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Semipseudosymmetric hyperideals in ternary semihypergroups

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Abstract. The main objective of this study is to introduce a new class of hyperideals called pseudosymmetric and semipseudosymmetric hyperideals in ternary semihypergroups. We try to investigate the properties of these hyperideals by giving some suitable examples for the same. In particular we prove that every pseudosymmetric hyperideal of a ternary semihypergroups is a semipseudosymmetric hyperideal. We study the fundamental relations among these hyperideals and provide few results.

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1. Introduction

Ternary semigroups are universal algebras with one associative operation. The theory of ternary algebraic system was introduced by D. H. Lehmer [13] in 1932. He investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. The notion of ternary semigroups was introduced by S. Banach (cf. [15]). He showed by an example that a ternary semigroup does not necessary reduce to an ordinary semigroup. In 1965, Sioson [28] studied ideal theory in ternary semigroups. He also introduced the notion of regular ternary semigroups and characterized them by using the notion of quasi-ideals. In [8, 9] Dudek et. al. studied the ideals in n-ary semigroups. In 1995, Dixit and Dewan [7] introduced and studied some properties of ideals and quasi-(bi-)ideals in ternary semigroups and in [27, 29, 30], some other results on ternary semigroups are provided.

Anjaneyulu [1, 2] was the first who examined the theory of pseudosymmetric ideals and pseudosymmetric semigroups. In [25, 26] the authors studied the pseudosymmetric ideals in semigroups. In [16, 17, 18] Sarala studied the pseudosymmetric ideals in ternary semigroups.

Hyperstructure theory was introduced in 1934, when F. Marty [20] defined hypergroups based on the notion of hyperoperation, began to analyze their properties and applied them to groups. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians. In a classical algebraic structure, the composition of two elements

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is an element, while in an algebraic hyperstructure, the composition of two elements is a set.

n-ary generalizations of algebraic structures is the most natural way for further development and deeper understanding of their fundamental properties. In [3], Davvaz and Vougiouklis introduced the concept of n-ary hypergroups as a generalization of hypergroups in the sense of Marty. Also, we can consider n-ary hypergroups as a nice generalization of n-ary groups. Davvaz and et. al. in [4] considered a class of algebraic hypersystems which represent a generalization of semigroups, hypersemigroups and n-ary semigroups. Ternary semihypergroups are algebraic structures with one associative ternary hyperoperation and they are a particular case of an n-ary semihypergroup (n-semihypergroup) for $n = 3$ (cf. [3, 4, 5, 6, 14]). Recently, Hila et al. [10, 11], Naka et al. [21, 22, 23, 24] and Mani et al. [19] have studied the ternary semihypergroups in terms of hyperideals and quasi(bi)-hyperideals.

In this article, the notions of pseudosymmetric and semipseudosymmetric hyperideals in ternary semihypergroups are introduced and studied. Some basic properties of them are investigated. Also, some characterizations of semipseudosymmetric hyperideals in ternary semihypergroup are provided. The interrelation between them is examined in ternary semihypergroups extending the related results for semigroups.

2. Preliminaries

Recall first the basic terms and definitions from the hyperstructure theory.

Definition 2.1. A map $\circ : H \times H \to \mathcal{P}^*(H)$ is termed as *hyperoperation* or *join operation* on the set H, where H is a non-empty set and $\mathcal{P}^*(H) = \mathcal{P}(H) \backslash \{\emptyset\}$ denotes the set of all non-empty subsets of H. A *hypergroupoid* is a pair $(H, \circ)$ where $\circ$ is a binary hyperoperation on the set H.

Let $* : \wp^*(H) \times \wp^*(H) \longrightarrow \wp^*(H) | (A, B) \longmapsto A * B = \bigcup\limits_{(a,b)\in A\times B} (a \circ b)$, for every $A, B \in \wp^*(H)$. This operation is well defined [12].

Definition 2.2. [12] A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all x, y, z of H, we have $(x \circ y) * \{z\} = \{x\} * (y \circ z)$, which means that

$$\bigcup_{u\in x\circ y} u \circ z = \bigcup_{v\in y\circ z} x \circ v$$

It is clear that $\{x\} * \{y\} = x \circ y$, for every $x, y \in H$.

A non-empty subset B of a semihypergroup H is termed as *sub-semihypergroup* of H if $B * B \subseteq B$ and H is termed as in this case *super-semihypergroup* of B. Let $(H, \circ)$ be a semihypergroup. Then H is termed as a *hypergroup* if it satisfies the reproduction axiom, for all $a \in H$, $\{a\} * H = H * \{a\} = H$. For the sake of simplicity, in the following writing $a * H$ we mean $\{a\} * H$.

Definition 2.3. A map $f : H \times H \times H \to \mathcal{P}^*(H)$ is termed as *ternary hyperoperation* on the set H, where H is a non-empty set.

A *ternary hypergroupoid* is a pair (H, f) where f is a ternary hyperoperation on the set H. Let

$$\begin{aligned} g : \wp^*(H) \times \wp^*(H) \times \wp^*(H) &\longrightarrow \wp^*(H) \\ (A, B, C) &\longmapsto g(A, B, C) = \bigcup_{(a,b,c) \in A \times B \times C} f(a, b, c), \end{aligned}$$

for every $A, B, C \in \wp^*(H)$. This operation is well defined.

A ternary hypergroupoid (H, f) is termed a *ternary semihypergroup* if for all $a_1, a_2, ..., a_5 \in H$, we have

$$g(f(a_1, a_2, a_3), a_4, a_5) = g(a_1, f(a_2, a_3, a_4), a_5) = g(a_1, a_2, f(a_3, a_4, a_5)). \qquad (*)$$

Since the set $\{x\}$ can be identified with the element x, any ternary semigroup is a ternary semihypergroup. It is clear that due to associative law in ternary semihypergroup (H, f), for any elements $x_1, x_2, ..., x_{2n+1} \in H$ and positive integers m, n with $m \leq n$, one may write

$$f(x_1, x_2, ..., x_{2n+1}) = g(x_1, ..., g(f(x_m, x_{m+1}, x_{m+2}), x_{m+3}, x_{m+4}), ..., x_{2n+1}).$$

Let (H, f) be a ternary semihypergroup. Then H is termed as a *ternary hypergroup* if for all $a, b, c \in H$, there exist $x, y, z \in H$ such that:

$$c \in f(x, a, b) \cap f(a, y, b) \cap f(a, b, z).$$

Let (H, f) be a ternary hypergroupoid. Then,

(1) (H, f) is $(1, 3)$-*commutative* if for all $a_1, a_2, a_3 \in H, f(a_1, a_2, a_3) = f(a_3, a_2, a_1)$;
(2) (H, f) is $(2, 3)$-*commutative* if for all $a_1, a_2, a_3 \in H, f(a_1, a_2, a_3) = f(a_1, a_3, a_2)$;
(3) (H, f) is $(1, 2)$-*commutative* if for all $a_1, a_2, a_3 \in H, f(a_1, a_2, a_3) = f(a_2, a_1, a_3)$;
(4) (H, f) is *commutative* if for all $a_1, a_2, a_3 \in H$ and for all $\sigma \in \mathbb{S}_3, f(a_1, a_2, a_3) = f(a_{\sigma(1)}, a_{\sigma(2)}, a_{\sigma(3)})$.

Let (H, f) be a ternary semihypergroup and T a non-empty subset of H. Then T is termed a *ternary subsemihypergroup* of H if and only if $g(T, T, T) \subseteq T$.

Different examples of ternary semihypergroups can be found in [3]-[6], [10, 11, 14], [16]-[18], [21]-[24].

Let (H, f) be a ternary semihypergroup. An element $e \in H$ is termed as *left identity* element of H if for all $a \in H$, $f(e, a, a) = \{a\}$. An element $e \in H$ is termed an *identity* element of H if for all $a \in H$, $f(a, a, e) = f(e, a, a) = f(a, e, a) = \{a\}$. It is clear that $f(e, e, a) = f(e, a, e) = f(a, e, e) = \{a\}$. A non-empty subset I of a ternary semihypergroup H is termed a *left* (*right, lateral*) *hyperideal* of H if $g(H, H, I) \subseteq I(g(I, H, H) \subseteq I, g(H, I, H) \subseteq I)$. A nonempty subset I of a ternary semihypergroup H is termed a *hyperideal* of H if it is a left, right and lateral hyperideal of H. A non-empty subset I of a ternary semihypergroup H is termed *two-sided hyperideal* of H if it is a left and right hyperideal of H.

For every element $a \in H$, the left, right, lateral, two-sided and hyperideal generated by a are, respectively, given by

$$\begin{aligned} \langle a \rangle_l &= \{a\} \cup g(H, H, a) \\ \langle a \rangle_r &= \{a\} \cup g(a, H, H) \\ \langle a \rangle_m &= \{a\} \cup g(H, a, H) \cup g(H, H, a, H, H) \\ \langle a \rangle_t &= \{a\} \cup g(H, H, a) \cup g(a, H, H) \cup g(H, H, a, H, H) \\ \langle a \rangle &= \{a\} \cup g(H, H, a) \cup g(a, H, H) \cup g(H, a, H) \cup g(H, H, a, H, H) \end{aligned}$$

We shall use the following abbreviated notation: the sequence $x_i, x_{i+1}, \dots, x_j$ will be denoted by x_i^j. For $j < i$, x_i^j is the empty symbol. In this convention,

$$f(x_1, \dots, x_i, y_{i+1}, \dots, y_j, z_{j+1}, \dots, z_m)$$

will be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^m)$, where $m = 2n+1, n \in N$. In the case when $y_{i+1} = ... = y_j = y$, the last expression will be written in the form $f(x_1^i, \overset{(j-i)}{y}, z_{j+1}^m)$.

Similarly, for the subsets $A_1, A_2, \dots, A_m$ of H we define

$$g(A_1^m) = g(A_1, A_2, \dots, A_m) = \bigcup\{f(x_1^m) | x_i \in A_i, i = 1, \dots, m\}.$$

3. Semipseudosymmetric hyperideals

In this section, we study the notions of pseudosymmetric and semipseudosymmetric hyperideals by providing various examples. The interrelations among completely prime, pseudosymmetric and semipseudo symmetric hyperideals were also established.

Definition 3.1. A hyperideal B of a ternary semihypergroup (H, f) is termed as a pseudosymmetric hyperideal, if $u, v, w \in H; f(u, v, w) \subseteq B \Rightarrow f(u, s, v, t, w) \subseteq B$, $\forall s, t \in H$.

Example 3.1. [11] Let Z^- be the set of all negative integers under ternary hyperoperation f defined as follows: $f(x, y, z) = x \cdot y \cdot z \cdot 2N$ where "$\cdot$" is the usual multiplication. Then (Z^-, f) is a ternary semihypergroup. Let $Q = \{6k \in Z^- | k \in Z^-\}$. Then Q is a pseudosymmetric hyperideal of Z^-.

Example 3.2. Let $H = \{0, a, b, c\}$. and $f(x, y, z) = (x \star y) \star z$ for all $x, y, z \in H$; where $\star$ is defined by the table:

$\star$	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	0	b	0
c	0	a	a	c

Then $\{H, f\}$ is a ternary semihypergroup. Clearly $I_1 = \{0\}$, $I_2 = \{0, d\}$, $I_3 = \{0, d, f\}$, $I_4 = \{0, d, e\}$ and $I_5 = H$ are all pseudosymmetric hyperideals.

Definition 3.2. A hyperideal B in a ternary semihypergroup (H, f) is termed as semipseudosymmetric hyperideal if for any odd natural number n, $y \in H$, $f(\overset{(n)}{y}) \subseteq B \Rightarrow g(<\overset{(n)}{y}>) \subseteq B$.

In the following theorem, we investigate the relation between pseudosymmetric and semipseudosymmetric hyperideals in ternary semihypergroups.

Theorem 3.1. *Every pseudosymmetric hyperideal of a ternary semihypergroup is a semipseudosymmetric hyperideal.*

Proof. Let B be a pseudosymmetric hyperideal of a ternary semihypergroup (H, f). Let us assume that $y \in H$ and $f(\overset{(n)}{y}) \subseteq B$ for some odd natural number. If $t \in g(\underbrace{<y>, <y>, <y>, ..., <y>}_{n \text{ times}})$, then $t \in f(s_1, y, s_2, y.....y, s_{n+1})$, where $s_i \in$

$H', i = 1, 2, , ...n+1$. Since B is a pseudosymmetric hyperideal, then $f(\overset{(n)}{y}) \subseteq B$ implies that $f(s_1, y, s_2, y.....y, s_{n+1}) \subseteq B$ and hence $t \in B$. Therefore $g(<\overset{(n)}{y}>) \subseteq B$. Thus we have B is a semipseudosymmetric hyperideal. □

Remark 3.1. We denote by (H', f) the ternary semihypergroup (H, f) with an identity adjoined.

Remark 3.2. The reverse of Theorem 1 does not hold i.e., a semipseudosymmetric hyperideal of a ternary semihypergroup may not be pseudosymmetric hyperideal.

Definition 3.3. Let (H, f) be a ternary semihypergroup. A proper hyperideal P of H is termed as *prime hyperideal* of H if $g(A, B, C) \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ for any three hyperideals A, B, C of H.

Example 3.3. Let $H = \{p, q, r, s, t, u\}$ and $f(a, b, c) = a \star b \star c$ for all $a, b, c \in H$; where $\star$ is defined by the table:

$\star$	p	q	r	s	t	u
p	p	$\{p, q\}$	r	$\{r, s\}$	t	$\{t, u\}$
q	q	q	s	s	u	u
r	r	$\{r, s\}$	r	$\{r, s\}$	r	$\{r, s\}$
s	s	s	s	s	s	s
t	t	$\{t, u\}$	r	$\{r, s\}$	t	$\{t, u\}$
u	u	u	s	s	u	u

Then $\{H, f\}$ is a ternary semihypergroup. Clearly $I_1 = \{r, s\}$, $I_2 = \{r, s, t, u\}$ and H are prime .

Definition 3.4. A proper hyperideal A of a ternary semihypergroup H is termed as *completely semiprime hyperideal* of H if $f(x, x, x) \subseteq A$ implies that $x \in A$ for any element $x \in A$.

Example 3.4. In the commutative ternary semihypergroup Z^- of all negative integers (where the hyperoperation is the usual operation of multiplication), the hyperideal $Q = \{6q : q \in Z^-\}$ is a completely semiprime hyperideal. For $y \in Z^-$, $f(\overset{(3)}{y}) = y \cdot y \cdot y \in Q \Leftrightarrow y^3$ is divisible by $6 \Leftrightarrow y$ is divisible by $6 \Leftrightarrow y = 6q$, for $q \in Z^-$ $\Leftrightarrow y \in Q$.

Definition 3.5. A proper hyperideal A of a ternary semihypergroup H is termed as *semiprime hyperideal* of H if $f(I, I, I) \subseteq A$ implies $I \subseteq A$ for any hyperideal I of H.

Example 3.5. Let $H = \{0, p, q, r, s, t, u\}$ and $f(x, y, z) = a \star b \star c$ for all $a, b, c \in H$ where $\star$ is defined by the table:

$\star$	0	p	q	r	s	t	u
0	0	0	0	0	0	0	0
p	0	p	$\{p, q\}$	r	$\{r, s\}$	t	$\{t, u\}$
q	0	q	q	s	s	u	u
r	0	r	$\{r, s\}$	r	$\{r, s\}$	r	$\{r, s\}$
s	0	s	s	s	s	s	s
t	0	t	$\{t, u\}$	r	$\{r, s\}$	t	$\{t, u\}$
u	0	u	u	s	s	u	u

Then $\{H, f\}$ is a ternary semihypergroup. Clearly $B_1 = \{0\}$, $B_2 = \{0, r, s\}$, $B_3 = \{0, r, s, t, u\}$, $B_4 = \{0, q, s, u\}$ are semiprime.

In the following theorem the relation between semiprime, completely semiprime, pseudosymmetric and semipseudosymmetric hyperideals were established.

Theorem 3.2. *If A is a hyperideal in a ternary semihypergroup (H, f), then the following statements are equivalent.*
(1) *H is completely semiprime.*
(2) *H is semiprime and pseudosymmetric.*
(3) *H is semiprime and semipseudosymmetric.*

Proof. (1) $\Rightarrow$ (2). Let us assume A is a completely semiprime hyperideal of a ternary semihypergroup (H, f). Let $x \in H$ and $g(<\overset{(n)}{x}>) \subseteq A$, for some $n \in N$, where n is odd. Now $f(\underbrace{x, x, x, ...x}_{n \text{ odd terms}}) \subseteq< f(\overset{(n)}{x}) >\subseteq g(<\overset{(n)}{x}>) \subseteq A$. This implies $f(\overset{(n)}{x}) \subseteq A \Rightarrow x \in A \Rightarrow< x >\subseteq A$. Therefore A is a semiprime hyperideal of (H, f). Let A be a completely semiprime hyperideal of the ternary semihypergroup (H, f). Let $u, v, w \in H$ and $f(u, v, w) \subseteq A$. Consider $g(f(v, \overset{(3)}{w}, u)) = g(f(v, w, u), f(v, w, u), f(v, w, u)) \subseteq g(v, w, f(u, v, w), f(u, v, w), u) \subseteq A$. We have $g(f(\overset{(3)}{v}, w, u)) \subseteq A$; A is a completely semiprime hyperideal $\Rightarrow f(v, w, u) \subseteq A$.
Similarly $g(f((\overset{(3)}{w}, u, v)) \subseteq A \Rightarrow f(w, u, v) \subseteq A$. If $s, t \in H'$, then $g(f(u, \overset{(3)}{s}, v, t, w)) = g(f(u, s, v, t, w), f(u, s, v, t, w), f(u, s, v, t, w)) \subseteq g(u, s, v, t, g(w, u, f(s, v, t)),$
$g(f(w, u, s), v, t), w) \subseteq A$. Therefore $g(f(u, s, \overset{(3)}{v}, t, w)) \subseteq A$; A is completely semiprime $\Rightarrow f(u, s, v, t, w) \subseteq A$. Therefore A is a pseudosymmetric hyperideal.

(2) $\Rightarrow$ (3). Let us suppose A is semiprime and pseudosymmetric. By Theorem 3.1, A is semipseudosymmetric hyperideal. Hence A is semiprime and semipseudosymmetric hyperideal.

(3) $\Rightarrow$ (1). Let us suppose A is semiprime and semipseudosymmetric. Let $y \in H$; $f(y, y, y) \subseteq A$. Since A is semipseudosymmetric, then it follows that $g(<\overset{(3)}{y}>) \subseteq A$. Since A is semiprime, then $g(<\overset{(3)}{y}>) \subseteq A \Rightarrow y \in A$. Therefore A is a completely semiprime hyperideal. □

Definition 3.6. An element z of a ternary semihypergroup (H, f) is termed as semisimple if $z \in g(<\overset{(3)}{z}>)$ i.e., $g(<\overset{(3)}{z}>) =< z >$.

Definition 3.7. A ternary semihypergroup H is called semisimple ternary semihypergroup if each element in H is semisimple.

By the The theorem 3.2, it follows

Corollary 3.3. *If I is a hyperideal of a semisimple ternary semihypergroup (H, f), then the following statements are equivalent*
(1) *I is completely semiprime.*
(2) *I is pseudosymmetric.*

(3) *I is semipseudosymmetric.*

Proof. Straightforward. □

Definition 3.8. A hyperideal I of a ternary semihypergroup (H, f) is called completely prime hyperideal of H if $x, y, z \in H$ and $f(x, y, z) \subseteq I$ implies either $x \in I$ or $y \in I$ or $z \in I$.

Example 3.6. [11] Let $H = N$ the set of natural numbers and $\cdot : H \times H \times H \to \mathcal{P}^*(H)$ defined as follows:

$$x \cdot y \cdot z = x + y + z + 5N$$

It can be easily seen that $(H, \cdot)$ is a ternary semihypergroup. Let $Q = \{k \in N | k \geq 5\} \subseteq H$. It can be easily verified that Q is a completely prime hyperideal of H.

Example 3.7. Let $H = \{0, p, q, r\}$. and $f(a, b, c) = (a \star b) \star c$ for all $a, b, c \in H$ where $\star$ is defined by the table:

$\star$	0	p	q	r
0	0	0	0	0
p	0	0	p	0
q	0	0	q	0
r	0	p	0	r

Then $\{H, f\}$ is a ternary semihypergroup. Clearly $A_1 = \{0, p, q\}$, $A_2 = \{0, p, r\}$, $A_3 = H$, are all completely prime hyperideals.

In the folowing theorem the relation between prime, completely prime, pseudosymmetric and semipseudosymmetric hyperideals were established.

Theorem 3.4. *If B is a hyperideal of a ternary semihypergroup (H, f), then the following statements are equivalent:*
(1) *B is completely prime.*
(2) *B is prime and pseudosymmetric.*
(3) *B is prime and semipseudosymmetric.*

Proof. (1) ⇒ (2). Let us suppose B is a completely prime hyperideal of a ternary semihypergroup (H, f). Let $x, y, z \in H$ and $g(< x >, < y >, < z >) \subseteq B$. Then $f(x, y, z) \subseteq B$. As B is a completely prime hyperideal, either $x \in B$ or $y \in B$ or $z \in B$. Thus B is a prime hyperideal of H. Let $x, y, z \in H$ and $f(x, y, z) \subseteq B$. Then $f(x, y, z) \subseteq B$, B is a completely prime hyperideal $\Rightarrow x \in B$ or $y \in B$ or $z \in B$ $\Rightarrow f(x, s, y, t, z) \subseteq B$, $\forall s, t \in H$. Thus B is a pseudosymmetric hyperideal.

(2) ⇒ (3). Let us suppose B is prime and pseudosymmetric. Since B is pseudosymmetric, then by Theorem 3.1, B is a semipseudosymmmetric hyperideal.

(3) ⇒ (1). Let us suppose B is prime and semipseudosymmetric. Let Y be a hyperideal of H such that $H \supseteq f(\overset{(3)}{Y}) \subseteq B$. Since B is prime, then $Y \subseteq B$. Hence B is semiprime. Since B is semiprime and semipseudosymmetric, then by Theorem 3.2, B is completely semiprime. Let $x, y, z \in H$ and $f(x, y, z) \subseteq B \Rightarrow g(< x >, < y >, < z >) \subseteq B \Rightarrow x \in B$ or $y \in B$ or $z \in B$. Hence B is a completely prime hyperideal. □

Theorem 3.5. *Let B be a semipseudosymmetric hyperideal of a ternary semihypergroup (H, f). Then the following sets are equal.*
(1) *$B_1 = \bigcap$ of all completely prime hyperideals of H which contains B.*

(2) $B_1^{'} = \bigcap$ *of all least completely prime hyperideals of* H *which contains* B.
(3) $B_1^{''}$ = *The least completely semiprime hyperideal of* H *relative to containing* B.
(4) $B_2 = \{y \in H/f(\overset{(n)}{y}) \subseteq B$ *for some odd natural number* $n\}$
(5) $B_3 = \bigcap$ *of all prime hyperideals of* H *which contains* B.
(6) $B_3^{'} = \bigcap$ *of all least prime hyperideals of* H *containing* B.
(7) $B_3^{''}$ = *The least semi prime hyperideal of* H *relative to containing* B.
(8) $B_4 = \{y \in H/g(<\overset{(n)}{y}>) \subseteq B$ *for some* $n \in N$ *where* n *is odd* $\}$.

Proof. Since completely prime hyperideals containing B, then the least completely prime hyperideals containing B and least completely semiprime hyperideal relative to containing B are identical, then it follows that $B_1 = B_1^{'} = B_1^{''}$. Since prime hyperideals which contains B, then the least prime hyperideals containing B and the least semiprime hyperideal relative to containing B are identical, then it follows that $B_3 = B_3^{'} = B_3^{''}$. Since B is a semipseudosymmetric hyperideal, then we have $B_2 = B_4$. Now, By Theorem 3.2, we have $B_1^{''} = B_3^{''}$. Therefore $B_1 = B_1^{'} = B_1^{''} = B_3 = B_3^{'} = B_3^{''}$ and $B_2 = B_4$. Hence the given sets are equal. □

Corollary 3.6. *Let* B *be a pseudosymmetric hyperideal of a ternary semihypergroup* (H, f)*, then the below sets are equal.*
(1) $B_1 = \bigcap$ *of all completely prime hyperideals of* H *containing* B.
(2) $B_1^{'} = \bigcap$ *of all least completely prime hyperideals of* H *containing* B.
(3) $B_1^{''}$ = *The least completely semiprime hyperideal of* H *relative to containing* B.
(4) $B_2 = \{y \in T/f(\overset{(n)}{y}) \subseteq B$ *for some* $n \in N$ *where* n *is odd* $\}$
(5) $B_3 = \bigcap$ *of all prime hyperideals of* H *which contains* B.
(6) $B_3^{'} = \bigcap$ *all the least prime hyperideals of* H *containing* B.
(7) $B_3^{''}$ = *The least semiprime hyperideal of* H *relative to containing* B.
(8) $B_4 = \{y \in T/g(<\overset{(n)}{y}>) \subseteq B$ *for some odd natural number* $n\}$.

Proof. By the Theorem 3.1, every pseudosymmetric hyperideal is a semipseudosymmetric hyperideal of H. Hence the proof follows from the Theorem 3.4. □

Theorem 3.7. *If* P *is a maximal hyperideal of a ternary semihypergroup* (H, f) *with* $P \neq H$*, then the following statements are equivalent:*
(1) P *is completely prime.*
(2) P *is completely semiprime.*
(3) P *is pseudosymmetric.*
(4) P *is semipseudosymmetric.*

Proof. (1) ⇒ (2). Let us assume that P is a completely prime hyperideal of a ternary semihypergroup (H, f). Suppose that $z \in H$ and $f(\overset{(3)}{z}) \subseteq P$. Since P is completely prime hyperideal of H, then $z \in P$. Therefore P is s completely semiprime hyperideal.

(2) ⇒ (3). Suppose P is completely semiprime. By Previous Theorem 3.2, P is pseudosymmetric.

(3) ⇒ (4). Suppose P is pseudosymmetric. By Theorem 3.1, P is semipseudosymmetric.

(4) ⇒ (1). Suppose P is semipseudosymmetric. By Theorem 3.3, P is completely prime. □

Definition 3.9. A ternary semihypergroup (H, f) is termed as semipseudosymmetric ternary semihypergroup if every hyperideal of H is a semipseudosymmetric hyperideal.

Theorem 3.8. *A ternary semihypergroup (H, f) is semipseudo symmetric if and only if every principal hyperideal is semipseudosymmetric hyperideal.*

Proof. Suppose a ternary semihypergroup (H, f) is semipseudosymmetric. Then, every hyperideal of H is semipseudosymmetric. Hence each principal hyperideal of H is semipseudosymmetric. Conversely, let us suppose that each principal hyperideal of (H, f) is semipseudosymmetric. Let B be any hyperideal of (H, f). For $y \in H, f(\overset{(n)}{y}) \subseteq B$ for some odd natural number n. Since (H, f) is a semipseudosymmetric hyperideal, then $g(<\overset{(n)}{y}>) \subseteq < f(\overset{(n)}{y}) >$. Now $g(<\overset{(n)}{y}>) \subseteq < f(\overset{(n)}{y}) > \subseteq B$, for $n \in N$, where n is odd. Therefore $g(<\overset{(n)}{y}>) \subseteq B$, for some $n \in N$ where n is odd. Therefore B is a semipseudosymmetric hyperideal. Hence (H, f) is a semipseudosymmetric ternary semihypergroup. □

4. Conclusion

In this paper we have introduced the semipseudosymmetric hyperideals in ternary semihypergroups and different properties of them were investigated. These are useful in order to analyse a new class of ternary semihypergroups. Also, we have introduced the notion of completely semiprime hyperideal and studied analogus results for ternary semihypergroups. Some further work can be done in the future characterizing different classes of TSHG through semipseudosymmetric hyperideals and completely semiprime hyperideals. Concrete examples have been constructed in support of our discussion. In particular we discussed the semipseudosymmetric hyperideals associated with ternary relations.

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