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Some notes on σ-algebraic soft sets

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Abstract. In this paper, we give some results for σ-algebraic soft sets. We define a negligible parameters for a σ-algebraic soft set and propose a new parameter reduction method and investigate its structural properties. Also, the concepts of soft null set and balanced soft set which are special σ-algebraic soft sets are introduced and some properties are given.

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1. Introduction

For centuries mankind wants to solve the problems they face in the world easily and reach the result quickly. Of course, we use mathematics to model the problems we encounter. But, it is not always easy to model an ambiguous phenomenon mathematically. It is very difficult to model uncertainties, especially with classical mathematical methods. For this reason, many mathematicians have discovered new mathematical concepts to model uncertainties. We can give the fuzzy set concept given by Zadeh as the pioneer of these concepts [19]. In the fuzzy set theory, we model the relative status by defining objects to the degree of membership in a universal set. Many scientists have studied on fuzzy sets both theoretically and practically until today. But Molodtsov pointed out that there are specific problems in fuzzy set theory, such as fitting membership functions, and the reason for these problems is deficiency of the parametrization. In [16], he has built the theory of soft set that is completely free from the membership degrees and he has defined a soft set as a parametrization of some subsets of a universal set. Molodtsov has applied soft sets to many fields such as analysis, game theory and operations research, stabilization, regularization and optimization, theoretically. Some basic mathematical concepts and structural properties of soft set theory such as set-theoretic operations, soft relations and functions, mappings between soft classes, similarity relation on soft sets and the concepts of soft group and soft topology was examined in [13, 2, 3, 12, 15, 1, 18, 11].There is a compact connections between soft sets and information systems. It is showed by Pei and Miao in [17]. They have stated that each soft set corresponds to an information system. Therefore, the applicability of the soft set theory to humanities and daily life problems is also expressed. One of the most important elements in human life is our decisions. In [10, 14, 6], the authors have given various decision-making methods using soft sets. It is very important to choose the right parameters and order them

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so that the decision can be given quickly and accurately in the decision-making process. Of course, there will be too many parameters that affect our decision, and it is obvious that this situation will affect the process negatively. So we need to select our parameters correctly and put them in the right order. For this reason, many scientists gave methods of parameter reduction in the given decision system. We can give some of these as example [5, 9].

In [9], the concept of σ-algebraic soft set is given and is investigated some basic properties. We have established a preference order and indiscernibility relation between the parameters using the measure theory. We define the notion of weight and impact of a parameter in the system. Immediately after these definitions, we gave the parametric weight concept of the system, which is a measure on σ-algebraic soft sets.

In this paper, we will define the concept of *soft null sets* as a special subclass of σ-algebraic soft sets and examine their structural properties. Next, we will determine *negligible parameters* in the system by using measure zero sets. We will discard the negligible parameters and obtain reduced system such that the parametric weight of the previous system and the reduced system is the same, and we will give an decision-making application using the reduced system. Finally, we will introduce the notion of balanced soft set which is also σ-algebraic soft set over a given initial measurable space. Besides, we obtain a metric which is called *imbalanced distance* between parameters, and will give some basic properties.

2. Preliminaries

Let's talk about the basic concepts we will use throughout this article. Let U be a set of objects, E be a set of parameters, $\mathcal{P}(U)$ be a family of all subsets of U i.e. is the power set of U.

Molodtsov says that we need to make an adequate parametrization to model the uncertainties in [16]. In order to do this modeling, the definition of soft set, a very important mathematical tool, is given as follows.

Definition 2.1. [16] A pair (F, E) is called a *soft set* over U if and only if F is a mapping of E into $\mathcal{P}(U)$.

Some basic set-theoretic operations is given by [2] and [13] as follows.

Let (F, A) and (G, B) be two soft sets over U. (F, A) is called *soft subset* of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$ for each $e \in A$, and denoted by $(F, A)\tilde{\subset}(G, B)$. Moreover, $(F, A) = (G, B)$ if and only if $(F, A)\tilde{\subset}(G, B)$ and $(G, B)\tilde{\subset}(F, A)$. *The soft union* of (F, A) and (G, B) is a soft set which is denoted by $(H, C) = (F, A)\tilde{\cup}(G, B)$ over U such that $C = A \cup B$ and for each $e \in C$

$$H(e) = \begin{cases} F(e) & , e \in A - B \\ G(e) & , e \in B - A \\ F(e) \cup G(e) & , e \in A \cap B \end{cases}$$

The soft intersection of (F, A) and (G, B) is a soft set which is denoted by $(H, C) = (F, A)\tilde{\cap}(G, B)$ over U such that $C = A \cap B \neq \varnothing$ and for each $e \in C$, $H(e) = F(e) \cap G(e)$. *The soft complement* of (F, A) is a soft set which is denoted by $(F, A)^c = (F^c, A)$ over U where $F^c : A \to \mathcal{P}(U)$ is a mapping such that $F^c(e) = U - F(e)$ for each $e \in A$. It is called that (F, A) is a *null soft set* over U if $F(e) = \varnothing$ for each $e \in A$ and denote by $\widetilde{\Phi}_A$. Similarly, (F,A) is a *universal soft set* over U if $F(e)=U$ for each $e\in A$ and denoted by $\widetilde{\mathcal{U}}_A$ [2, 13].

Two interesting operation which are called **And** and **Or** between two soft sets are defined by Maji et al. in [13]. Let (F,A) and (G,B) be two soft sets. Then $(F,A)\textbf{And}(G,B)$ is the soft set $(H,A\times B)$ over U and defined by $H(e,e')=F(e)\cap G(e')$ for each $(e,e')\in A\times B$. Similarly, $(F,A)\textbf{Or}(G,B)$ is the soft set $(H,A\times B)$ over U and defined by $H(e,e')=F(e)\cup G(e')$.

Min gave the concept of similarity between soft sets in [15] as follows.

Definition 2.2. [15] Let (F,A) and (G,B) be soft sets over U. It is called that (F,A) is *similar* to (G,B) if there exists a bijection $\phi:A\to B$ such that $F=G\circ\phi$, and denoted by $(F,A)\cong(G,B)$.

We know that σ-algebras are very important for measure theory. We will use the measure theory and σ-algebras as in [9]. For the basic concepts of measure theory and σ-algebras we propose [7] and [8] as references.

Now, we will refer to σ-algebraic soft sets, which is mentioned in [9] and which we will examine some other properties in this article.

Definition 2.3. [9] Let U be a universal set and E be a set of parameters and $\mathcal{A}$ be a σ-algebra on U. A soft set (F,A) is called a *σ-algebraic soft set* over U where $A\subseteq E$ if $F(e)\in\mathcal{A}$ for all $e\in A$.

The collection of all σ-algebraic soft set over U via E is denoted by $\sigma\mathcal{S}(U;E)$.

Since each element of the σ-algebra is said to be a measurable set, a σ-algebraic soft set becomes a parametrization of some measurable sets on U. Suppose that $\mu:\mathcal{A}\to\mathbb{R}\cup\{\infty\}$ be a measure where $\mathcal{A}$ is a σ-algebra. Then, we gain that two relation which is called *preference* and *indiscernibility relations* on the parameter set as mentioned in [9]. Let (F,A) be a σ-algebraic soft set over U. For each $e_1,e_2\in A$, it is called that e_1 is *less preferred* than e_2 which is denoted by $e_1\preceq e_2$ if $\mu(F(e_1))\le\mu(F(e_2))$ and e_1 is *indiscernible* to e_2 which is denoted by $e_1\sim e_2$ if $\mu(F(e_1))=\mu(F(e_2))$. We also gave the definition of the *weight* and *impact* of a parameter using the measure of the set. $\mu(F(e))$ is called *weight* of e which is denoted by $w(e)$ in (F,A) and it is defined that $W(F,A)$ is the *parametric weight* of (F,A) such that

$$W(F,A)=\sum_{e\in A}w(e). \tag{1}$$

The ratio of the weight of a parameter to the parametric weight of the system is called impact of it, i.e. if (F,A) is a σ-algebraic soft set and $e\in A$ then, the *impact* of e is defined by

$$\mathfrak{i}(e)=\frac{w(e)}{W(F,A)}.$$

Besides, we showed that the parametric weight $W(F,A)$ of a soft set (F,A) is a measure on σ-algebraic soft sets in [9].

3. Results

3.1. Soft Null Sets. In this section, we give some results for σ-algebraic soft sets defined on a measurable universe. Let $(U,\mathcal{A},\mu)$ be a measurable space as a universe where U is non-empty set, $\mathcal{A}$ is a σ-algebra and μ is a measure.

We know from measure theory that the subset X of U is called *measure zero* (or null set) if $\mu(X) = 0$ [7, 8].

Now, we give the following definition for σ-algebraic soft sets.

Definition 3.1. Let (F, A) be a σ-algebraic soft set over U. Then, it is called that (F, A) is a *soft null set* over U if $\mu(F(e)) = 0$ for each $e \in A$.

Example 3.2. Let $\mathbb{R}$ be a set of real numbers and μ be a standart Lebesgue measure on it. $E = \{☺, ☹, ☻, ☼\}$ be the set of parameters. We define the soft set over $\mathbb{R}$ as $(F, A) = \{☺ = \mathbb{Q}, ☹ = \{\sqrt{2}\}\}$ where $A \subseteq E$. We know that $\mu(\mathbb{Q}) = 0$ and $\mu(\{\sqrt{2}\}) = 0$. Thus (F, A) is soft null set over $\mathbb{R}$.

Note that, the null soft set is a soft null set, but the opposite is not true as seen in the above example.

Theorem 3.3. Let (F, A) be a soft null set, (G, B) be a σ-algebraic soft set and $(G, B)\widetilde{\subset}(F, A)$. Then, (G, B) is a soft null set.

Proof. Since $(G, B)\widetilde{\subset}(F, A)$, we have $B \subset A$ and $G(e) \subset F(e)$ for each $e \in B$. By hypothesis, (F, A) and (G, B) are σ-algebraic soft sets, then we have $\mu(G(e)) \leq \mu(F(e))$ for each $e \in B$. Since (F, A) is a soft null set, then we obtain that $\mu(G(e)) \leq \mu(F(e)) = 0$. Hence $\mu(G(e)) = 0$ for every $e \in B$. □

Theorem 3.4. Intersection of two soft null sets is a soft null set.

Proof. Let (F, A) and (G, B) be soft null sets over U. Let say $(F, A)\widetilde{\cap}(G, B) = (H, C)$ where $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for each $e \in C$. Since (F, A) and (G, B) are soft null sets, we have $\mu(F(e)) = 0$ and $\mu(G(e')) = 0$ for each $e \in A$, $e' \in B$, respectively. Since $H(e) \subseteq F(e)$ and $H(e) \subseteq G(e)$ for each $e \in C$, then we have $\mu(H(e)) = 0$. □

Theorem 3.5. Union of two soft null sets is a soft null set.

Proof. Since μ is a measure, then it is sub-additive i.e.

$$\mu\left(\bigcup_{i \in \mathbb{N}} X_i\right) \leq \sum_{i \in \mathbb{N}} \mu(X_i).$$

Suppose that $(F, A)\widetilde{\cup}(G, B) = (H, C)$. Then,

$$H(e) = \begin{cases} F(e) & ,\text{if } e \in A - B \\ G(e) & ,\text{if } e \in B - A \\ F(e) \cup G(c) & ,\text{if } e \in A \cap B \end{cases}$$

for each $e \in C = A \cup B$. By definition, if $H(e) = F(e)$, then $\mu(H(e)) = 0$, and if $H(e) = G(e)$, $\mu(H(e)) = 0$. If $e \in A \cap B$, then $H(e) = F(e) \cup G(e)$. Since μ sub-additive, then we have $\mu(H(e)) \leq \mu(F(e)) + \mu(G(e)) = 0 + 0$. Hence, $\mu(H(e)) = 0$. □

Similar to Theorem 3.4 and Theorem 3.5, the following theorem can be obtained.

Theorem 3.6. Let (F, A) and (G, B) be soft null sets. Then
(i) (F, A)**And**(G, B) is also soft null set.
(ii) (F, A)**Or**(G, B) is also soft null set.

Theorem 3.7. Let (F, A) and (G, B) be σ-algebraic soft sets and $(F, A) \cong (G, B)$. If (F, A) is a soft null set, then (G, B) is also soft null set.

Proof. Suppose that $(F,A) \cong (G,B)$. Then, we have a bijective function $\phi : B \to A$ such that $G(e) = (F \circ \phi)(e)$ for all $e \in B$. Since (F,A) is a soft null set, then it is obtained that $G(e) = F(\phi(e)) = 0$ for each $e \in B$. Thus, (G,B) is a soft null set. □

In [9], it is proved that the parametric weight of a soft set is a measure on σ-algebraic soft sets. Then we can express the following theorems for measure zero soft sets.

Theorem 3.8. If (F,A) is a soft null set, then its parametric weight $W(F,A)$ is zero.

Proof. Let (F,A) be a soft null set. We have that $\mu(F(e)) = 0$ for each $e \in A$. Since the parametric weight of (F,A) is

$$W(F,A) = \sum_{e \in A} \mu(F(e)),$$

then, we obtain that $W(F,A) = 0$. □

Let (F,A) be a σ-algebraic soft set. If the parametric weight of (F,A) is zero, then (F,A) is called *measure zero soft set* with respect to its parametric weight.

Theorem 3.9. Let (F,A) and (G,B) be σ-algebraic soft sets, and (G,B) be a soft null set. Then

$$W((F,A)\tilde{\cup}(G,B)) = W(F,A).$$

Proof. Suppose that $(F,A)\tilde{\cup}(G,B) = (H,C)$ such that $C = A \cup B$ and

$$H(e) = \begin{cases} F(e) & ,\text{if } e \in A - B \\ G(e) & ,\text{if } e \in B - A \\ F(e) \cup G(c) & ,\text{if } e \in A \cap B \end{cases}$$

for each $e \in C$. Since (G,B) is a soft null set, then we have that

$$\begin{aligned} W(H,C) &= \sum_{e \in A-B} \mu(F(e)) + \sum_{e \in B-A} \mu(G(e)) + \sum_{e \in A\cap B} \mu(F(e) \cup G(e)) \\ &= \sum_{e \in A-B} \mu(F(e)) + 0 + \sum_{e \in A\cap B} \mu(F(e) \cup G(e)) \\ &= \sum_{e \in A-B} \mu(F(e)) + \sum_{e \in A\cap B} \mu(F(e) \cup G(e)) \end{aligned} \tag{2}$$

We know that $\mu(F(e) \cup G(e)) \le \mu(F(e)) + \mu(G(e)) = \mu(F(e))$, then we have $\mu(F(e) \cup G(e)) = \mu(F(e))$. Hence, we obtain that

$$\sum_{e \in A-B} \mu(F(e)) + \sum_{e \in A\cap B} \mu(F(e))$$

from third step of Equation 2. Thus, we have proven that $W(H,C) = W(F,A)$. □

3.2. Negligible Parameters in σ-Algebraic Soft Sets. In a decision-making process, the excess of parameters is decelerated and aggravated the process. Therefore, we ignore the parameters which are generally useless, unnecessary and ineffective. This parameter reduction process, of course, affects the speed of decision. It is clear that the theory of soft set is one of the influential mathematical tool used in the decision-making process. Now, we suggest another parameter reduction method using σ-algebraic soft sets and the notion of measure and measure zero sets. So, we can give the following definition.

Definition 3.10. Let (F, A) be a σ-algebraic soft set over U. The parameter $e \in A$ is a *negligible parameter* if $\mu(F(e)) = 0$.

Of course, obviously, if $e \in A$ is a negligible parameter, then its weight and impact in (F, A) are $w(e) = 0$ and $\mathfrak{i}(e) = 0$, respectively.

Theorem 3.11. If (F, A) is a soft null set, then all parameters in A are negligible.

Proof. It is obvious. □

If there are negligible parameters, we can discard them in the system. So we get a reduced system from the previous system. Of course, these two systems have the same parametric weight. Namely, if (F, A) is a σ-algebraic soft set and $\mu(F(e_0)) = 0$ for fixed $e_0 \in A$, then we obtain reduced soft set $(G, A - \{e_0\})$ such that $W(F, A) = W(G, B)$ where $B = A - \{e_0\}$. So instead of deciding with (F, A), we can use the reduced form (G, B). We have already proposed a parameter reduction method in [9]. We propose another parameter reduction method for σ-algebraic soft sets in this paper. Obviously,

Theorem 3.12. Let (F, A) be a σ-algebraic soft set, and if there is an element $e_0 \in A$ such that e_0 is a negligible parameter, then $W(F, A) = W(G, B)$ where $G = F|_B$ and $B = A - \{e_0\}$.

The σ-algebraic soft set (G, B) obtained in the above theorem is called *reduced form* of the σ-algebraic soft set (F, A).

Example 3.13. Let $U = \{a, b, c\}$ and the σ-algebra $\mathcal{A}$ be the power set $\mathcal{P}(U)$ of U. For each $X \in \mathcal{A}$ and $a \in U$,

$$\mu(X) = \begin{cases} 1 & , a \in X \\ 0 & , a \notin X \end{cases}$$

be the measure on U. So, $(U, \mathcal{A}, \mu)$ is a measurable universe. Let $(F, E) = \{1 = \varnothing, 2 = \{a, b\}, 3 = \{b, c\}, 4 = \{a\}, 5 = U\}$ be the σ-algebraic soft set over U where $E = \{1, 2, 3, 4, 5\}$ is the parameter set. Since $\mu(F(1)) = 0$, $\mu(F(2)) = 1$, $\mu(F(3)) = 0$, $\mu(F(4)) = 1$ and $\mu(F(5)) = 1$, the parameters 1 and 3 are negligible. So, we can discard them on the parameter set. Hence we get the σ-algebraic soft set $(G, A) = \{2 = \{a, b\}, 4 = \{a\}, 5 = U\}$ where $A = \{2, 4, 5\} \subset E$. Moreover, we have

$$\begin{aligned} W(F, E) &= \sum_{e \in E} \mu(F(e)) \\ &= \mu(F(1)) + \mu(F(2)) + \mu(F(3)) + \mu(F(4)) + \mu(F(5)) = 3 \end{aligned}$$

and

$$\begin{aligned} W(G, A) &= \sum_{e \in A} \mu(G(e)) \\ &= \mu(G(2)) + \mu(G(4)) + \mu(G(5)) = 3. \end{aligned}$$

Thus $W(F, E) = W(G, A)$.

Furthermore, weights of 1 and 3 in the system (F, A) are zero, and so are the impacts.

Theorem 3.14. Let (F, A) and (G, B) be σ-algebraic soft sets and $(F, A) \cong (G, B)$. The system (F, A) have a negligible parameter, then (G, B) is too.

Proof. It is obvious. □

According to Theorem 3.14, we reach the result that (F,A) and (G,B) can be reduced at the same time. So we can give following theorem.

Theorem 3.15. Let (F,A) and (G,B) be σ-algebraic soft sets and (F,A) have negligible parameters. If $(F,A) \cong (G,B)$, then their reduced forms are similar.

Proof. Suppose that $(F,A) \cong (G,B)$. Then we have a bijective function $\phi : A \to B$ such that $F(e) = G(\phi(e))$ for each $e \in A$. Since ϕ is a bijection, then there is an element $e' \in B$ for each $e \in A$. If (F,A) have negligible parameters then (G,B) is too from Theorem 3.14. Discard every negligible parameters from A and so in B. The more we discard from A, the more at B. Let (F',A') be a reduced form of (F,A) and (G',B') be reduced form of (G,B). So we have a bijective function $\phi' : A' \to B'$ which is restricted function of ϕ. Thus, $(F',A') \cong (G',B')$, obviously. □

Using the notion of parametric weight of a σ-algebraic soft sets, we can set an equivalence relation on the $\sigma\mathcal{S}(U;E)$.

Define the relation $\approx$ is as

$$(F,A) \approx (G,B) :\Leftrightarrow W(F,A) = W(G,B). \tag{3}$$

Obviously, this relation is an equivalence relation on all σ-algebraic soft sets over U.

If the system (F,A) has any negligible parameter, then its parametric weight and parametric weight of its reduced form are same from Theorem 3.12. Therefore, they are equivalent with respect to the relation $\approx$.

In Theorem 3.36 in [9], we have stated that if (F,A) is similar to (G,B) then $W(F,A) = W(G,B)$. Then we clearly get the following result from this theorem.

Theorem 3.16. If $(F,A) \cong (G,B)$, then $(F,A) \approx (G,B)$.

But the opposite of the theorem is not true. For example,

Example 3.17. Let $U = \{a,b,c,d,e,f\}$ be a universe, $E = \{1,2,3,4,5,6\}$ be a set of parameters, $\mathcal{A} = \mathcal{P}(U)$ be a σ-algebra and μ be the counting measure on $\mathcal{A}$. Consider the σ-algebraic soft set

$$(F,A) = \{1 = \{a,b,c\}, 2 = \{b\}\}$$

and

$$(G,B) = \{3 = \{a,d,e,f\}\}.$$

Then the parametric weight of (F,A) is $W(F,A) = 4$ and the parametric weight of G,B is $W(G,B) = 4$. Thus $W(F,A) = W(G,B)$ i.e. $(F,A) \approx (G,B)$ but (F,A) is not similar to (G,B).

In measure theory, a finite nonnegative measure on a σ-algebra can be turned out a metric on σ-algebra. We shall assume as a nonnegative finite measure any given measure unless otherwise claimed. Let $\mathcal{A}$ be a σ-algebra on U and define the function

$$d : \mathcal{A} \times \mathcal{A} \to \mathbb{R}, d(A,B) = \mu(A \vartriangle B) \tag{4}$$

where $\vartriangle$ is a symmetric difference of A and B. Of course, d is a pseudo-metric on $\mathcal{A}$. It is defined that the relation between elements of $\mathcal{A}$ such that

$$A \simeq B \Leftrightarrow \mu(A \vartriangle B) = 0 \tag{5}$$

for each $A, B \in \mathcal{A}$ i.e. A and B are same set on $\mathcal{A}$. Note that, the relation $\simeq$ is an equivalence relation on $\mathcal{A}$. Thus,

$$\overline{d} : (\mathcal{A}/\simeq) \times (\mathcal{A}/\simeq) \to \mathbb{R}, \overline{d}([A],[B]) = d(A,B) \tag{6}$$

is a metric where $[A]$, $[B]$ are equivalence class of A and B, respectively [4].

Lemma 3.18. [8] Let $(U, \mathcal{A}, \mu)$ be a measure space and $A, B \in \mathcal{A}$. If $\mu(A \triangle B) = 0$, then $\mu(A) = \mu(B)$.

Now, we define a special σ-algebraic soft set over the measure space $(U, \mathcal{A}, \mu)$.

Definition 3.19. Let (F, A) be a σ-algebraic soft set over the measure space $(U, \mathcal{A}, \mu)$. We call that (F, A) is a *balanced soft set* over U with respect to A if $\mu(F(e) \triangle F(e')) = 0$ for each $e, e' \in A$ where $F(e) \triangle F(e')$ is the symmetric difference of $F(e)$ and $F(e')$.

From Equation 5, if (F, A) is balanced soft set, then we have that all e-approximated sets are same i.e. $F(e) \simeq F(e')$ for each $e, e' \in A$.

Example 3.20. Consider the measurable space $(U, \mathcal{A}, \mu)$ from Example 3.17, let $(F, A) = \{2 = \{a, b\}, 5 = \{a, b\}\}$ be σ-algebraic soft set over U. Then we have

$$F(1) \triangle F(1) = F(1) \triangle F(5) = F(5) \triangle F(1) = F(5) \triangle F(5) = \varnothing$$

and

$$\mu(F(1) \triangle F(1)) = \mu(F(1) \triangle F(5)) = \mu(F(5) \triangle F(1)) = \mu(F(5) \triangle F(5)) = 0.$$

Thus, (F, A) is a balanced soft set over U.

Theorem 3.21. If (F, A) is a universal soft set or null soft set, then it is balanced.

Proof. It is obvious. □

Theorem 3.22. If (F, A) is balanced soft set, then $e \sim e'$ for each $e, e' \in A$.

Proof. Suppose that (F, A) is balaced soft set over $(U, \mathcal{A}, \mu)$. Then we have $\mu(F(e) \triangle F(e')) = 0$ for each $e, e' \in A$. Therefore, it is obtained that $\mu(F(e)) = \mu(F(e'))$ for any $e, e' \in A$ from Lemma 3.18. Thus $e \sim e'$ for each $e, e' \in A$ i.e. all parameters in A are indiscernible from Definition 3.22 in [9]. □

Theorem 3.23. If $(F, A) \cong (G, B)$ and (F, A) is balanced, then (G, B) is balanced.

Proof. Assume that (F, A) is similar to (G, B), then we have a bijective function $\phi : A \to B$ such that $F(e) = (G \circ \phi)(e)$ for each $e \in A$ from Definition 2.2. Since ϕ is a bijection, there is an inverse $\phi^{-1} : B \to A$ such that $(F \circ \phi^{-1})(e_*) = G(e_*)$ for each $e_* \in B$ where $\phi^{-1}(e_*) = e$ for each $e \in A$. Therefore, we have

$$\begin{aligned} \mu(G(e_*) \triangle G(e'_*)) &= \mu((F \circ \phi^{-1})(e_*) \triangle (F \circ \phi^{-1})(e'_*)) \\ &= \mu(F(e) \triangle F(e')) = 0. \end{aligned}$$

Hence (G, B) is a balanced soft set. □

We can derive a metric on the parameter set using the metric given in Equation 6. Let (F, A) be a σ-algebraic soft set over the measurable space $(U, \mathcal{A}, \mu)$. If we define the function $d^* : A \times A \to \mathbb{R}$ such that

$$d^*(e, e') = \overline{d}([F(e)], [F(e')]) = d(F(e), F(e')), \tag{7}$$

then it is a metric on A.

Definition 3.24. The metric given in Equation 7 is called *the imbalance distance* of the parameters in A.

Example 3.25. Consider the measurable space and σ-algebraic soft set (F, E) in Example 3.13. The imbalance distance of the parameters 2 and 3 is

$$d^*(2,3) = \overline{d}([F(2)],[F(3)]) = d(F(2),F(3)) = \mu(F(2) \triangle F(3)) = 1.$$

The imbalance distance of 4 and 5 is

$$d^*(4,5) = \overline{d}([F(4)],[F(5)]) = d(F(4),F(5)) = \mu(F(4) \triangle F(5)) = 0.$$

We conclude that $2 \nsim 3$ and $4 \sim 5$ with respect to given measurable space.

Obviously, we obtain following theorem for balanced soft sets.

Theorem 3.26. If (F, A) is a balanced soft set, then $d^*(e, e') = 0$ for each $e, e' \in A$.

4. Conclusion

Decisions in a decision-making process depend on the parameters, e.g. the beauty and cost of a house in a simple home purchase problem. Of course, there are many parameters that can influence the decision. The multiplicity of parameters affect the decision process in the negative direction. It is therefore essential to determine the most appropriate and most effective parameters for the decision. Soft sets which is a parameterization of some subsets of a given inital universe is an effective mathematical tool for decision-making process. It is also important to measure of the phenomenon which we make decisions. In [9], we gave the notion of σ-algebraic soft set by using measure theory to make more realistic decisions. With these concepts, we compared the parameters that will affect our decisions. In this paper, we have defined the concept of soft null sets which is a special case of a σ-algebraic soft set given in [9]. Then we discussed the basic properties. Afterwards, we define the concept of negligible parameter using the notion of measure zero set and we have made a parameter reduction different from [9]. In addition, we gave the concept of parametric weight of a soft set which is a measure on σ-algebraic soft sets. In here, we show that the parametric weights of a σ-algebraic soft set and its reduced form with respect to negligible parameters are same. In this way we can build a decision system with the same weight as the first by reducing the multiplicity of parameters in the decision making process. In the future, the effects of σ-algebraic soft sets on information systems and data mining can be explored. Besides, we have defined the concept of balanced soft set as an another special σ-algebraic soft set. Therewithal, we have obtained a metric for the parameters. Because the decision process depends on the parameters, the metric obtained can be used for reduction the parameters. In addition to future work, an application can be made to the information systems of balanced soft sets. Because, in [17], it has been shown that each information system is a soft set and each soft set is an information system over related universal set.

The author hope that this article shed light on the scientist which is working in this area.

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