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Pattern recognition based on multiple attribute decision making in intuitionistic fuzzy environment

Ion Iancu

Abstract. In this paper we adapt an algorithm from the multiple attribute decision making field in order to be used in pattern recognition, working with intuitionistic fuzzy sets and intuitionistic fuzzy multi sets. Our method is implemented in two versions: using the score matrix (that is characteristic of decision making problems) and using a similarity measure (that is characteristic of pattern recognition problems). Firstly, the method is built to work with intuitionistic fuzzy sets and after it is extended for intuitionistic fuzzy multi sets. Experimental results demonstrate the superiority of the second versions in pattern recognition problems. For each example we compare our results with those given by other measures whose accuracy has been validated by the respective examples and we conclude that our method can be successfully used in pattern recognition instead of some specific methods in this area.

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1. Introduction

Intuitionistic fuzzy sets (IFSs) have been proposed by Atanassov ([3], [4], [5]) as a generalization of the traditional fuzzy sets introduced by Zadeh in 1965 ([42]). The main advantage of IFSs is the property to incorporate the uncertainty of the information. The IFSs offer a new possibility to represent imperfect knowledge and, therefore, to describe many real problems in a more adequate way. Such problems appear when we face with human opinions involving two or more answers of the type: Yes, No, I do not know, I'm not sure, etc.

Pattern recognition under intuitionistic fuzzy sets environment had been applied to many areas as data analysis, artificial intelligence, and decision making problems. Distance and similarity measures between intuitionistic fuzzy sets play an important role in pattern recognition problems ([9], [24],[25],[26],[41]).

Another important application of IFSs is multiple attribute decision making (MADM). Chen and Tan ([8]) introduced a score function and utilised it and the minimum and maximum operations to develop a technique for handling MADM problem based on IFSs. Another technique for handling MADM problems under complete weight information were developed by Hong and Choi ([13]).

Atanassov et al. ([6]) provided a tool to solve the multi-person multi-attribute decision making problems in which the attribute weights are given as exact numerical values and the alternative score is expressed in intuitionistic fuzzy numbers. Xu and

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Yager ([36]) developed some geometric aggregation operators and gave an application of these operators to MADM based on IFSs.

All the above approaches based on the intuitionistic fuzzy information only consider the situations where the information about attribute weights is completely known. However, in some cases, this information may be completely unknown ([35]). This problem is solved by Xu and Hu ([37]) by development of some entropy-based procedures.

In order to solve MADM problems various types of fuzzy sets can be used: interval intuitionistic fuzzy sets ([23]), interval type-2 fuzzy sets ([12]) , interval intuitionistic trapezoidal fuzzy numbers ([15]) , hesitant fuzzy sets ([40]).

In this paper we change and adapt the technique used by Xu and Hu ([37]) in order to be used in pattern recognition for both IFSs and intuitionistic fuzzy multisets (IFMSs). In this way we show that the MADM technique is of a general nature and can substitute other methods commonly used in applications other than decisions making. The considered numerical examples will demonstrate the superiority of our proposed technique compared to that of Xu and Hu ([37]) if it is used in pattern recognition.

Further, this paper is organized as follows. In the second section we present the basic concepts of intuitionistic fuzzy sets and intuitionistic fuzzy multisets, an intuitionistic fuzzy similarity measure and an entropy based procedure for decision making; all these notions are used in the next sections. In Section 3 we modify the procedure for decision making in order to be used, successfully, in pattern recognition, both IFSs as well as IFMSs. The Section 4 is devoted to numerical experiments; we compare the results given by our procedure with those obtained by standard procedures. The conclusions are discussed in the last section.

2. Preliminary results

2.1. Intuitionistic fuzzy sets. The notion of intuitionistic fuzzy set is defined ([3]) as follows:

Definition 2.1. An IFS A in X is defined as $A = \{(x, \mu_A(x), \nu_A(x))/x \in X\}$ where $\mu_A, \nu_A : X \to [0,1]$ satisfy the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1 \quad \forall x \in X.$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and non-membership of x to A, respectively. Obviously, a fuzzy set A corresponds to the following IFS $A = \{(x, \mu_A(x), 1 - \mu_A(x))/x \in X\}$. For each IFS A in X,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is called the intuitionistic fuzzy index of x in A; it is a hesitancy degree of x to A ([3],[4],[5]) and satisfies the inequality

$$0 \le \pi_A(x) \le 1 \quad \forall x \in X.$$

Therefore, if we want to describe an intuitionistic fuzzy set we must use any two functions from the triplet: (membership function, non-membership function, intuitionistic fuzzy index).

Definition 2.2. A function $T : [0,1]^2 \to [0,1]$ is a t-norm iff it is commutative, associative, non-decreasing and $T(x,1) = x\ \forall x \in [0,1]$.

Definition 2.3. A function $S : [0,1]^2 \to [0,1]$ is a t-conorm iff it is commutative, associative, non-decreasing and $S(x,0) = x\ \forall x \in [0,1]$.

Definition 2.4. A function $N : [0,1] \to [0,1]$ is a strong negation iff it is an involutive and continuous decreasing function from $[0,1]$ to itself.

The relation between the t-norms, t-conorms and negations is given in the next theorem.

Theorem 2.1. ([2]) *If T is a t-norm and N is a strong negation then $S(x,y) = N(T(N(x),N(y)))$ is a t-conorm and reciprocally, $T(x,y) = N(S(N(x),N(y)))$; namely T and S are N-dual.*

T-operators are used in order to define the generalized operations on intuitionistic fuzzy sets ([11]):

Definition 2.5. For two intuitionistic fuzzy sets A and B in X, the generalized intersection and union are defined as:

$$A \cap_{T,S} B = \{(x, T(\mu_A(x), \mu_B(x)), S(\nu_A(x), \nu_B(x)))/x \in X\}$$

$$A \cup_{T,S} B = \{(x, S(\mu_A(x), \mu_B(x)), T(\nu_A(x), \nu_B(x)))/x \in X\}$$

where T denotes a t-norm and S a t-conorm.

For a pair (T,S) of t-operators N-dual with respect to $N(x) = 1-x$, the generalized intersection and union are intuitionistic fuzzy sets ([11]).

2.2. Intuitionistic fuzzy multisets. Consider a finite universal set $X = \{x_1, x_2, ..., x_n\}$. A crisp multiset M of X is an expression such as any element of X can appear more than once in M. For instance, $M = \{a,a,b,c,c,c\}$ is a crisp multiset; anoter notation is $M = \{2/a, 1/b, 3/c\}$.

Definition 2.6. ([38]) Let X be a nonempty set. A fuzzy multiset (FMS) A in X is characterized by the count membership function $MC_A : X \to Q$, where Q is the set of all crisp multisets in $[0,1]$. Hence, for any $x \in X$, $MC_A(x)$ is a crisp multiset from $[0,1]$. A FMS A is given by

$$A = \{(x, (\mu_A^1(x), ..., \mu_A^n(x)))/x \in X\}$$

where the membership sequence

$$(\mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x))$$

satisfies the inequality

$$\mu_A^1(x) \geq \mu_A^2(x) \geq .... \geq \mu_A^n(x).$$

Definition 2.7. ([28]) An intuitionistic fuzzy multiset A in X is characterized by two functions namely count membership function CM_A and count non-membership function CN_A such that $CM_A, CN_A : X \to Q$, where Q is the set of all crisp multisets in $[0,1]$. For any $x \in X$, the membership sequence denoted by

$$(\mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x))$$

is defined as a decreasingly ordered sequence of elements in $CM_A(x)$

$$\mu_A^1(x) \geq \mu_A^2(x) \geq .... \geq \mu_A^n(x)$$

and the corresponding non-membership sequence of elements in $CN_A(x)$ is defined as

$$(\nu_A^1(x), \nu_A^2(x), ..., \nu_A^n(x))$$

such that $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i \in \{1, 2, ..., n\}$.

The membership sequence is arranged in decreasing order, but the corresponding non-membership sequence may not be ordered.

Definition 2.8. Length of an element x in an IFMS A, denoted by $L(x : A)$, is defined as the cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ that is $L(x : A) = |CM_A(x)| = |CN_A(x)|$.

We can make $L(x : A) = L(x : B)$ by appending sufficient number of 0's and 1's with the membership and non-membership values respectively.

2.3. Intuitionistic fuzzy similarity measures. An intuitionistic fuzzy similarity measure $S(A, B)$ serves to match the two IFSs A and B.

Definition 2.9. $S(A, B)$ is said a similarity measure between IFSs if it satisfies the following properties, for every IFSs A, B, C:

(P1) $0 \leq S(A, B) \leq 1$
(P2) $S(A, B) = 1$ if and only if $A = B$
(P3) $S(A, B) = S(B, A)$
(P4) $S(A, C) \leq S(A, B)$ and $S(A, C) \leq S(B, C)$ if $A \subseteq B \subseteq C$.

Various types of similarity measures can be find in the papers ([7],[9],[19],[24],[25],[26],[41]). We will work with the similarity measure defined in [9].

Let x_1 and x_2 be two intuitionistic fuzzy values whose intervals are defined as $[\mu(x_1), 1-\nu(x_1)]$ and $[\mu(x_2), 1-\nu(x_2)]$, respectively. The similarity measure $M(x_1, x_2)$ between x_1 and x_2 is defined as follows ([9])

$$M(x_1, x_2) = ur(x_1, x_2) * ms(x_1, x_2) + (1 - ur(x_1, x_2)) * hs(x_1, x_2)$$

where

$$ur(x_1, x_2) = \frac{min(\pi(x_1), \pi(x_2)) + 1}{max(\pi(x_1), \pi(x_2)) + 1},$$

$$hs(x_1, x_2) = 1 - max(|\mu(x_1) - \mu(x_2)|, |\nu(x_1) - \nu(x_2)|),$$

$$ms(x_1, x_2) = 1 - \frac{|\mu(x_1) - \mu(x_2) + \nu(x_2) - \nu(x_1)|}{2}.$$

Let

$$A = \{(x_i, \mu_A(x_i), \nu_A(x_i))/x_i \in X, 1 \leq i \leq n\}$$

and

$$B = \{(x_i, \mu_B(x_i), \nu_B(x_i))/x_i \in X, 1 \leq i \leq n\}$$

be two intuitionistic fuzzy sets in the universe $X = \{x_1, x_2, ..., x_n\}$; the intuitionistic fuzzy value of the element x_i in the intuitionistic fuzzy set A is represented as $[\mu_A(x_i), 1 - \nu_A(x_i)]$ and in the intuitionistic fuzzy set B is represented as $[\mu_B(x_i), 1 - \nu_B(x_i)]$, where $1 \leq i \leq n$.

The similarity measure $S_{CR}(A,B)$ between the intuitionistic fuzzy sets A and B is defined as ([9])

$$S_{CR}(A,B) = (1 - \frac{(S_M(A,B) - \frac{S_M(A,B)}{n})^2}{n})\frac{S_M(A,B)}{n}$$

with

$$S_M(A,B) = \sum_{i=1}^{n} M([\mu_A(x_i), 1-\nu_A(x_i)], [\mu_B(x_i), 1-\nu_B(x_i)]).$$

2.4. An entropy based procedure for decision making. Given an IFS $A = \{(x_i, \mu_A(x_i), \nu_A(x_i))\}$ we denote $a_i = (\mu_A(x_i), \nu_A(x_i))$; then $s(a_i) = \mu_A(x_i) - \nu_A(x_i) \in [0,1]$ is called ([8]) the score of a_i. We consider a decision making problem characterized by: a finite set of alternatives $A = \{A_1, A_2, ..., A_n\}$, a finite set of attributes $G = \{G_1, G_2, ..., G_m\}$ and a weight vector

$$w = (w_1, w_2, ..., w_m)^T, w_i \geq 0, \sum_{i=1}^{m} w_i = 1$$

of the attributes. Let $R = (r_{ij})_{m\times n}$ be an intuitionistic decision matrix, where $r_{ij} = (\mu_{ij}, \nu_{ij})$, μ_{ij} indicates the degree that the alternative A_j satisfies the attribute G_i and ν_{ij} indicates the degree that the alternative A_j does not satisfy the attribute G_i. For the matrix $R = (r_{ij})_{m\times n}$ we consider its score matrix $S = (s_{ij})_{m\times n}$, where $s_{ij} = s(r_{ij})$, $i \in \{1, 2, ..., m\}$, $j \in \{1, 2, ..., n\}$.

Xu and Hu ([37]) presented an entropy-based procedure for multiple attribute decision making in intuitionistic fuzzy environment; this procedure (called, later on, MADM) involves the following steps.

Step1. The score matrix $S = (s_{ij})_{m\times n}$ is transformed into the normalized score matrix $\overline{S} = (\overline{s}_{ij})_{m\times n}$ with

$$\overline{s}_{ij} = \frac{s_{ij} - min\{s_{ij}/j \in \{1, 2, ..., n\}\}}{max\{s_{ij}/j \in \{1, 2, ..., n\}\} - min\{s_{ij}/j \in \{1, 2, ..., n\}\}}$$

Step2. Normalize each line of $\overline{S}$ and get the normalized matrix $\widehat{S} = (\widehat{s}_{ij})_{m\times n}$ where

$$\widehat{s}_{ij} = \frac{\overline{s}_{ij}}{\sum_{j=1}^{n} \overline{s}_{ij}}, i \in \{1, 2, ..., m\}, j \in \{1, 2, ..., n\}$$

Step3. Calculate the entropy associated with the attribute G_i

$$E_i = -\frac{1}{ln(n)} \sum_{j=1}^{n} \widehat{s}_{ij} ln(\widehat{s}_{ij}), i \in \{1, 2, ..., m\}$$

Step4. The entropy weight with respect to the attribute G_i is computed as

$$w_i = \frac{1 - E_i}{\sum_{i=1}^{m} E_i}, i \in \{1, 2, ..., m\}$$

Step5. The additive weighted averaging (AWA) operator ([18]) is used to fuse the normalized scores $\overline{s}_{ij}$ into overall scores $\overline{s}_j$ of the alternatives A_j

$$\overline{s}_j = \sum_{i=1}^{m} w_i \overline{s}_{ij}, j \in \{1, 2, ..., n\}.$$

The scores $\{\overline{s}_j\}$ are used to order the alternatives $A_j, j \in \{1, 2, ..., n\}$.

3. Pattern recognition based on decision-making techniques

Firstly we adapt the previous procedure to be used in pattern recognition. We consider a set of patterns $A = \{A_1, A_2, ..., A_n\}$ representing intuitionistic fuzzy sets in $X = \{x_1, x_2, ..., x_m\}$; the sets A and X will be interpreted as the set of alternatives and the set of attributes, respectively, from multiple attribute decision making methods.

Assume that a sample

$$B = \{(x_1, \mu_B(x_1), \nu_B(x_1)), ..., (x_m, \mu_B(x_m), \nu_B(x_m))\}$$

is given and we want to determine the pattern A_i, defined as

$$\{(x_1, \mu_{A_i}(x_1), \nu_{A_i}(x_1)), ..., (x_m, \mu_{A_i}(x_m), \nu_{A_i}(x_m))\},$$

which is similar to B. Generally, this problem is solved using distance or similarity measures between two IFSs. Let $R = (r_{ij})_{m\times n}$ be the matrix defined by $r_{ij} = (\mu_{A_j}(x_i), \nu_{A_j}(x_i))$. In the following we propose two possibility to use the MADM procedure:

a) using the score matrix.
Each column from the matrix R is intersected with the sample that must be recognized, obtaining a new matrix R'; then the matrix score S associated with the matrix R' is calculated. Further we use the steps 1 to 5 from MADM procedure and name RSc-MADM this version (Recognition based on Score and MADM procedure).

b) using a similarity measure.
We note $b_i = (\mu_B(x_i), \nu_B(x_i))$ and then the matrix score from MADM procedure is computed as

$$s_{ij} = SM(r_{ij}, b_i), 1 \leq i \leq m, 1 \leq j \leq n$$

where SM is a similarity measure; in our case the similarity measure is computed between two IFSs r_{ij} and b_i with cardinality 1 (named singleton intuitionistic fuzzy sets). Further we use the steps 1 to 5 from MADM procedure and name RSm-MADM this version (Recognition based on Similarity measure and MADM procedure).

In order to extend the RSc-MADM and RSm-MADM procedures for working with intuitionistic fuzzy multi sets we need the following notions. For an intuitionistic fuzzy set $A = \{(x_i, \mu_A(x_i), \nu_A(x_i))/x_i \in X\}$ on the finite universe $X = \{x_1, x_2, ..., x_n\}$ we denote $A_j = (\mu_j, \nu_j), j \in \{1, 2, ..., n\}$ where $\mu_j = \mu_A(x_j)$ and $\nu_j = \nu_A(x_j)$. The set of these singleton intuitionistic fuzzy sets is denoted by F. According to [8] and [13] the score function s is $s(A_j) = \mu_j - \nu_j$ and an accuracy function δ is defined by $\delta(A_j) = \mu_j + \nu_j$. Let A and B be two singleton intuitionistic fuzzy sets. According to their scores and accuracies, the ranking order of A and B is stipulated as follows ([22]):

(1) if $s(A) > s(B)$ then A is greater than B, denoted by $A > B$
(2) if $s(A) < s(B)$ then A is smaller than B, denoted by $A < B$
(3) if $s(A) = s(B)$ then
  (3a) if $\delta(A) = \delta(B)$ then A is equal to B, denoted by $A = B$
  (3b) if $\delta(A) < \delta(B)$ then A is smaller than B, denoted by $A < B$
  (3c) if $\delta(A) > \delta(B)$ then A is greater than B, denoted by $A > B$.

Definition 3.1. ([22]) Let $A_j = (\mu_j, \nu_j), j \in \{1, 2, ..., n\}$, be singleton intuitionistic fuzzy sets. A mapping $f_w^O : F^n \to F$ is called an intuitionistic fuzzy OWA operator if it satisfies

$$f_w^O(A_1, A_2, ..., A_n) = \sum_{i=1}^{n} w_i B_i$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector associated with the mapping f_w^O which satisfies the normalized conditions:

$$w_i \in [0, 1] \quad and \quad \sum_{i=1}^{n} w_i = 1;$$

$B_i = (\overline{\mu}_i, \overline{\nu}_i)$ is the i-th largest of the n intuitionistic fuzzy sets $A_j, j \in \{1, 2, ..., n\}$ which is determined through using some ranking method such as the above scoring function ranking method.

Theorem 3.1. ([22]) *Assumed that $A_j = (\mu_j, \nu_j)$, with $j \in \{1, 2, ..., n\}$ are singleton intuitionistic fuzzy sets. Then, the aggregation result through using the intuitionistic fuzzy OWA operator f_w^O is an intuitionistic fuzzy set and*

$$f_w^O(A_1, A_2, ..., A_n) = (1 - \prod_{i=1}^{n}(1 - \overline{\mu}_i)^{w_i}, \prod_{i=1}^{n} \overline{\nu}_i^{w_i})$$

when $B_i = (\overline{\mu}_i, \overline{\nu}_i)$ is the i-th largest of the n intuitionistic fuzzy sets $A_j, j \in \{1, 2, ..., n\}$ which is determined through using some method of intuitionistic fuzzy sets.

4. Numerical examples

In this section, using the similarity measure presented in the previous section, we apply the two procedures RSc-MADM and RSm-MADM for a set of examples used in various papers to test the correctness of some new similarity measures. For intersection we use the relations from Definition 5 with $T(x, y) = xy$ and $S(x, y) = x + y - xy$. In the next tables we denote by Sc the score calculated in Step5 from the MADM procedure.

4.1. Numerical examples for intuitionistic fuzzy sets. To highlight the superiority of our method we will test it on some examples that use other techniques.
Example 1 ([24]). Assume that there are three patterns denoted with IFSs in $X = \{x_1, x_2, x_3\}$. The three patterns are denoted as follows:

$$A_1 = \{(x_1, 0.3, 0.3), (x_2, 0.2, 0.2), (x_3, 0.1, 0.1)\},$$
$$A_2 = \{(x_1, 0.2, 0.2), (x_2, 0.2, 0.2), (x_3, 0.2, 0.2)\},$$
$$A_3 = \{(x_1, 0.4, 0.4), (x_2, 0.4, 0.4), (x_3, 0.4, 0.4)\}.$$

Assume that a sample

$$B = \{(x_1, 0.3, 0.3), (x_2, 0.2, 0.2), (x_3, 0.1, 0.1)\}$$

is given. To interpret the notions of these patterns, we borrow the idea from [34]. Given three kinds of mineral fields, each is featured by the content of three minerals and contains one kind of typical hybrid minerals. The three kinds of typical hybrid minerals are represented by IFSs A_1, A_2, A_3 in X, respectively.

Given another kind of hybrid mineral B, to which field does this kind of mineral B most probably belong to? From this data, it is evident that $B = A_1$. The RSm-MADM procedure recognizes that B and A_1 are identical (the score is 1) while the RSc-MADM procedure gives an incorrect result (see Table 1).

Table 1. Results for Example 1.

	$Sc(A_1)$	$Sc(A_2)$	$Sc(A_3)$
RSm-MADM	**1.000**	0.551	0.000
RSc-MADM	0.821	0.890	0.000

Example 2 ([24]). Assume that there are three patterns denoted with IFSs in $X = \{x_1, x_2, x_3\}$. The three patterns are denoted as follows:

$$A_1 = \{(x_1, 0.1, 0.1), (x_2, 0.5, 0.1), (x_3, 0.1, 0.9)\},$$

$$A_2 = \{(x_1, 0.5, 0.5), (x_2, 0.7, 0.3), (x_3, 0.0, 0.8)\},$$

$$A_3 = \{(x_1, 0.7, 0.2), (x_2, 0.1, 0.8), (x_3, 0.4, 0.4)\}.$$

Assume that the following sample is given

$$B = \{(x_1, 0.4, 0.4), (x_2, 0.6, 0.2), (x_3, 0.0, 0.8)\}.$$

The RSm-MADM procedure gives the same result as in other works that have analyzed this example ([16], [17], [24], [25]): $B = A_2$ but the RSc-MADM procedure provides an incorrect recognition (see Table 2).

Table 2. Results for Example 2.

	$Sc(A_1)$	$Sc(A_2)$	$Sc(A_3)$
RSm-MADM	0.865	**0.999**	0.000
RSc-MADM	0.358	0.321	0.747

Example 3 ([9], [21], [39]). Assume that there are three patterns denoted with IFSs in $X = \{x_1, x_2, x_3\}$. Three alternatives are denoted as follows:

$$A_1 = \{(x_1, 1, 0.0), (x_2, 0.8, 0.0), (x_3, 0.7, 0.1)\},$$

$$A_2 = \{(x_1, 0.8, 0.1), (x_2, 1, 0.0), (x_3, 0.9, 0.0)\},$$

$$A_3 = \{(x_1, 0.6, 0.2), (x_2, 0.8, 0.0), (x_3, 1, 0.0)\}.$$

Assume that a reference

$$B = \{(x_1, 0.5, 0.3), (x_2, 0.6, 0.2), (x_3, 0.8, 0.1)\}$$

is given.

According to Table 3, the RSm-MADM procedure gives a valid result: $B = A_3$, that is the same as in the cited papers [9], [21], [39] while the RSc-MADM procedure gives the greatest score for A_2.

Table 3. Results for Example 3.

	$Sc(A_1)$	$Sc(A_2)$	$Sc(A_3)$
RSm-MADM	0.651	0.365	**0.657**
RSc-MADM	0.234	0.832	0.210

Example 4 ([20], [25]). We consider the patterns P_1, P_2, P_3 and the sample Q from Refs. 20 and 25, where the membership and non-membership functions are given in graphical form ([25]) or as mathematical expressions defined on interval $[1, 3]$ ([20]). Using these expressions we consider the patterns and the sample denoted with IFSs in $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ with

$$x_1 = 1.3, x_2 = 1.6, x_3 = 1.9, x_4 = 2.2, x_5 = 2.5, x_6 = 2.8$$

The three patterns are denoted as

$$\begin{aligned} A_1 = \{&(x_1, 0.94, 0.0), (x_2, 0.88, 0.0), (x_3, 0.82, 0.0), \\ &(x_4, 0.78, 0.02), (x_5, 0.75, 0.05), (x_6, 0.72, 0.08)\}, \\ A_2 = \{&(x_1, 0.86, 0.07), (x_2, 0.92, 0.04), (x_3, 0.98, 0.01), \\ &(x_4, 0.98, 0.0), (x_5, 0.95, 0.0), (x_6, 0.92, 0.0)\}, \\ A_3 = \{&(x_1, 0.66, 0.14), (x_2, 0.72, 0.08), (x_3, 0.78, 0.02), \\ &(x_4, 0.84, 0.0), (x_5, 0.9, 0.0), (x_6, 0.96, 0.0)\}, \end{aligned}$$

and the sample is represented as the IFS

$$\begin{aligned} B = \{&(x_1, 0.53, 0.27), (x_2, 0.56, 0.24), (x_3, 0.59, 0.21), \\ &(x_4, 0.64, 0.18), (x_5, 0.7, 0.15), (x_6, 0.76, 0.12)\}. \end{aligned}$$

As it results from Table 4, the RSm-MADM procedure gives the same results as in Julian et al. ([20]): the sample Q is similar with the pattern P_3.

Table 4. Results for Example 4.

	$Sc(A_1)$	$Sc(A_2)$	$Sc(A_3)$
RSm-MADM	0.596	0.079	**0.623**
RSc-MADM	0.372	0.915	0.344

Example 5 - Medical recognition. In this example we applied our procedures for cancer pattern recognition. We analyze this problem in the context of colorectal cancer diagnosis as used in other papers ([27], [41], [43]). The patient, who is in the follow-up program, may fall into any of the following states: metastasis, recurrence, bad and well. If the state of a particular patient can be correctly decided, then the state information can be utilized to choose an appropriate treatment. A physician can subjectively judge the belongingness of each patient in the output classes.

Let A be an attributes set of a patient and the main 5 characters (the change of habit and character of stool, bellyache, ictus sileus, chronic sileus, anemia) used to quantify the attribute, respectively denoted x_1, x_2, x_3, x_4, x_5. These characters usually are language variables with values defined as IFSs.

Let a colorectal cancer patient whose 5 characters quantify as

$$B = \{(x_1, 0.3, 0.5), (x_2, 0.4, 0.4), (x_3, 0.6, 0.2), (x_4, 0.5, 0.1), (x_5, 0.9, 0.0)\},$$

and A_1, A_2, A_3 and A_4 are the character sets of the samples denoted metastasis, recurrence, bad (metastasis and recurrence simultaneously) and well:

$$A_1 = \{(x_1, 0.4, 0.4), (x_2, 0.3, 0.3), (x_3, 0.5, 0.1), (x_4, 0.5, 0.2), (x_5, 0.6, 0.2)\},$$

$$A_2 = \{(x_1, 0.2, 0.6), (x_2, 0.3, 0.5), (x_3, 0.2, 0.3), (x_4, 0.7, 0.1), (x_5, 0.8, 0.0)\},$$

$$A_3 = \{(x_1, 0.1, 0.9), (x_2, 0.0, 1.0), (x_3, 0.2, 0.7), (x_4, 0.1, 0.8), (x_5, 0.2, 0.8)\},$$

$$A_4 = \{(x_1, 0.8, 0.2), (x_2, 0.9, 0.0), (x_3, 1, 0.0), (x_4, 0.7, 0.2), (x_5, 0.6, 0.4)\}.$$

From the Table 5 we see that the RSm-MADM procedure gives the same result as those obtained in Refs. 41 and 43: the stage of disease is metastasis. The procedure RSc-MADM indicates "well" which is an incorrect result.

Table 5. Results for Example 5.

	$Sc(A_1)$	$Sc(A_2)$	$Sc(A_3)$	$Sc(A_4)$
RSm-MADM	**0.953**	0.805	0.064	0.337
RSc-MADM	0.656	0.580	0.000	0.905

Example 6 - Medical diagnosis. IFSs have used by many researchers in order to perform medical disgnosis. To test our method, we consider the data consisting of: ([10], [26], [30], [31], [33]) a set of patients $P = \{Bob, Joe, Ted\}$, a set of diagnoses $D = \{Viral\ fever, Malaria, Typhoid, Stomach\ problem, Chest\ problem\}$ and a set of symptoms $S = \{Temperature, Headache, Stomach\ pain, Cough, Chest\ pain\}$.

The Tables 6 and 7 contain the symptoms characteristics for the diagnoses considered and the symptoms for each patient, respectively. Each element of this tables is given as a pair (membership degree μ, non-membership degree ν).

Table 6. Symptoms characteristics for the diagnoses considered.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.4, 0.0)	(0.7, 0.0)	(0.3, 0.3)	(0.1, 0.7)	(0.1, 0.8)
Headache	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)	(0.0, 0.8)
Stomach pain	(0.1, 0.7)	(0.0, 0.9)	(0.2, 0.7)	(0.8, 0.0)	(0.2, 0.8)
Cough	(0.4, 0.3)	(0.7, 0.0)	(0.2, 0.6)	(0.2, 0.7)	(0.2, 0.8)
Chest pain	(0.1, 0.7)	(0.1, 0.8)	(0.1, 0.9)	(0.2, 0.7)	(0.8, 0.1)

Table 7. Symptoms characteristics for the patients considered.

	Temperature	Headache	Stomach pain	Cough	Chest pain
Bob	(0.0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)	(0.1, 0.8)
Joe	(0.8, 0.1)	(0.8, 0.1)	(0.0, 0.6)	(0.2, 0.7)	(0.0, 0.5)
Ted	(0.6, 0.1)	(0.5, 0.4)	(0.3, 0.4)	(0.7, 0.2)	(0.3, 0.4)

As it results from the Tables 8 and 9, the RSm-MADM procedure gives, for all patients, the same results as in Refs. [10], [26], [30], [31], [33] while the RSc-MADM procedure gives incorrect results.

Table 8. Results for Example 6 given by RSm-MADM procedure.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Bob	0.495	0.257	0.579	**0.945**	0.483
Joe	0.695	0.617	**0.761**	0.439	0.299
Ted	**0.870**	0.684	0.530	0.372	0.138

Table 9. Results for Example 6 given by RSc-MADM procedure.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Bob	0.436	0.437	0.382	0.392	0.303
Joe	0.440	0.474	0.319	0.391	0.248
Ted	0.411	0.457	0.306	0.369	0.290

4.2. Numerical examples for intuitionistic fuzzy multiset sets. Using the intuitionistic fuzzy OWA operator, before to apply our proposed procedures, we transform an IFMS

$$A = \{(x, (\mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x)), (\nu_A^1(x), \nu_A^2(x), ..., \nu_A^n(x)))/x \in X\}$$

into an IFS $A' = \{(x, \mu_A(x), \nu_A(x))/x \in X\}$; for this we use the weight vector $w = (0.5, 0.3, 0.2)^T$.

Example 7 ([28]) - Medical diagnosis. Most of human reasoning, such as medical diagnosis, involves the use of linguistic variables. But the description of a linguistic variable in terms of membership function only is not adequate; there is chance of existing of a non-null complement. IFS can be used in this context for representing both membership and non-membership of an element to a set. But there are situations that each element has different membership values and in such situations IFMS is more adequate to be used.

The our aim in this example is to establish the diagnostic for a set of patients considering a set of symptoms. Let $P = \{P1, P2, P3, P4\}$ be the set of patients, $S = \{Temperature, Cough, Throat\ pain, Headache, Body\ pain\}$ be the set of symptoms, $D = \{Viral\ fever, Tuberculosis, Typhoid, Throat\ disease\}$ be the set of diseases.

Only by taking one time inspection it is possible we do not arrive a conclusion that a particular person has a disease or not. Sometimes, a patient may show symptoms of different diseases also.

One solution to establish the disease is to examine the patient at different time intervals. We consider 3 different times in a day: 7 AM, 1 PM and 7 PM. Tables 10 and 11 give the Symptoms characteristics for the diagnoses considered and the Symptoms characteristics for the patients considered. In the Table 11 an IFMS

$$A = \{(\mu_A^1(x), \mu_A^2(x), \mu_A^3(x)), (\nu_A^1(x), \nu_A^2(x), \nu_A^3(x))\}$$

is represented as

$$((\mu_A^1(x), \nu_A^1(x)), (\mu_A^2(x), \nu_A^2(x)), (\mu_A^3(x), \nu_A^3(x)))^T.$$

The IFMS representation from Table 11 is transformed into the IFS representation, given in Table 12.

Table 10. Symptoms characteristics for the diagnoses considered.

	Viral fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.8, 0.1)	(0.2, 0.7)	(0.5, 0.3)	(0.1, 0.7)
Cough	(0.2, 0.7)	(0.9, 0.0)	(0.3, 0.5)	(0.3, 0.6)
Throat pain	(0.3, 0.5)	(0.7, 0.2)	(0.2, 0.7)	(0.8, 0.1)
Headache	(0.5, 0.3)	(0.6, 0.3)	(0.2, 0.6)	(0.1, 0.8)
Body pain	(0.5, 0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.1, 0.8)

Table 11. Symptoms characteristics for the patients considered - IFMS representation.

	Temperature	Cough	Throat pain	Headache	Body pain
P1	(0.6, 0.2)	(0.4, 0.3)	(0.1, 0.7)	(0.5, 0.4)	(0.2, 0.6)
	(0.7, 0.1)	(0.3, 0.6)	(0.2, 0.7)	(0.6, 0.3)	(0.3, 0.4)
	(0.5, 0.4)	(0.4, 0.4)	(0.0, 0.8)	(0.7, 0.2)	(0.4, 0.4)
P2	(0.4, 0.5)	(0.7, 0.2)	(0.6, 0.3)	(0.3, 0.7)	(0.8, 0.1)
	(0.3, 0.4)	(0.6, 0.2)	(0.5, 0.3)	(0.6, 0.3)	(0.7, 0.2)
	(0.5, 0.4)	(0.8, 0.1)	(0.4, 0.4)	(0.2, 0.7)	(0.5, 0.3)
P3	(0.1, 0.7)	(0.3, 0.6)	(0.8, 0.0)	(0.3, 0.6)	(0.4, 0.4)
	(0.2, 0.6)	(0.2, 0.0)	(0.7, 0.1)	(0.2, 0.7)	(0.3, 0.7)
	(0.1, 0.9)	(0.1, 0.7)	(0.8, 0.1)	(0.2, 0.6)	(0.2, 0.7)
P4	(0.5, 0.4)	(0.4, 0.5)	(0.2, 0.7)	(0.5, 0.4)	(0.4, 0.6)
	(0.4, 0.4)	(0.3, 0.3)	(0.1, 0.6)	(0.6, 0.3)	(0.5, 0.4)
	(0.5, 0.3)	(0.4, 0.5)	(0.0, 0.7)	(0.3, 0.6)	(0.4, 0.3)

Table 12. Symptoms characteristics for the patients considered - IFS representation.

	Temperature	Cough	Throat pain	Headache	Body pain
P1	(0.638, 0.162)	(0.381, 0.376)	(0.133, 0.719)	(0.638, 0.259)	(0.334, 0.434)
P2	(0.435, 0.428)	(0.741, 0.141)	(0.536, 0.318)	(0.457, 0.458)	(0.729, 0.153)
P3	(0.151, 0.681)	(0.213, 0.000)	(0.783, 0.000)	(0.252, 0.619)	(0.334, 0.529)
P4	(0.481, 0.346)	(0.352, 0.387)	(0.133, 0.668)	(0.522, 0.376)	(0.452, 0.398)

According to Tables 13 and 14, the RSm-MADM procedure gives the same result as in Ref. [28] for the patients P1, P2 and P4 while for P3 it indicates another disease; for all patients, RSc-MADM procedure indicates an incorrect result.

Table 13. Results for Example 7 given by RSm-MADM procedure.

	Viral fever	Tuberculosis	Typhoid	Throat disease
P1	0.810	0.220	**0.873**	0.118
P2	0.384	**0.802**	0.548	0.195
P3	0.213	0.333	0.722	0.666
P4	0.668	0.303	**0.890**	0.126

Table 14. Results for Example 7 given by RSc-MADM procedure.

	Viral fever	Tuberculosis	Typhoid	Throat disease
P1	0.545	0.736	0.354	0.212
P2	0.538	**0.737**	0.347	0.215
P3	0.551	0.722	0.368	0.214
P4	0.547	0.735	0.354	0.210

Example 8 ([1], [29]) - Multi robot system. The multi robot system ([1]) considered consists of a central controller and four patrolling robots in a large area. Each robot is equipped with ultrasonic sensor, accelerometer sensor, cliff sensor, bump sensor and temperature sensor and is wirelessly controlled by the controller. The controller makes decisions depending upon the sensor readings. For example, if the cliff sensor value in Robot1 indicates the presence of a cliff, the controller can change the commands that are sent to the Robot1; that is, the controller can direct the Robot1 towards the right, left or backward directions. Similar is the case with every other sensor reading.

Let $R = \{R1, R2, R3, R4\}$ be a set of four robots, $C = \{Fire$, $Obstacle$, $Bump$, $Cliff$, $Shock/Vibration\}$ be a set of situations and $S = \{Temperature\ sensor$, $Ultrasonic\ sensor$, $Bump\ sensor$, $Cliff\ sensor$, $Accelerometer\ sensor\}$ be a set of sensors deployed on each Robot. A single robot can be assigned different membership and non-membership values for the five different sensor readings. Whether from a single reading can we conclude what are the situations faced by the robots? The sensor readings from the robots have to be monitored for a particular time, say for three minutes. If for example, the ultrasonic sensor in Robot1 indicates an obstacle, it sends a message to the controller so that the corrective measure could be taken. The controller has to make sure whether the Robot1 is really faced with an obstacle or not. For that purpose, the controller monitors the ultrasonic sensor reading for three minutes. Depending upon the consistency of the readings, the controller identifies the situation.

In Table 15 each sensor reading is described by two numbers: membership μ, non - membership ν. The objective is to make a proper decision for each Robot. Hence the readings are monitored for a particular interval time (3 minutes). Tables 16 and 17 show the sensor readings monitored for 3 minutes, one reading per minute, using IFMS representation and FMS representation, respectively.

Table 15. Description of each sensor reading.

	Fire	Obstacle	Bump	Cliff	Shock/ Vibration
Temperature sensor	(0.8, 0.1)	(0.2, 0.7)	(0.1, 0.7)	(0.2, 0.5)	(0.5, 0.2)
Ultrasonic sensor	(0.2, 0.7)	(0.8, 0.1)	(0.6, 0.3)	(0.2, 0.7)	(0.1, 0.7)
Bump sensor	(0.1, 0.7)	(0.1, 0.7)	(0.9, 0.1)	(0.1, 0.7)	(0.2, 0.5)
Cliff sensor	(0.2, 0.5)	(0.1, 0.7)	(0.1, 0.7)	(0.7, 0.1)	(0.1, 0.7)
Accelerometer sensor	(0.1, 0.7)	(0.2, 0.5)	(0.1, 0.7)	(0.1, 0.7)	(0.8, 0.2)

From Table 18 we see that the RSm-MADM procedure gives the same results as in Ref. [29]: correct situations for the robots R1, R2, R3 and R4 are Obstacle, Shock/Vibration, Bump and Cliff, respectively. The RSc-MADM procedure gives the correct result only for robot 2 (see Table 19).

5. Conclusions

The paper proposes a possibility to unify multiple attribute decision making with pattern recognition. For this, starting from intuitionistic fuzzy decision matrix used in MADM methods, we construct the "score" matrix in two ways: based on the score function and based on a similarity measure.

We apply our procedure to various problems from pattern recognition modeled with intuitionistic fuzzy sets and intuitionistic fuzzy multisets. Comparing our results with those obtained by different similarity measures one obtaines: the RSm-MADM procedure gives correct results in 15 of the 16 cases considered in Section 4 while the

Table 16. Sensor readings monitored for 3 minutes - IFMS representation.

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Acceleromet sensor
R1	(0.8, 0.1)	(0.8, 0.1)	(0.1, 0.9)	(0.2, 0.8)	(0.3, 0.6)
	(0.7, 0.2)	(0.8, 0.1)	(0.2, 0.7)	(0.1, 0.6)	(0.3, 0.4)
	(0.9, 0.0)	(0.9, 0.1)	(0.0, 0.8)	(0.0, 0.7)	(0.4, 0.4)
R2	(0.4, 0.5)	(0.3, 0.7)	(0.1, 0.7)	(0.2, 0.6)	(0.8, 0.1)
	(0.3, 0.4)	(0.2, 0.6)	(0.2, 0.6)	(0.5, 0.4)	(0.7, 0.2)
	(0.6, 0.3)	(0.3, 0.1)	(0.4, 0.4)	(0.2, 0.7)	(0.6, 0.3)
R3	(0.1, 0.8)	(0.6, 0.4)	(0.8, 0.1)	(0.1, 0.9)	(0.2, 0.7)
	(0.2, 0.6)	(0.2, 0.0)	(0.7, 0.1)	(0.2, 0.7)	(0.3, 0.7)
	(0.1, 0.9)	(0.1, 0.7)	(0.8, 0.1)	(0.2, 0.6)	(0.2, 0.7)
R4	(0.1, 0.7)	(0.3, 0.6)	(0.2, 0.7)	(0.8, 0.2)	(0.1, 0.7)
	(0.4, 0.4)	(0.3, 0.3)	(0.1, 0.6)	(0.6, 0.3)	(0.5, 0.4)
	(0.5, 0.3)	(0.4, 0.5)	(0.0, 0.7)	(0.9, 0.0)	(0.4, 0.3)

Table 17. Sensor readings monitored for 3 minutes - IFS representation.

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Acceleromet sensor
R1	(0.847, 0.000)	(0.859, 0.100)	(0.133, 0.775)	(0.113, 0.675)	(0.352, 0.434)
R2	(0.352, 0.485)	(0.271, 0.253)	(0.291, 0.505)	(0.368, 0.505)	(0.741, 0.153)
R3	(0.151, 0.709)	(0.421, 0.000)	(0.783, 0.100)	(0.181, 0.681)	(0.252, 0.700)
R4	(0.406, 0.387)	(0.332, 0.402)	(0.133, 0.668)	(0.838, 0.000)	(0.406, 0.410)

Table 18. Results for Example 8, given by RSm-MADM procedure.

	Fire	Obstacle	Bump	Cliff	Shock/ Vibration
R1	0.585	**0.746**	0.418	0.276	0.398
R2	0.263	0.334	0.282	0.250	**0.777**
R3	0.179	0.510	**1.000**	0.196	0.229
R4	0.284	0.421	0.142	**0.724**	0.269

Table 19. Results for Example 8, given by RSc-MADM procedure.

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
R1	0.199	0.222	0.352	0.281	0.387
R2	0.185	0.231	0.354	0.275	**0.382**
R3	0.189	0.227	0.356	0.276	0.380
R4	0.188	0.228	0.349	0.282	0.389

RSc-MADM procedure gives correct results only in two cases. Therefore, we conclude that the RSm-MADM procedure (based on similarity measures) can be successfully used both in decision making and in pattern recognition.

The method can be improved to be an efficient tool for medical diagnosis and the physician's decision by adding a communication interface in natural language ([14]). In a future paper we intend to compare our method with other computational intelligence algorithms often successfully used for decision making in medicine ([32]).

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(Ion Iancu) Department of Computer Science, University of Craiova, 13 A.I. Cuza Street, Craiova, 200585, Romania
E-mail address: ion.iancu@edu.ucv.ro