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Properties of derivations in a Semantic Schema

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Abstract. The concept of semantic schema was introduced in [2]. The inference process was modeled there by means of a relation which is named derivation. In this paper we study several properties of the derivations in such a structure. These properties will be used in a future paper to design a knowledge manager based on semantic schemas.

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1. Introduction

The concept of semantic schema was introduced in [2]. We describe there a simple mechanism by which we can represent and process the knowledge. A semantic schema is a tuple of entities, each of which specifying some features of the representation process. This concept is an abstract structure which becomes a real description of a knowledge piece if some interpretation is considered. Various interpretations can be used for the same semantic schema. The concepts and results were applied in a client-server technology, trying to model some aspects concerning the use of the distributed knowledge in the domain of logic programming with constraints ([2]).

Two aspects are relieved in connection with a semantic schema $\mathcal{S}$:

1) A *formal aspect* in $\mathcal{S}$ by which some formal computations in a Peano algebra are obtained.

2) An *evaluation aspect* with respect to some interpretation. The entities obtained in the previous step get values from a space, which is named the *semantic space*.

In this paper we give several algebraic properties for the formal aspect of the computations in a semantic schema. These computations are based on a specific relation defined in a semantic schema and this is named *derivation*. In Section 2 we review this concept. Several algebraic properties of a derivation are given in Section 3. These properties are useful to continue this research work, as we mention in the last section of this paper.

2. Semantic schema

Consider a symbol θ of arity 2 and a finite non-empty set A_0. We denote by $\overline{A}_0$ the Peano θ-algebra ([1]) generated by A_0, therefore $\overline{A}_0 = \bigcup_{n\geq 0} A_n$ where A_n are defined recursively as follows ([1]):

$$A_{n+1} = A_n \cup \{ \ \theta(u, v) \ \mid \ u, v \in A_n\}, \quad n \geq 0 \tag{1}$$

For every $\alpha \in \overline{A_0}$ we define $trace(\alpha)$ as follows:

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(1) if $\alpha \in A_0$ then $trace(\alpha) =< \alpha >$
(2) if $\alpha = \theta(u, v)$ then $trace(\alpha) =< p, q >$, where $trace(u) =< p >$ and $trace(v) =< q >$.

If $E \subseteq A_1 \times \ldots \times A_n$ and $i \in \{1, \ldots, n\}$ then we denote

$$pr_i E = \{x \in A_i \mid \exists (x_1, \ldots, x_{i-1}, x, x_{i+1}, x_n) \in E\}$$

Definition 2.1. *A* **semantic θ-schema** *is a system $\mathcal{S} = (X, A_0, A, R)$ where*
- *X is a finite non-empty set of symbols and its elements are named object symbols*
- *A_0 is a finite non-empty set of elements named label symbols*
- *$A_0 \subseteq A \subseteq \overline{A}_0$, where $\overline{A}_0$ is the Peano θ-algebra generated by A_0*
- *$R \subseteq X \times A \times X$ is a non-empty set which fulfills the following conditions*

$$(x, \theta(u, v), y) \in R \Longrightarrow \exists z \in X : (x, u, z) \in R, (z, v, y) \in R \tag{2}$$

$$\theta(u, v) \in A, (x, u, z) \in R, (z, v, y) \in R \Longrightarrow (x, \theta(u, v), y) \in R \tag{3}$$

$$pr_2 R = A \tag{4}$$

In the remainder of this paper we say shortly θ-schema instead of semantic θ-schema. We denote

$$R_0 = R \cap (X \times A_0 \times X) \tag{5}$$

Let $\mathcal{S} = (X, A_0, A, R)$ be a semantic schema. We consider a symbol h of arity 1, a symbol σ of arity 2 and take the set:

$$M = \{h(x, a, y) \quad | \quad (x, a, y) \in R_0\}$$

We denote by $\mathcal{H}$ the Peano σ-algebra generated by M.

We denote by Z the alphabet which includes the symbol σ, the elements of X, the elements of A, the left and right parentheses, the symbol h and comma. We denote by Z^* the set of all words over Z. As in the case of a rewriting system we define two rewriting rules in the next definition.

Definition 2.2. *Let be $w_1, w_2 \in Z^*$. We define the binary relation $\Rightarrow$ as follows:*
- *If $(x, a, y) \in R_0$ then $w_1(x, a, y)w_2 \Rightarrow w_1 h(x, a, y)w_2$*
- *Let be $(x, \theta(u, v), y) \in R$. If $(x, u, z) \in R$ and $(z, v, y) \in R$ then*

$$w_1(x, \theta(u, v), y)w_2 \Rightarrow w_1 \sigma((x, u, z), (z, v, y))w_2$$

The relation $\Rightarrow$ is named the **direct derivation** *relation over Z^*. We denote by $\Rightarrow^*$ and $\Rightarrow^+$ the reflexive and transitive closure of the relation $\Rightarrow$, respectively the transitive closure. The relation $\Rightarrow^*$ will be called simply the* **derivation** *relation over Z^*.*

Definition 2.3. *For each $w \in Z^*$ where $w = w_1 \ldots w_n$ with $w_i \in Z, i \in \{1, ..., n\}$, $n \geq 1$, we denote $first(w) = w_1$ and $last(w) = w_n$.*

Definition 2.4. *The* **mapping generated** *by $\mathcal{S}$ is the mapping*

$$\mathcal{G}_{\mathcal{S}} : R \longrightarrow 2^{\mathcal{H}}$$

defined as follows:
- *$\mathcal{G}_{\mathcal{S}}(x, a, y) = \{h(x, a, y)\}$ for $a \in A_0$*
- *$\mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y) = \{w \in \mathcal{H} \mid (x, \theta(u, v), y) \Rightarrow^* w\}$*

3. Properties of the derivation relation

Proposition 3.1. *Suppose* $(x, \theta(u, v), y) \in R$. *If* $(x, \theta(u, v), y) \Rightarrow^+ w$ *then:*
i) There is $z \in X$ *such that* $(x, \theta(u, v), y) \Rightarrow \sigma((x, u, z), (z, v, y)) \Rightarrow^* w$
ii) There are α *and* β *such that:*
1) $w = \sigma(\alpha, \beta)$
2) $(x, u, z) \Rightarrow^* \alpha$, $(z, v, y) \Rightarrow^* \beta$

Proof. The assertion $i)$ is obviously true. We verify by induction on $n \geq 1$ that if $\sigma((x, u, z), (z, v, y)) \Rightarrow^n w$ then $ii)$ is true and moreover, $last(\alpha) \in \{)\}$ and $first(\beta) \in \{(, \sigma, h\}$.
For $n = 1$ the following cases can be encountered:
1) $u \in A_0$ and $w = \sigma(h(x, u, z), (z, v, y))$. In that case $\alpha = h(x, u, z)$, $\beta = (z, v, y)$ and $(x, u, z) \Rightarrow h(x, u, z)$.
2) $v \in A_0$ and $w = \sigma((x, u, z), h(z, v, y))$. We have $\alpha = (x, u, z)$, $\beta = h(z, v, y)$ and $(z, v, y) \Rightarrow h(z, v, y)$.
3) $u = \theta(u_1, v_1)$, $w = \sigma(\sigma((x, u_1, z_1), (z_1, v_1, z)), (z, v, y))$, $\alpha = \sigma((x, u_1, z_1), (z_1, v_1, z))$, $\beta = (z, v, y)$ for some $z_1 \in X$.
4) $v = \theta(u_2, v_2)$, $w = \sigma((x, u, z), \sigma((z, u_2, z_2), (z_2, v_2, y)))$, $\alpha = (x, u, z)$, $\beta = \sigma((z, u_2, z_2), (z_2, v_2, y))$ for some $z_2 \in X$.
We observe that the assertion is true for these cases. Suppose the assertion is true for n and consider a derivation:

$$\sigma((x, u, z), (z, v, y)) \Rightarrow^n w_1 \Rightarrow w$$

By the inductive assumption, there are α_1 and β_1 such that
$w_1 = \sigma(\alpha_1, \beta_1)$,
$(x, u, z) \Rightarrow^* \alpha_1$, $(z, v, y) \Rightarrow^* \beta_1$,
$last(\alpha_1) \in \{)\}$ and $first(\beta_1) \in \{(, \sigma, h\}$
We have $w_1 \Rightarrow w$, therefore the following cases can be encountered:
$i_1)$ $\sigma(\alpha_1, \beta_1) = \omega_1(x_1, a, y_1)\omega_2 \Rightarrow \omega_1 h(x_1, a, y_1)\omega_2 = w$, $a \in A_0$
$i_2)$ $\sigma(\alpha_1, \beta_1) = \omega_1(x_1, \theta(u_1, v_1), y_1)\omega_2 \Rightarrow \omega_1\sigma((x_1, u_1, z_1), (z_1, v_1, y_1))\omega_2 = w$ for some $z_1 \in X$
Let us take into consideration the assumption $last(\alpha_1) \in \{)\}$ and $first(\beta_1) \in \{(, \sigma, h\}$. It follows that the word

$$last(\alpha_1), first(\beta_1)$$

can be only one of the following words:

$$), ($$

$$), \sigma$$

$$), h$$

therefore either α_1 is a subword of ω_1 or β_1 is a subword of ω_2.
The following cases are taken into consideration:
$a)$ Suppose α_1 is a subword of ω_1.
From $i_1)$ and $i_2)$ we deduce that (x_1, a, y_1) or $(x_1, \theta(u_1, v_1), y_1)$ is a subword of β_1.

- If (x_1, a, y_1) is a subword of β_1 then $\beta_1 = \mu_1(x_1, a, y_1)\mu_2$ for some words μ_1 and μ_2. In that case, from $i_1)$ we deduce that

$$\sigma(\alpha_1, \beta_1) = \sigma(\alpha_1, \mu_1(x_1, a, y_1)\mu_2) \Rightarrow \sigma(\alpha_1, \mu_1 h(x_1, a, y_1)\mu_2) = w$$

therefore $w = \sigma(\alpha, \beta)$ for $\alpha = \alpha_1$ and $\beta = \mu_1 h(x_1, a, y_1)\mu_2$. But

$$(x, u, z) \Rightarrow^* \alpha_1 \Rightarrow^* \alpha$$

$$(z, v, y) \Rightarrow^* \beta_1 \Rightarrow \beta$$

$last(\alpha) = last(\alpha_1) \in \{)\}$, $first(\beta) = first(\mu_1) = first(\beta_1)$ if μ_1 is a non-empty word and $first(\beta) = h$ if μ_1 is the empty word.

- Let us suppose that $(x_1, \theta(u_1, v_1), y_1)$ is a subword of β_1. In that case we obtain $\beta_1 = \mu_1(x_1, \theta(u_1, v_1), y_1)\mu_2$ and from i_2) we deduce that
  $\sigma(\alpha_1, \beta_1) = \sigma(\alpha_1, \mu_1(x_1, \theta(u_1, v_1), y_1)\mu_2)$
  $\sigma(\alpha_1, \mu_1(x_1, \theta(u_1, v_1), y_1)\mu_2) \Rightarrow \sigma(\alpha_1, \mu_1\sigma((x_1, u_1, z_1), (z_1, v_1, y_1))\mu_2)$
  $\sigma(\alpha_1, \mu_1\sigma((x_1, u_1, z_1), (z_1, v_1, y_1))\mu_2) = w$
  therefore $w = \sigma(\alpha, \beta)$ for $\alpha = \alpha_1$ and $\beta = \mu_1\sigma((x_1, u_1, z_1), (z_1, v_1, y_1))\mu_2$.

b) Suppose now that β_1 is a subword of ω_2. ¿From i_1) and i_2) we deduce that (x_1, a, y_1) or $(x_1, \theta(u_1, v_1), y_1)$ is a subword of α_1. Suppose that (x_1, a, y_1) is a subword of α_1, therefore $\alpha_1 = \mu_1(x_1, a, y_1)\mu_2$. From i_1) we deduce that $\sigma(\alpha_1, \beta_1) = \sigma(\mu_1(x_1, a, y_1)\mu_2, \beta_1) \Rightarrow \sigma(\mu_1 h(x_1, a, y_1)\mu_2, \beta_1) = w$, therefore $w = \sigma(\alpha, \beta)$ for $\alpha = \mu_1 h(x_1, a, y_1)\mu_2$ and $\beta = \beta_1$. But $(x, u, z) \Rightarrow^* \alpha_1$ and $\alpha_1 \Rightarrow \alpha$, therefore $(x, u, z) \Rightarrow^* \alpha$. We have also $(z, v, y) \Rightarrow^* \beta_1$ and $\beta_1 = \beta$, therefore $(z, v, y) \Rightarrow^* \beta$. In addition, $first(\beta) = first(\beta_1)$ and $last(\alpha) \in \{)\}$ if μ_2 is the empty word. If μ_2 is a non-empty word, then $last(\alpha) = last(\mu_2) = last(\alpha_1)$.

Thus the proposition is proved. □

Proposition 3.2. *If $(x, u, y) \Rightarrow^+ \alpha$ and $\alpha \in (\{\sigma\} \cup M)^*$ then $\alpha \in \mathcal{H}$.*

Proof. We prove by induction on n that if $(x, u, y) \Rightarrow^n \alpha$ and $\alpha \in (\{\sigma\} \cup M)^*$ then $\alpha \in \mathcal{H}$. We verify this property for n=1. If $(x, u, y) \Rightarrow \alpha$ then two cases are possible:

1) $u \in A_0$ and $\alpha = h(x, u, y)$. In that case we have $\alpha \in \mathcal{H}$.

2) $u \in A \setminus A_0$, therefore $u = \theta(u_1, v_1)$. In that case $\alpha = \sigma((x, v_1, z_1), (z_1, v_2, y))$ for some $z_1 \in X$. This case is not possible because $\alpha \notin (\{\sigma\} \cup M)^*$.

Suppose the assertion is true for $n \in \{1, ..., k\}$ and take a derivation $(x, u, y) \Rightarrow^{k+1} \alpha$ such that $\alpha \in (\{\sigma\} \cup M)^*$. Because $k+1 \geq 2$ and $\alpha \in (\{\sigma\} \cup M)^*$ we have $u = \theta(v_1, v_2)$ for some $v_1, v_2 \in A$. Really, if by contrary we suppose that $u \in A_0$ then we have:

$$(x, u, y) \Rightarrow h(x, u, y) \Rightarrow^k h^k(x, u, y) = \alpha$$

therefore $\alpha \notin (\{\sigma\} \cup M)^*$.

The derivation $(x, u, y) \Rightarrow^{k+1} \alpha$ can be written as follows:

$$(x, \theta(v_1, v_2), y) \Rightarrow \sigma((x, v_1, z), (z, v_2, y)) \Rightarrow^k \alpha$$

for some $z \in X$. Applying Proposition 3.1 we deduce that there are β_1, β_2 such that $(x, v_1, z) \Rightarrow^* \beta_1$, $(z, v_2, y) \Rightarrow^* \beta_2$ and $\alpha = \sigma(\beta_1, \beta_2)$. Because $\alpha \in (\{\sigma\} \cup M)^*$ we have $\beta_1, \beta_2 \in (\{\sigma\} \cup M)^*$. Applying the inductive assumption we have $\beta_1, \beta_2 \in \mathcal{H}$, therefore $\alpha = \sigma(\beta_1, \beta_2) \in \mathcal{H}$. □

Proposition 3.3. *Suppose that $w \in \mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y)$ and denote by α and β those elements of $\mathcal{H}$, uniquely determined, such that $w = \sigma(\alpha, \beta)$. There is $z \in X$, such that*

$(x, \theta(u, v), y) \Rightarrow \sigma((x, u, z), (z, v, y)) \Rightarrow^* w$

$\alpha \in \mathcal{G}_{\mathcal{S}}(x, u, z)$ *and* $\beta \in \mathcal{G}_{\mathcal{S}}(z, v, y)$

Proof. We have $(x, \theta(u, v), y) \Rightarrow^+ w$ and $w \in \mathcal{H}$ because $w \in \mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y)$. But $\mathcal{H}$ is a Peano σ-algebra, therefore w is written as $w = \sigma(\alpha, \beta)$ for $\alpha, \beta \in \mathcal{H}$ uniquely determined. By Proposition 3.1 there is $z \in X$ such that:

$$(x, \theta(u, v), y) \Rightarrow \sigma((x, u, z), (z, v, y)) \Rightarrow^* w$$

and there are α_1, β_1 such that $w = \sigma(\alpha_1, \beta_1)$, $(x, u, z) \Rightarrow^* \alpha_1$, $(z, v, y) \Rightarrow^* \beta_1$. By Proposition 3.2 we obtain $\alpha_1 \in \mathcal{H}$ and $\beta_1 \in \mathcal{H}$. But $w = \sigma(\alpha, \beta) = \sigma(\alpha_1, \beta_1)$, where $\alpha, \beta, \alpha_1, \beta_1 \in \mathcal{H}$. By the property of the Peano σ-algebra $\mathcal{H}$, we have $\alpha = \alpha_1$ and $\beta = \beta_1$. In conclusion, the proposition is proved. □

Remark 3.1. *Finally we shall prove that just one element z satisfies the conditions of the previous proposition.*

Proposition 3.4. *If $(x, u, z) \Rightarrow^* \alpha$ and $(z, v, y) \Rightarrow^* \beta$ then $\sigma((x, u, z), (z, v, y)) \Rightarrow^* \sigma(\alpha, \beta)$*

Proof. There are the following derivations:
$(x, u, z) \Rightarrow \omega_1 \Rightarrow \omega_2 \Rightarrow \ldots \Rightarrow \omega_k \Rightarrow \alpha$
$(z, v, y) \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \ldots \Rightarrow w_r \Rightarrow \beta$
We know that if $\mu \Rightarrow \nu$ is a direct derivation and $w \in Z^*$ then $w\mu \Rightarrow w\nu$ and $\mu w \Rightarrow \nu w$. Based on this property we obtain the following derivations:
$\sigma((x, u, z), (z, v, y)) \Rightarrow \sigma(\omega_1, (z, v, y)) \Rightarrow \ldots \Rightarrow \sigma(\alpha, (z, v, y))$
$\sigma(\alpha, (z, v, y)) \Rightarrow \sigma(\alpha, w_1) \Rightarrow \ldots \Rightarrow \sigma(\alpha, \beta)$
and the proposition is proved. □

Corollary 3.1.

$$\mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y) = \bigcup_{z \in X} \mathcal{G}_{\mathcal{S}}(x, u, z) \otimes_\sigma \mathcal{G}_{\mathcal{S}}(z, v, y)$$

where $P \otimes_\sigma Q = \{\sigma(u, v) \mid u \in P, v \in Q\}$.

Proof. By Proposition 3.3, if $w \in \mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y)$ then $w \in \mathcal{G}_{\mathcal{S}}(x, u, z) \otimes_\sigma \mathcal{G}_{\mathcal{S}}(z, v, y)$. Conversely, consider $w = \sigma(\alpha, \beta)$, where $\alpha \in \mathcal{G}_{\mathcal{S}}(x, u, z)$ and $\beta \in \mathcal{G}_{\mathcal{S}}(z, v, y)$.

It follows that $(x, u, z) \Rightarrow^* \alpha$, $(z, v, y) \Rightarrow^* \beta$ and $\alpha \in \mathcal{H}$, $\beta \in \mathcal{H}$. On the other hand, if $(x, u, z) \Rightarrow^* \alpha$ and $(z, v, y) \Rightarrow^* \beta$ then

$$\sigma((x, u, z), (z, v, y)) \Rightarrow^* \sigma(\alpha, \beta) = w \tag{6}$$

as is stated in Proposition 3.4. But $\theta(u, v) \in A$, $(x, \theta(u, v), y) \in R$, $(x, u, z) \in R$ and $(z, v, y) \in R$. It follows that:

$$(x, \theta(u, v), y) \Rightarrow \sigma((x, u, z), (z, v, y))$$

therefore using (6) we deduce $(x, \theta(u, v), y) \Rightarrow^* w$. We recall that $\alpha, \beta \in \mathcal{H}$ and $w = \sigma(\alpha, \beta)$, therefore $w \in \mathcal{H}$. In this way we have $w \in \mathcal{G}_{\mathcal{S}}(x, \theta(u, v), y)$ and the proposition is proved. □

Definition 3.1. *We define:*
$H(h(x, a, y)) = < h(x, a, y) >$ *for* $h(x, a, y) \in M$
$H(\sigma(\alpha, \beta)) = < p, q >$, *where* $H(\alpha) = < p >$ *and* $H(\beta) = < q >$, $\sigma(\alpha, \beta) \in \mathcal{H}$, $\alpha \in \mathcal{H}$, $\beta \in \mathcal{H}$.

Proposition 3.5. *Let be $u \in A$ such that $trace(u) = < a_1, \ldots, a_n >$. For every $\alpha \in \mathcal{G}_{\mathcal{S}}(x_1, u, z_1)$ there are $y_1, \ldots, y_{n-1} \in X$ such that $H(\alpha) = < h(x_1, a_1, y_1), h(y_1, a_2, y_2), \ldots, h(y_{n-1}, a_n, z_1) >$ for $n \geq 2$ and $H(\alpha) = < h(x_1, u, z_1) >$ for $n = 1$.*

Proof. We proceed by induction on n. For n=1 we have $trace(u) =< a_1 >$, therefore $u = a_1 \in A_0$. If α is an arbitrary element of $\mathcal{G}_\mathcal{S}(x_1, u, z_1)$ then $(x_1, u, z_1) \Rightarrow^* \alpha$ and $\alpha \in \mathcal{H}$. This derivation is a direct one, that is $(x_1, u, z_1) \Rightarrow h(x_1, u, z_1) = \alpha$. It follows that $H(\alpha) =< h(x_1, u, z_1) >$ and the property is verified for n=1.

Consider $k \geq 1$ and suppose the proposition is true for $n \in \{1, \ldots, k\}$. Take an element $u \in A$ such that $trace(u) =< a_1, \ldots, a_{k+1} >$. There is $u_1, v_1 \in A$ such that $u = \theta(u_1, v_1)$. Take an element $\alpha \in \mathcal{G}_\mathcal{S}(x_1, u, z_1) = \mathcal{G}_\mathcal{S}(x_1, \theta(u_1, v_1), z_1)$. By Corollary 3.1 we deduce that there is $z \in X$ such that $\alpha = \sigma(\alpha_1, \beta_1)$, where $\alpha_1 \in \mathcal{G}_\mathcal{S}(x_1, u_1, z)$ and $\beta_1 \in \mathcal{G}_\mathcal{S}(z, v_1, z_1)$. We use the inductive assumption. Because $u = \theta(u_1, v_1)$ and $trace(u) =< a_1, \ldots, a_{k+1} >$, it follows that there is $i \in \{1, \ldots, k\}$ such that $trace(u_1) =< a_1, \ldots, a_i >$ and $trace(v_1) =< a_{i+1}, \ldots, a_{k+1} >$.

By the inductive assumption we have the following properties:

1) there are $y_1, \ldots, y_{i-1} \in X$ such that $H(\alpha_1) =< h(x_1, a_1, y_1), h(y_1, a_2, y_2), \ldots, h(y_{i-1}, a_i, z) >$

2) there are $t_1, \ldots, t_{k-i} \in X$ such that $H(\beta_1) =< h(z, a_{i+1}, t_1), h(t_1, a_{i+2}, t_2), \ldots, h(t_{k-i}, a_{k+1}, z_1) >$

But $\alpha = \sigma(\alpha_1, \beta_1)$, therefore $H(\alpha)$ is the following system:

$$< h(x_1, a_1, y_1), h(y_1, a_2, y_2), \ldots, h(y_{i-1}, a_i, z), h(z, a_{i+1}, t_1), \ldots, h(t_{k-i}, a_{k+1}, z_1) >$$

and the proposition is proved. □

Corollary 3.2. *If $\mathcal{G}_\mathcal{S}(x_1, u, z_1) \cap \mathcal{G}_\mathcal{S}(x_2, v, z_2) \neq \emptyset$ then $x_1 = x_2$, $trace(u) = trace(v)$ and $z_1 = z_2$.*

Proof. If $\alpha \in \mathcal{G}_\mathcal{S}(x_1, u, z_1) \cap \mathcal{G}_\mathcal{S}(x_2, v, z_2)$ and $trace(u) =< a_1, \ldots, a_n >$, $trace(v) =< b_1, \ldots, b_k >$ then by Proposition 3.5 there are $y_1, \ldots, y_{n-1}, t_1, \ldots, t_{k-1} \in X$ such that:

$H(\alpha) =< h(x_1, a_1, y_1), h(y_1, a_2, y_2), \ldots, h(y_{n-1}, a_n, z_1) >$

$H(\alpha) =< h(x_2, b_1, t_1), h(t_1, b_2, t_2), \ldots, h(t_{k-1}, b_k, z_2) >$

therefore $n = k$, $a_1 = b_1$, …, $a_n = b_k$, $x_1 = x_2$, $y_1 = t_1$, …, $y_{n-1} = t_{k-1}$ and $z_1 = z_2$. Thus, $x_1 = x_2$, $trace(u) = trace(v)$ and $z_1 = z_2$. □

Corollary 3.3. *The element $z \in X$ from Proposition 3.3 is uniquely determined.*

Proof. If $\alpha \in \mathcal{G}_\mathcal{S}(x, u, z_1) \cap \mathcal{G}_\mathcal{S}(x, u, z_2)$ then $z_1 = z_2$ by Corollary 3.2. □

4. Open problems

The following open problems are relieved:

- Study the case when the component A of a semantic schema is an infinite set. The corresponding set R is also an infinite set. Give an example of knowledge piece which can be modeled by such structures.
- Embed two distinct semantic schemas in a semantic schema.
- Introduce a partial order between two semantic schemas and find the least semantic schema which contains some semantic schemas.
- Combine two semantic schemas such that the reasoning by analogy can be performed.
- Design a knowledge manager which uses the previous concepts and can process the distributed knowledge.

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