Annals of University of Craiova, Math. Comp. Sci. Ser.
Volume 30(2), 2003, Pages 34–39
ISSN: 1223-6934

GBS operators of Schurer-Stancu type

Dan Bărbosu

In the memory of Professor E. Dobrescu

Abstract. If $p \geq 0, q \geq 0$ are given positive integers and $\alpha_1, \beta_1, \alpha_2, \beta_2$ are real parameters satisfying $0 \leq \alpha_1 \leq \beta_1$, $0 \leq \alpha_2 \leq \beta_2$, in ([9]) was constructed the bivariate Schurer-Stancu operator $\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} : C([0,1+p]\times[0,1+q]) \to C([0,1]\times[0,1])$ defined for any $f \in C([0,1+p]\times[0,1+q])$ and any $m,n\in\mathbb{N}$ by

$$\left(\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f\right)(x,y) = = \sum_{k=0}^{m+p}\sum_{j=0}^{n+q}\widetilde{p}_{mk}(x)\widetilde{p}_{nj}(y) f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right)$$

where $\widetilde{p}_{m,k}(n), \widetilde{p}_{nj}(y)$ are the fundamental Schurer polynomials and approximation properties of this operator were established.

Denoting by $C_b([0,1+p]\times[0,1+q])$ the space of B-continuous real valued functions defined on $[0,1+p]\times[0,1+q]$ the GBS operator associated to $\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}$ is constructed. This operator, denoted by $\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}$ applies the space $C_b([0,1+p]\times[0,1+q])$ into $C_b([0,1]\times[0,1])$ and it is defined for any $f\in C_b([0,1+p]\times[0,1+q])$ and any $m,n\in\mathbb{N}$ by

$$\left(\widetilde{U}^{(\alpha_1,\beta_1,p_1,q_1)}_{m,n,p,q} f\right)(x,y) = \sum_{k=0}^{m}\sum_{j=0}^{n}\widetilde{p}_{m,k}(x)\widetilde{p}_{nj}(y)\times \times\{f(k/m,y)+f(x,j/n)-f(k/m,j/n)\}$$

Some approximation properties (concerning a convergence theorem and the approximation order, in terms of mixed modulus of smoothness), for the sequence $\left\{\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f\right\}_{m,n\in\mathbb{N}}$ are established.

Note that for $p=q=0$ and $\alpha_1=\beta_1=\alpha_2=\beta_2=0$ our GBS operator reduces to the GBS operator of Bernstein type, constructed in 1966 by E. Dobrescu and I. Matei ([15]).

The paper is devoted to the memory of the GREAT Romanian Mathematician, Professor Eugen Dobrescu, disappeared premature in 1993.

2000 Mathematics Subject Classification. 41A36, 41A63.

Key words and phrases. positive linear operator, Bernstein operator, Schurer operator, Stancu operator, B-continuous function, parametric extension, GBS operator, mixed modulus of smoothness.

1. Preliminaries

Let $p \geq 0$ be a given integer. In 1962 F. Schurer (see ([18])) constructed and studied the positive and linear operator $\widetilde{B}_{m,p} : C([0,1+p]) \to C([0,1])$, which associates to any function $f \in C([0,1+p])$ the polynomial $\widetilde{B}_{m,p}f$ defined by

$$\left(\widetilde{B}_{m,p}f\right)(x,y) = \sum_{k=0}^{m+p}\widetilde{p}_{m,k}(x) f(k/m) \tag{1}$$

Received: 30 October 2003.

where $\widetilde{p}_{m,k}(x)$ are the fundamental Schurer polynomials.
Extensions of the operator (1.1) to the case of bivariate functions were studied in our earlier papers ([8]), ([9]) and ([10]).

In 1968, D.D. Stancu ([19]) constructed and studied a linear and positive operator depending on two non-negative real parameters α and β which satisfy the condition $0 \le \alpha \le \beta$. This operator, denoted by $P_m^{(\alpha,\beta)}$, associates to any function $f \in C([0,1])$ the polynomial $P_m^{(\alpha,\beta)} f$, defined by

$$\left(P_m^{(\alpha,\beta)} f\right)(x) = p_{m,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right) \tag{2}$$

where $p_{m,k}(x)$ are the fundamental Bernstein polynomials.
The operator (1.2) is known in mathematical literature as "the Bernstein-Stancu operator".
Extensions of the operator (1.2) to the case of bivariate functions were constructed by F. Stancu ([23]) and D. Bărbosu ([5], [6], [9]).

Considering a given integer $p \ge 0$ and two real parameters α and β which satisfy the condition $0 \le \alpha \le \beta$, in our recent paper ([11]) was constructed the linear and positive operator $\widetilde{S}_{m,p}^{(\alpha,\beta)}$, defined for any $f \in C([0,1+p])$ and any $m \in \mathbb{N}$ by

$$\left(\widetilde{S}_{m,p}^{(\alpha,\beta)} f\right)(x) = \sum\nolimits_{k=0}^{m+p} \widetilde{p}_{m,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right) \tag{3}$$

The operator (1.3) was called "Schurer-Stancu type operator", because for $\alpha = \beta = 0$ it reduces to the operator (1.1) and for $p = 0$, it reduces to the operator (1.2). If $p = 0$, $\alpha = \beta = 0$, the operator (1.3) is the classical Bernstein operator.

Considering two given intergers $p \ge 0, q \ge 0$ and four real parameters $\alpha_1, \beta_1, \alpha_2, \beta_2$ satisfying the conditions $0 \le \alpha_1 \le \beta_1$ and $0 \le \alpha_2 \le \beta_2$, in the paper ([9]) we constructed the bivariate operator of Schurer-Stancu type $\widetilde{S}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)} : C_1([0,1+p] \times [0,1+q]) \to C([0,1] \times [0,1])$, defined for any $f \in C([0,1+p] \times [0,1+q])$ and any $m, n \in \mathbb{N}$ by

$$\begin{aligned}\left(\widetilde{S}_{m,n,p,q}^{(\alpha_1,\beta_1,p_1,q_1)} f\right)(x,y) &= \sum\nolimits_{k=0}^{m+p} \sum\nolimits_{j=0}^{n+q} \widetilde{p}_{mk}(x) \widetilde{p}_{n,j}(y) \times \\ &\cdot \quad f\left(\frac{k+\alpha_1}{m+\beta_1}, \frac{j+\alpha_2}{n+\beta_2}\right)\end{aligned} \tag{4}$$

Some approximation properties of (1.3) were studied in the same paper ([9]).
Clearly, for $p = q = 0$ the operator (1.4) reduces to the Stancu bivariate operator, studied by F. Stancu ([23)] and D. Bărbosu ([5]).
For $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$, the operator (1.4) is the bivariate Schurer type operator studied in our earlier paper ([10]).

The aim of the present paper is to extend the operator (1.4) to the case B-continuous (Bögel continuous) functions. More exactly, we shall present a GBS (Generalized Boolean Sum) operator of Schurer-Stancu type and some approximation properties of this operator.
The term of "B-continuous function" was introduced by K. Bögel (see ([12]), [13])). One of the first result concerning the approximation of this kind of functions is due to E. Dobrescu and I. Matei ([15]).

An important "test function theorem", (the analogous of the well known Korovkin theorem), for approximation of B-continuous functions using GBS-operators is due to C. Badea, I. Badea and H.H. Gonska ([2]).

The analogous of first order modulus of smoothness for univariate functions is the "mixed modulus of smoothness", introduced by I. Badea ([4]). This modulus is used for evaluating the approximation order of B-continuous functions using GBS operators. The analogous of well-known Shisha-Mond theorem ([17]) for B-continuous functions was established by H.H. Gonska ([16]), C. Badea and C. Cottin ([3]).

2. GBS operators of Schurer-Stancu type

Let $p \geq 0, q \geq 0$ be given integers and let us to denote by $C_b([0,1+p]\times[0,1+q])$ the space of real valued functions B-continuous on $[0,1+p]\times[0,1+q]$.

Next, we consider four non-negative parameters $\alpha_1,\beta_1,\alpha_2,\beta_2$ satisfying the conditions $0\leq\alpha_1\leq\beta_1$, $0\leq\alpha_2\leq\beta_2$. The parametric extensions of the Schurer-Stancu type operators (1.4) are defined respectively by

$$\left({}_x\widetilde{S}^{(\alpha_1,\beta_1)}_{m,p}f\right)(x,y) = \sum\nolimits_{k=0}^{m+p}\widetilde{p}_{m,k}(x)f(k/m,y) \tag{5}$$

$$\left({}_y\widetilde{S}^{(\alpha_2,\beta_2)}_{n,q}f\right)(x,y) = \sum\nolimits_{j=0}^{n+q}\widetilde{p}_{n,j}(y)f(x,j/n) \tag{6}$$

It is easy to see that ${}_x\widetilde{S}^{(\alpha_1,\beta_1)}_{m,p}$ and ${}_y\widetilde{S}^{(\alpha_2,\beta_2)}_{n,q}$ are linear and positive operator (see ([9])). They commute on $C([0,1+p]\times[0,1+q])$ and their product is the bivariate Schurer-Stancu type operator $S^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}: C([0,1+p]\times[0,1+q])\to C([0,1]\times[0,1])$, defined for any $f\in C([0,1+p]\times[0,1+q])$ and any $m,n\in\mathbb{N}$ by

$$\begin{aligned}\left(S^{\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}f\right)(x,y) &= \sum\nolimits_{k=0}^{m+p}\sum\nolimits_{j=0}^{n+q}\widetilde{p}_{m,k}(x)\widetilde{p}_{n,j}(y)\times\\ &\times\ f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right)\end{aligned} \tag{7}$$

In ([9]) were proved, among others, the following properties of the operator (2.3).

Lemma 2.1. *The operator* (2.3) *is linear and positive.*

Lemma 2.2. *If* $e_{ij}(s,t)=s^it^j$ $(i,j\in\mathbb{N},\ 0\leq i+j\leq 2)$ *are the test functions, the operator* (2.3) *verifies*

$$\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{00};x,y)=1 \tag{8}$$

$$\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{10};x,y)=\frac{m+p}{m+\beta_1}\,x+\frac{\alpha_1}{m+\beta_2} \tag{9}$$

$$\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{0,1};x,y)=\frac{n+q}{n+\beta_2}\,y+\frac{\alpha_2}{n+\beta_2} \tag{10}$$

$$\begin{aligned}\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{20};x,y) &= \frac{1}{(m+\beta_1)^2}\left\{(m+p)^2x^2+(m+p)x(1-x)+\right.\\ &+\left.2\alpha_1\frac{m(m+p)}{m+\beta_1}\,x+\frac{\alpha_1^2(3m+\beta_1)}{m+\beta_1}\right\}\end{aligned} \tag{11}$$

$$\begin{aligned}\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{02};x,y) &= \frac{1}{(n+\beta_2)^2}\left\{(n+q)^2y^2+(n+q)y(1-y)+\right.\\ &+\left.2\alpha_2\frac{n(n+q)}{n+\beta_2}\,y+\frac{\alpha_2^2(3n+\beta_2)}{n+\beta_2}\right\}\end{aligned} \tag{12}$$

Definition 2.1. Let $\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} : C_b([0,1+p]\times[0,1+q]) \to C_b([0,1]\times[0,1])$ be the boolean sum of (2.1) and (2.2), i.e.

$$\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} =_x \widetilde{S}^{(\alpha_1,\beta_1)}_{m,p} +_y \widetilde{S}^{\alpha_2,\beta_2}_{n,q} - \widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} \tag{13}$$

The operator (2.9) will be called GBS operator of Schurer-Stancu type.

Lemma 2.3. *The GBS operator of Schurer-Stancu type is defined for any* $f \in C_b([0,1+p]\times[0,1+q])$ *by*

$$\left(\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f\right)(x,y) = = \sum_{k=0}^{m+p}\sum_{j=0}^{n+q} \widetilde{p}_{m,k}(x)\widetilde{p}_{n,j}(y)\left\{ f\left(\frac{k+\alpha_1}{m+p}, y\right) + f\left(x, \frac{j+\alpha_2}{n+q}\right) - - f\left(\frac{k+\alpha_1}{m+p}, \frac{j+\alpha_2}{n+q}\right)\right\} \tag{14}$$

Proof. The assertion follows by direct computation from (2.9), taking into account of Lemma 2.2 (the identity (2.4)). □

Remark 2.1.
(1) For $p = q = 0$, the operator (2.10) is the GBS operator of Stancu type, introduced in our paper ([6])
(2) For $\alpha = \beta = 0$, the operator (2.10) is the GBS operator of Schurer type, introduced in our paper ([8])
(3) For $\alpha = \beta = 0$ and $p = q = 0$, the operator (2.10) is the GBS operator of Bernstein type, introduced by E. Dobrescu and I. Matei ([15]).

Theorem 2.1. *For any* $f \in C_b([0,1+p]\times[0,1+q])$ *the sequence* $\left\{\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f\right\}_{m,n\in\mathbb{N}}$ *converges to* f *uniformly on* $[0,1]\times[0,1]$ *as* m *and* n *tend to infinity.*

Proof. From Lemma 2.1 and Lemma 2.2 (the identity (2.4)) follows that $\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}$ is a linear positive operator, reproducing the constant functions.
Taking into account of Lemma 2.2 (the identities (2.5), (2.6), (2.7) and (2.8)) we get:

$$\begin{aligned} \lim_{m,n\to\infty} \widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{10};x,y) &= x \\ \lim_{m,n\to\infty} \widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{01};x,y) &= y \\ \lim_{m,n\to\infty} \widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}(e_{20}+e_{02};x,y) &= x^2+y^2, \end{aligned}$$

uniformly on $[0,1]\times[0,1]$.
We can apply the "test functions theorem" due to C. Badea, I. Badea and H.H. Gonska ([2]) and we arrive to desired result. □

In what follows ω_{mixed} denotes the "mixed modulus of smoothness" (see ([4]), ([3]), ([16])) and we suppose known the variant of Shisha-Mond theorem for B-continuous functions (see ([3]), ([6])).

Theorem 2.2. *For any* $f \in C_b([0,1+p])\times[0,1+q])$*, in each point* $(x,y)\in[0,1]\times[0,1]$*, the operator (2.10) verifies*

$$\left|\left(\widetilde{U}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f\right)(x,y) - f(x,y)\right| \le 4\omega_{mixed}(\delta_{m,p,\alpha_1,\beta_1,x}\ \delta_{n,q,\alpha_2,\beta_2,y}) \tag{15}$$

where

$$\begin{aligned} \delta_{1,m,p,\alpha_1,\beta_1,x} &= \frac{(p-\beta_1)^2}{(m+\beta_1)^2} + \frac{(m+p)}{(m+\beta_1)^2}\, x(1-x) + \\ &+ \frac{2\alpha_1(mp-2m\beta_1-\beta_1^2)}{(m+\beta_1)^3}\, x + \frac{\alpha_1^2(3m+p)}{(m+\beta_2)^3} \end{aligned} \tag{16}$$

$$\begin{aligned} \delta_{2,n,q,\alpha_2,\beta_2,y} &= \frac{(q-\beta_2)^2}{(n+\beta_2)^2} + \frac{(n+q)}{(n+\beta_2)^2}\, y(1-y) + \\ &+ \frac{2\alpha_2(nq-2n\beta_2-\beta_2)}{(n+\beta_2)^2}\, y + \frac{\alpha_2^2(3n+q)}{(n+\beta_2)^2} \end{aligned} \tag{17}$$

Proof. Applying the Shisha-Mond type theorem for B-continuous functions (see ([3]), ([15])) we get

$$\begin{aligned} &\left|\left(\widetilde{U}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)} f\right)(x,y)\right| \le \\ &\le \Bigg(1 + \delta_1^{-1}\sqrt{L_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}((\cdot - x)^2; x,y)} + \delta_2^{-1}\sqrt{L_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}((* - y)^2; x,y)} + \\ &\quad + \delta_1^{-1}\delta_2^{-2}\sqrt{L_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}((\cdot - x)^2(* - y)^2; x,y)}\Bigg)\, \omega_{mixed}(\delta_1,\delta_2) \end{aligned} \tag{18}$$

for any $\delta_1 > 0, \delta_2 > 0$.
Next, taking into account of Lemma 2.2 and choosing $\delta_1 = \delta_{m,p,\alpha_1,\beta_1,x}$, $\delta_2 = \delta_{n,q,\alpha_2,\beta_2,y}$ in (2.15), we arrive to the desired inequality (2.11). □

Corollary 2.3. *For any $f \in C_b([0,1+p]\times[0,1+q])$, any $(x,y) \in [0,1]\times[0,1]$, the GBS operator of Schurer-Stancu type verify:*

$$\left|\left(\widetilde{U}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)} f\right)(x,y) - f(x,y)\right| \le 4\,\omega_{mixed}(\delta_1,\delta_2) \tag{19}$$

where

$$\delta_1 = \max_{x\in[0,1]} \delta_{m,p,\alpha_1,\beta_1,x}, \quad \delta_2 = \max_{y\in[0,1]} \delta_{n,q,\alpha_2,\beta_2,y} \tag{20}$$

and β_1, β_2 *satisfy* (2.14).

Proof. The assertion follows from (2.11), taking into account that the mixed modulus of smoothness is monotonous increasing with respect the natural order relation from $\mathbb{R}^2$, i.e.
$(\forall)\ (\delta_1,\delta_2),\ (\delta_1',\delta_2') \in [0,b-a]\times[0,d-c]$, $\delta_1 < \delta_1'$, $\delta_2 < \delta_2' \Rightarrow$
$\Rightarrow \omega_{mixed}(\delta_1,\delta_2) \le \omega_{mixed}(\delta_1',\delta_2')$. □

Remark 2.2.

(i) The theorem 2.2 give us the order of local approximation (in each point $(x,y) \in [0,1]\times[0,1]$) while Corollary 2.3 give the order of global approximation of B-continuous function f by $\widetilde{U}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}$

(ii) Naturally, the inequalities (2.11) and (2.16) can be more detailed, depending on the relations between the parameters $\alpha_1, \beta_1, \alpha_2, \beta_2, p, q$

(iii) As consequences of Theorem 2.1 and Theorem 2.2, for $p = q = 0$, we obtain approximation properties of the GBS operator of Stancu type, introduced and studied in ([6])

(iv) For $\alpha_1 = \beta_1 = 0$, $\alpha_2 = \beta_2 = 0$, as consequences of Theorem 2.1 and Theorem 2.2, we get approximation properties of the GBS operator of Schurer type, introduced and studied in ([8])

(v) For $\alpha_1 = \beta_1 = 0$, $\alpha_2 = \beta_2 = 0$, $p = q = 0$, we get approximation properties of the GBS operator of Bernstein type, introduced by E. Dobrescu and I. Matei (see ([15])) and studied also by I. Badea (see ([3])) and many others.

References

[1] O. Agratini, *Aproximare prin operatori liniari*, Cluj-Napoca, Presa Universitară Clujeană, (2000) (Romanian).

[2] C. Badea, I. Badea, H.H. Gonska, A test function theorem and approximation by pseudopolynomials, *Bull. Austral. Math. Soc.*, **34**, 53-64 (1986).

[3] C. Badea, C. Cottin, Korovkin-type theorems for Generalized Boolean Sum operators, *Colloquia Mathematica Societatis Janos Bolyai*, **58**, Approximation Theory, Kecskemet (Hungary), 51-68 (1990).

[4] I. Badea, Modul de continuitate în sens Bögel şi unele aplicaţii în aproximarea printr-un operator Bernstein, *Studia Univ. "Babeş-Bolyai", Ser. Math-Mech*, **18**(2), 69-78 (1973) (Romanian).

[5] D. Bărbosu, *Aproximarea funcţiilor de mai multe variabile prin sume booleene de operatori liniari de tip interpolator*, Cluj-Napoca, Risoprint, (2002) (Romanian).

[6] D. Bărbosu, Aproximation properties of a bivariate Stancu type operator, *Studia Univ. "Babeş-Bolyai", Matematica*, **XLVII**(4), 13-18 (2002).

[7] D. Bărbosu, GBS operators of Bernstein-Schurer type (to appear in *Matematica*, Cluj-Napoca).

[8] D. Bărbosu, Bivariate operators of Schurer-Stancu type (to appear in *Anal. Şt. Univ. "Ovidius"*, Constanţa).

[9] D. Bărbosu, Bivariate operators of Bernstein-Schurer type (to appear in *Rev. Anal. Num. Theor. Approx.*).

[10] D. Bărbosu, Schurer-Stancu type operators (to appear in *Studia Univ. "Babeş-Bolyai"*).

[11] K. Bögel, Mehrdimensionale Differention von Funktionen mehrer Väränderlicher, *J. Reine Angew. Math.*, **170**, 197-217 (1934).

[12] K. Bögel, Über die mehrdimensionale Differentiation Integration und beschränkte Variation, *J. Reine Angew. Math.*, **173**, 5-29 (1935).

[13] F.J. Delvos, W. Schempp, *Boolean Methods in Interpolation and Approximation*, Harlow, UK: Longman Scientific & Technical (1989).

[14] E. Dobrescu, I. Matei, Aproximarea prin polinoame de tip Bernstein a funcţiilor bidimensional continue, *Anal. Univ. Timişoara, Seria Ştiinţe matematice-fizice*, **IV**, 85-90 (1966).

[15] H.H. Gonska, *Quantitative approximation in $C(X)$*, Habilitationsschrift, Universitaät Duisburg (1985).

[16] O. Shisha, B. Mond, The degree of convergence of linear operators, *Acad. Sci. U.S.A.*, **60**, 1196-1200 (1968).

[17] F. Schurer, *Linear positive operators in approximation theory*, Math. Inst. Techn. Univ. Delft: Report, 1962.

[18] D.D. Stancu, Approximation of functions by a new class of linear polynomial operators, *Rev. Roum. Math. Pures et Appl.*, **13**(8), 1173-1194 (1968).

[19] D.D. Stancu, Asupra unei generalizări a polinoamelor lui Bernstein, *Studia Univ. "Babeş-Bolyai"*, **14**, 31-45 (1969) (Romanian).

[20] D.D. Stancu, *Curs şi culegere de probleme de analiză numerică*, **I**, Cluj-Napoca, Lito. Univ. "Babeş-Bolyai", (1977) (Romanian).

[21] D.D. Stancu, Gh. Coman, O. Agratini, R. Trîmbiţaş, *Analiză numerică şi teoria aproximării*, **I**, Cluj-Napoca, Presa Universitară Clujeană, (2001) (Romanian).

[22] F. Stancu, *Aproximarea funcţiilor de două şi mai multe variabile cu ajutorul operatorilor liniari pozitivi*, Cluj-Napoca, Ph.D. Thesis, 1984 (Romanian).

(Dan Bărbosu) Department of Mathematics and Computer Science
Faculty of Sciences
North University of Baia Mare
Victoriei 76, 4800 Baia Mare, ROMANIA
E-mail address: dbarbosu@ubm.ro, danbarbosu@yahoo.com