (Support Vector Machine), the nearest neighbour classifiers, and the generative nature of the Bayesian networks, are combined together in order to benefit from their complementarity. The experimental results of each single features extraction method and each single classification method are compared with our approach to show its robustness. To approve the claimed hypothesis and results, the experiments are conducted using databases of Handwritten Tifinagh Characters.

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1. Introduction

The development of any language depends today on its level of integration in NTIC. The Royal Institute of the Amazigh Culture (IRCAM) at Rabat in morocco was created in order to promote and lead research related to the Amazigh culture, especially the computerization of the Tifinagh alphabet adopted by IRCAM [1] which is composed from thirty-three characters representing consonants and vowels as shown in Fig. 1. The Optical Character Recognition (OCR) is one of the most active field which can contribute in the development of Amazigh Culture. The objective of this research paper is to contribute by increasing the recognition rate of Tifinagh Optical Character Recognition (OCR).

The rest of the paper is organized as follows. Section 2 reports the related works of Tifinagh Optical Character Recognition (OCR) while the Section 3 presents the proposed approach and describes different stage of the Optical Character Recognition system. The Section 4 presents the results and discussion for the proposed recognition system. Finally, the conclusion and a future work are given in the section 5.
2. Related works

In the recent years, numerous researchers have annually published valuable publications in the field of Optical Character Recognition (OCR) especially the recognition of Tifinagh characters. In their paper, El Ayachi and all [2] have developed an automatic system for recognition of Tifinagh characters using a multilayer neural networks in classification step and Walsh transform in the extraction phase. Bencharef and all [3] have used the geodesic descriptors based on the calculation of the Riemannian metric. They indicated that these descriptors are known for their reliability towards the change of scale, the existence of noise and geometric distortions. For the classification of Tifinagh characters, SVM and neural networks were used. Amrouch and all [4] have used a Hidden Markov model (HMM), and a Haugh transform for the recognition of amazigh character. A Markov model for each character complicates the integration of this system. Es Saady and all [5] have treated the printed and isolated Amazigh characters using an automatic character recognition based on the formalism of finite automata. They encountered integration difficulties for the 33 references characters. Many other works were published in the same area but none of them have exceeded and achieved a recognition rate of the proposed approach [6, 7, 8, 9]. Many other research works related to the Optical Character Recognition (OCR) of tifinagh character can be found in the survey proposed by Ouadid and all in [10]. The generative nature of some used classifiers and the discriminative power of some others led us to wisely combine them together in order to benefit from their complementarity. Also, the fact that some descriptors give a very good result with some classifiers in some situation leads us to conclude that their fusion and combination can be a good idea to develop a very efficient system for character recognition especially Tifinagh characters. Therefore, in this paper, a new method based on the fusion and combination of multiple descriptors and classifiers is proposed in order to increase considerably the recognition rates of tifinagh characters.

3. Proposed approach

In this work, the proposed system (Fig. 2) is based on the combination of multiple descriptors and classifiers. It contains three phases to recognize isolated printed Tifinagh characters:

- Pre-processing phase,
- Feature Extraction phase,
3.1. **Pre-processing phase.** In the pre-processing phase, the input image is transformed to binary system in order to reduce and remove the noisy pixels. After that, the normalization is applied to remove unwanted areas using the method of histogram; in this phase, we first calculate the horizontal and vertical histograms, then the histogram is scanned horizontally in both directions: respectively from the top to the bottom and from the bottom to the top until finding the first black pixels, thereafter, the vertical histogram is traversed in both directions: respectively from the left to the right and from the right to the left until finding the first black pixels. Finally, after determining the positions of the first black pixels, we eliminate the unwanted areas.
3.2. Feature Extraction phase. Many Feature Extraction method are used to characterize the tifinagh characters in order to classify them using a recognition system. The results depend partly on the efficiency of the used descriptor. To calculate the feature descriptor of Tifinagh characters, Krawtchouk, Chebyshev and Hermite moments are used in this phase due their invariance to many geometric transformations.

3.2.1. Krawtchouk moments. Krawtchouk polynomials were proposed by Soviet Ukrainian mathematician M.P. Krawtchouk [11] as an extension of Hermite polynomials. Krawtchouk moments were introduced into image processing area by Yap et al. [12, 13]. For a discrete image of M x N pixels with intensity function \( f(x, y) \), Krawtchouk moments are defined as

\[
K_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} K^t_p(x, M - 1)K^t_q(y, N - 1)f(x, y),
\]

where \( K^t_p(x, M) \) are the weighted (normalized) Krawtchouk polynomials [12] which can be obtained by the following recurrence formula

\[
\begin{align*}
K^t_0(x, M) &= \sqrt{w(x, t, M)}, \\
K^t_1(x, M) &= 1 - \frac{x}{M} \sqrt{w(x, t, M)}, \\
K^t_{p+1}(x, M) &= A \frac{Mt - 2pt + p - x}{(M - p)t} K^t_p(x, M) - B p(1 - t) (M - p) t K^t_{p-1}(x, M),
\end{align*}
\]

where

\[
A = \sqrt{\frac{1 - t}{t} \frac{p + 1}{M - p}}, \quad B = \sqrt{\frac{(1 - t)^2 p(p + 1)}{t^2 (M - p)(M - p + 1)}},
\]

and, the weight function

\[
w(x, t, M) = C^x_M t^x (1 - t)^{M-x}.
\]

The parameter \( t \) determines the localization of the polynomial in the interval \([0, M]\). If we know a priori where in the image our region of interest is located, we can choose such \( t \) that the central part of the polynomial is shifted to this area. Having no prior knowledge, a common choice is \( t = 0.5 \).

3.2.2. Chebyshev moments. The discrete Chebyshev moments, suitable for discrete digital image of M x N pixels with intensity function \( f(x, y) \), are

\[
C_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} T_p(x, M - 1)T_q(y, N - 1)f(x, y),
\]

where \( T_p(x, M) \) are the normalized discrete Chebyshev polynomials [14, 15] defined by recurrence as

\[
\begin{align*}
T_0(x, M) &= \frac{1}{\sqrt{M}}, \\
T_1(x, M) &= (2x + 1 - M)\sqrt{\frac{3}{M(M^2 - 1)}}, \\
T_p(x, M) &= (\alpha_1 x + \alpha_2)T_{p-1}(x, M) + \alpha_3 T_{p-2}(x, M),
\end{align*}
\]
where
\[
\alpha_1 = \frac{2}{p} \sqrt{\frac{4p^2 - 1}{M^2 - p^2}},
\]
\[
\alpha_2 = \frac{1 - M}{p} \sqrt{\frac{4p^2 - 1}{M^2 - p^2}},
\]
\[
\alpha_3 = \frac{p - 1}{p} \sqrt{\frac{2p + 1}{2p - 3}} \sqrt{\frac{M^2(p - 1)^2}{M^2 - p^2}}.
\]

3.2.3. Gaussian-Hermite moments. Although named after Hermite, these polynomials were firstly defined by Laplace and studied in detail by Chebyshev. Hermite rediscovered them in [16]. 2D Gaussian-Hermite (GH) moments are a very useful tool for image description. They are defined for digital image of M x N pixels with intensity function f(x, y), by
\[
G_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} H_p(x, \sigma_1) H_q(y, \sigma_2) f(x, y),
\]
where \(H_p(x, \sigma)\) are the normalized and weighted Gaussian-Hermite polynomials defined by
\[
H_p(x, \sigma) = \exp \left( -\frac{x^2}{2\sigma^2} \right) \frac{1}{\sqrt{p!2^p\sigma \sqrt{\pi}}} H_p \left( \frac{x}{\sigma} \right),
\]
where \(H_p(x)\) are the normal Gaussian-Hermite polynomials defined by recurrence
\[
\begin{align*}
H_0(x) &= 1, \\
H_1(x) &= 2x, \\
H_p(x) &= 2xH_{p-1}(x) - 2(p - 1)H_{p-2}(x),
\end{align*}
\]
or by the general explicit expression as
\[
H_p(x) = p! \sum_{k=0}^{p/2} \frac{(-1)^k (2x)^{p-2k}}{k!(p-2k)!}.
\]
The scale parameter \(\sigma\) should be set up such that the image is sufficiently covered by the central part of the Gaussian. Large \(\sigma\) leads to ineffective modulation, while small \(\sigma\) results in too much suppression of boundary regions. The choice of \(\sigma\) influences the performance of the moments, but there is no exact rule how to set it up. Finding appropriate \(\sigma\) is a heuristic which depends on the image size and content.

3.3. Classification phase. The robustness of the recognition system is based on the decision given by the classification phase. Many classifiers are used singly to classify the tifinagh characters. They can be combined together with descriptors in order to increase the recognition rates of amazigh characters. In this paper, the generative nature of the naive Bayes classifier is combined with the discriminative power of the neural networks, the multiclass SVM (Support Vector Machine), the nearest neighbour classifiers in order to benefit from their complementarity.
3.3.1. Naïve Bayes. A Bayesian network is a graphical probabilistic model representing the random variable as a directed acyclic graph. It is defined by [17]:

- $G = (X, E)$, Where $X$ is the set of nodes and $E$ is the set of edges, $G$ is a Directed Acyclic Graph (DAG) whose vertices are associated with a set of random variables $X = \{X_1, X_2, ..., X_n\}$;
- $\theta = \{P(X_i|Pa(X_i))\}$ is a conditional probabilities of each node $X_i$ relative to the state of his parents $Pa(X_i)$ in $G$.

The graphical part of the Bayesian network indicates the dependencies between variables and gives a visual representation tool of knowledge more easily understandable by users. Pearl and all [18] have also shown that Bayesian networks allow to compactly representing the joint probability distribution over all the variables:

$$P(X) = P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i|Pa(X_i)),$$

where $Pa(X_i)$ is the set of parents of the node $X_i$ in the graph $G$ of the Bayesian network.

In the case of the classification, the Bayesian network can have a class node $C_i$ and many attribute nodes $X_j$. The naive Bayes classifier is used in this paper due to its robustness and simplicity. The Fig. 4 illustrates its graphical structure.

To estimate the Bayesian network parameters and probabilities, Gaussian distributions are generally used. The conditional distribution of a node relative to its parent is a Gaussian distribution whose mean is a linear combination of the parent’s value and whose variance is independent of the parent’s value [19]:

$$P(X_i|Pa(X_i)) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left\{ \frac{-1}{2\sigma_i^2} (x_i - (u_i + \sum_{j=1}^{n} \frac{\sigma_{ij}}{\sigma_j^2} (x_j - u_j)))^2 \right\},$$

where $Pa(X_i)$ is the set of parents of the node $X_i$ in the graph $G$ of the Bayesian network.

Hence,

- $Pa(X_i)$ Are the parents of $X_i$;
- $u_i, u_j, \sigma_i$ and $\sigma_j$ are the means and variances of the attributes $X_i$ and $X_j$ respectively without considering their parents;
- $n_i$ is the number of parents;
- $\sigma_{ij}$ is the regression matrix of weights.

After the parameter and structure learning of a Bayesian network, the Bayesian inference is used to calculate the probability of any variable in a probabilistic model from the observation of one or more other variables. So, the chosen class $C_i$ is the
one that maximizes these probabilities [20, 21]:

\[
P(X_i|Pa(X_i)) = \begin{cases} 
P(C_i) \prod_{i=1}^{n} P(X_i|Pa(X)), & \text{if } X_j \text{ has parent} \\ 
P(C_i) \prod_{i=1}^{n} P(X_i|X_j), & \text{else.} \end{cases} \tag{11}
\]

For the naive Bayes classifier, the absence of parents and the variables independence assumption are used to write the posterior probability of each class as given in the following equation [22]

\[
P(C_i|X) = P(C_i) \prod_{i=1}^{n} P(X_i|Pa(X)). \tag{12}
\]

Therefore, the decision rule \(d\) of an attribute \(X\) is given by:

\[
d(X) = \arg\max_{c_i} P(C_i|X) = \arg\max_{c_i} P(C_i) \prod_{i=1}^{n} P(X_i|Pa(X)). \tag{13}
\]

The class with maximum probability leads to the suitable character for the input image.

### 3.3.2. Neural networks

A multilayer neural network consists of an input layer including a set of input nodes, one or more hidden layers of nodes, and an output layer of nodes. Fig. 5 shows an example of a three-layer network used in this paper, having input layer formed by \(M\) nodes, one hidden layer formed by \(L\) nodes, and output layer formed by \(N\) nodes [23, 24]. This neural network is trained to classify inputs according to target classes. The training input data are loaded from the reference database while the target data should consist of vectors of all zero values except for a one element, where its index is the class they are to represent. The transfer function used in this tree layer neural network is hyperbolic tangent sigmoid transfer function defined by:

\[
tsig(x) = \frac{2}{1 + \exp(-2x)} - 1. \tag{14}
\]

According to authors in [25], the number of neurons in the hidden layer is approximately equal to:

\[
L = E(1 + \sqrt{M(N + 2)}), \tag{15}
\]

where
- \(E(x)\) denotes the integer part of \(x\),
- \(M\) and \(N\) are respectively the number of neurons in the input and output layers.
3.3.3. Multiclass SVM. Support vector machine (SVM) were originally designed for binary classification. SVM is a classification method which is based on finding a hyper-plan that separates data sets into two classes. Several methods have been proposed to construct a multi-class classifier [26] by combining one-against-one binary classifiers or one-against-all binary classifiers. In this paper, the one-against-one and the one-against-all multiclass classifier are used. Those classifiers are based on the Gaussian kernel function defined by:

\[ k(x, y) = \exp \left( -\frac{||x - y||^2}{2\sigma^2} \right), \]

where

\( \bullet \sigma > 0 \) equal to 1.

Many other kernel functions can be used for each binary classifier.

3.3.4. One-against-one multiclass SVM. From N class in data sets, the one-against-one multiclass SVM method constructs N(N-1)/2 binary classifier where each one is trained on data from two classes. The Structure of the one-against-one multiclass SVM classifier can be represented by the Fig. 6. To design and extend SVM binary classifier into a one-against-one multiclass SVM, two groups of data examples are constructed from two classes. The obtained SVM binary classifier is trained to decide if the class is from the first class or it belongs to the second group of classes. This process is repeated for another couple of classes until finishing all the possible couples of the classes from data sets. So, by following this way, multiclass SVM is transformed to a multiple N(N-1)/2 SVM binary classifier. Each SVM binary classifier is trained using a matrix of training data, where each row corresponds to the features extracted as an observation from a class. When classifying an object with an input features
vector, each binary classifier from the multiclass SVM one-against-one model decides and votes for only one class. The class with the majority votes is the correct class which the object belongs to.

3.3.5. *One-against-all multiclass SVM*. The one-against-all multiclass SVM classifier contains N binary classifier, where N is the number of class in data sets. The ith binary SVM is trained with all of the data examples in the ith class with positive labels, and all other data examples with negative labels. To construct a one-against-all multiclass SVM model from binary classifier, the classes are divided into two groups: the first group is formed by one class, and the second group is all the other classes. The obtained SVM binary classifier is trained to decide if the class is from the first group or it belongs to the second groups of classes. This process is repeated for the second group that contains more than two classes until having only one class for each group. The process must stop there. So, by following this way, multiclass SVM is transformed to a multiple SVM binary classifier. Each SVM binary classifier is trained using a matrix of training data, where each row corresponds to features extracted as an observation from a class. After training phase, the multiclass SVM model is able to decide the correct class for an input features vector. To classify an object, its input features vector is presented iteratively to the ith against all binary classifier from the first to the Nth classifier while the result is negative. When the ith binary classifier gives a positive result, the process is stopped. This means that the object belongs to the ith class. The Structure of the one-against-all multiclass SVM classifier is given by the Fig. 7.
3.3.6. Nearest neighbour. The nearest neighbour classifier is used to compare the feature vector of the input image and the feature vectors stored in the database. It is obtained by finding the distance between the prototype image and the database. The class is found by measuring the distance between a feature vector of input image and feature vectors of images in reference database. The Euclidean distance measurement is used in this paper, but other distance measurement can be also used [27].

Let $X_1, \ldots, X_k$ be the $k$ class features vectors in the database and $X_q$ the feature vector of the query image. The feature vector with the minimum distance is found to be the closest matching vector. It is given by:

$$d(X_q|X_j) = \min_{j=1,\ldots,k} \left\{ \sqrt{\sum (x_q(i) - x_j(i))^2} \right\}.$$  \hspace{1cm} (17)

The nearest neighbour classifier does not need any training phase.

4. Results and discussion

To test the accuracy of the combined classifiers and descriptors, the recognition rate of each classifiers and descriptors are calculated separately. Then the recognition rate of the descriptors fusion with each classifier are calculated. Finally, the result of the proposed approach based on combination of descriptors and classifiers are calculated.

We conducted the tests on database containing a set of 1376 image of hanwrittren tifinagh characters [28]. Figure 8 shows some examples of image characters from database. The proposed system has been implemented and tested on a core i7 personnel computer using Matlab software.

In experiments, voting rule classifier combination schemes are used. Then, for each descriptor, each one of the combined classifiers votes for the appropriate character. The character with maximum votes is selected and considered to be the suited character. The total number of the votes is given by the product of the number of the used descriptors and the number of the used classifiers. The robustness of the recognition system is based on the decision given in the classification phase. The efficient description of the character image is also very important for the accuracy of the recognition system.

The computation of recognition and error rates with execution time is presented for each descriptors and each classification approaches in the following Table 1:
Table 1. Recognition rate and error rate of single classifier and descriptor.

<table>
<thead>
<tr>
<th>Descriptors</th>
<th>Recognition rate (%)</th>
<th>Error rate (%)</th>
<th>Execution time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Neural Network</td>
<td>SVM One</td>
<td>SVM All</td>
</tr>
<tr>
<td>Hermite Moments</td>
<td>83.33</td>
<td>92.93</td>
<td>75.25</td>
</tr>
<tr>
<td>Chebyshev Moments</td>
<td>89.90</td>
<td>90.40</td>
<td>79.29</td>
</tr>
<tr>
<td>Krawtchouk Moments</td>
<td>91.41</td>
<td>75.25</td>
<td>70.20</td>
</tr>
<tr>
<td>Fusion</td>
<td>92.93</td>
<td>75.25</td>
<td>90.40</td>
</tr>
<tr>
<td>Hermite Moments</td>
<td>16.67</td>
<td>07.07</td>
<td>24.75</td>
</tr>
<tr>
<td>Chebyshev Moments</td>
<td>10.10</td>
<td>09.60</td>
<td>20.71</td>
</tr>
<tr>
<td>Krawtchouk Moments</td>
<td>08.59</td>
<td>24.75</td>
<td>29.80</td>
</tr>
<tr>
<td>Fusion</td>
<td>07.07</td>
<td>24.75</td>
<td>09.60</td>
</tr>
<tr>
<td>Hermite Moments</td>
<td>38.24</td>
<td>62.99</td>
<td>39.91</td>
</tr>
<tr>
<td>Chebyshev Moments</td>
<td>17.86</td>
<td>47.59</td>
<td>18.84</td>
</tr>
<tr>
<td>Krawtchouk Moments</td>
<td>147.49</td>
<td>180.96</td>
<td>150.83</td>
</tr>
<tr>
<td>Fusion</td>
<td>205.24</td>
<td>234.87</td>
<td>203.44</td>
</tr>
</tbody>
</table>

The recognition rate and error rate of the proposed method are presented in the following Table 2:

Table 2. Recognition rate and error rate of the proposed method.

<table>
<thead>
<tr>
<th>Descriptor and classifier combination</th>
<th>Recognition rate (%)</th>
<th>Error rate (%)</th>
<th>Execution time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>96.46</td>
<td>3.54</td>
<td>2092.93</td>
</tr>
</tbody>
</table>

From the results in Table 1 and Table 2, the recognition rate of the proposed system is improved in the recognition field of the Isolated Handwritten Tifinagh Characters as shown in Fig. 7, despite of the processing time which is significantly increased.

In order to show the robustness of the proposed approach, we give more details by calculating the confusion matrix of some method as presented in Fig. 8 and Fig. 9. We can see from the Figures of confusion matrix (Fig. 10, Fig. 11 and Fig.12) that the misclassified characters (indicated by red color in the confusion matrix), in the case of using Hermite moment as single descriptor and SVM one versus one as a classifier or in the case of using Krawtchouk moment as single descriptor and Neural network as single classifier, are reduced in the case of using our proposed approach based on combination of many descriptors and classifiers. The obtained results show that some characters have a relatively low recognition rate compared to others. The misclassifications are due to 2 factors. The first one is the structural similarity of characters. The second factor is bad writing of some characters in the database whose classification is difficult even for a human operator.

To showcase the proposed method, we compared our obtained results with the other best approaches in term of recognition rate (Table 3). It should be noted that M. Amrouch and all system [2] reports the highest recognition rate of about 97.89 % in handwritten Amazigh language OCR by using continuous HMMs.
Figure 9. Comparison of recognition rates and execution time of single descriptor and classifier with recognition rates and execution time of the proposed method.
Figure 10. Confusion matrix of Hermite Moments descriptor and SVM one versus one classifier.

Conclusion

Generally, the recognition system uses single method to calculate the feature descriptor of the input image and one approach in the classification phase. Even if it is very fast in execution, the recognition rate is still very modest. In order to increase the recognition rate, three feature extraction methods are combined together
in the extraction phase with four classification approaches in order to profit from their complementarity each other.

The obtained results show the robustness of the system based on the proposed approach in the recognition field of the Isolated Handwritten Tifinagh Characters. The use of other fast descriptors and classifiers may increase the recognition rate of Tifinagh characters. The enhancement of the pre-processing and post processing phases can also improve the robustness of the recognition system.

Figure 11. Confusion matrix of Krawtchouk Moments descriptor and Neural network classifier.
Figure 12. Confusion matrix of the proposed method.

Table 3. Comparison with other approaches.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Recognition rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M. Amrouch and al [29]</td>
<td>97.89</td>
</tr>
<tr>
<td>Y. Es Saady and al [30]</td>
<td>96.32</td>
</tr>
<tr>
<td>A. Djematene and al [31]</td>
<td>92.30</td>
</tr>
<tr>
<td>S. Gounane and al [32]</td>
<td>91.05</td>
</tr>
<tr>
<td>Proposed method</td>
<td>96.46</td>
</tr>
</tbody>
</table>

References


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