Annals of the University of Craiova, Mathematics and Computer Science Series
Volume 53(1), 2026, Pages 163–180, DOI: 10.52846/ami.v53i1.2129
ISSN: 1223-6934

Comparative Analysis of CFKHDM and CFTSM for the Solutions of Time-fractional Newell-Whitehead-Segel Equations

Ali Khalouta

Abstract. Recently, the author presented an efficient technique to obtain a solution to the Newell-Whitehead-Segel equation of fractional order where the fractional derivative is in the sense of Caputo-Fabrizio [14]. The objective of this paper is to explore a novel analytical methods for solving the same equation but involving a conformable fractional derivative operator. The methods are denoted as the conformable fractional Khalouta decomposition method (CFKHDM) and the conformable fractional Taylor series method (CFTSM). Additionally, a comparison between CFKHDM and CFTSM is performed. The accuracy and efficiency of the proposed methods are demonstrated by solving three numerical tests of the conformable time-fractional Newell-Whitehead-Segel equation. All numerical calculations were performed using the MATLAB 6 software package. These tests show that CFKHDM and CFTSM are simple, efficient and very powerful methods for finding an analytical solution to such equations. Accordingly, we believe that in the future, the current methods can be applied to compute approximate analytical solutions for a large class of conformable fractional partial differential equations.

2020 *Mathematics Subject Classification.* Primary 35R11, 26A33; Secondary 74G10, 34K28.
Key words and phrases. Fractional Newell-Whitehead-Segel equation, Conformable fractional derivative, Conformable fractional Khaouta transform, Adomian decomposition method, Taylor series method, analytical method.

1. Introduction

In recent decades, the theory of fractional differential equations has played an important role in many practical applications, being largely developed to describe many real-world problems that cannot be covered in the classical mathematical literature. For this reason, many researchers have provide may definitions of fractional derivative operators and applied them to study solutions of fractional differential equations, for example Riemann–Liouville [18], Liouville-Caputo [15], Hadamard [16], Hilfer [10], Caputo–Fabrizio [9] and Atangana–Baleanu [6].

In 2014, Khalil introduced a novel fractional derivative operator, known as conformable fractional derivative [12], which contains many features that cannot be satisfied with existing fractional derivative operators.

Recently, many authors have focused on developing numerical and analytical methods for conformable fractional differential equations. For instance, conformable fractional Laplace transform method (CFLTM) [19], conformable fractiona natural transform method (CFNTM) [4], Shehu conformable fractional transform method (SCFTM)

Received March 14, 2025. Accepted December 27, 2025.

[7], conformable fractional power series method (CFPSM) [17], conformable fractional Sumudu decomposition method (CFSDM) [8], conformable fractional reduced differential transform method (CFRDTM) [20], conformable fractional variational iteration method (CFVIM) [21], conformable homotopy analysis method (CHAM) [2], conformable sine-Gordon expansion method (CSGEM) [11].

This work proposes two new analytical methods: the conformable fractional Khalouta decomposition method (CFKHDM) and the conformable fractional Taylor series method (CFTSM). The two proposed approaches are used to solve nonlinear conformable frational partial differential equations without perturbation or discretizing them, with less computation, which leads to a more realistic representation. The main motivation of this paper is to perform a comparative analysis for CFKHDM and CFTSM solutions for the following conformable time-fractional Newell-Whitehead-Segel equation

$$\mathcal{T}_{\varsigma}^{\varpi}\left(\xi\right)\left(\varrho,\varsigma\right)=a\xi_{\varrho\varrho}(\varrho,\varsigma)+b\xi(\varrho,\varsigma)+c\xi^{p}(\varrho,\varsigma), \tag{1}$$

subject to

$$\xi(\varrho,0)=\xi_{0}(\varrho), \tag{2}$$

where $\mathcal{T}_{\varsigma}^{\varpi}$ is the conformable time-fractional derivative operator of order $0<\varpi\leq 1$ for the function $\xi(\varrho,\varsigma)$ with $(\varrho,\varsigma)\in\mathbb{R}\times\mathbb{R}^{+}$, a, b and c are real numbers with $a, b>0$ and p is a positive integer.

The framework of this paper is as follows. In the second part, we introduce the definition and associated properties of the conformable fractional derivative and the conformable fractional Khalouta transform. In the third and fourth parts, we present the basic idea of CFKHDM and CFTSM to solve the equations (1)-(2). To demonstrate the effectiveness and accuracy of the proposed methods, three numerical tests are presented in the fifth part using the results of the third and fourth parts. The numerical simulation results are presented and discussed in the sixth part. Finally, in the seventh part, we conclude this paper with some remarks.

2. Definitions and properties

This part recalls the definition and the related properties of the conformable fractional derivative and the conformable fractional Khalouta transform.

Definition 2.1. [12] Let $0<\varpi\leq 1$ and $\xi:[0,+\infty)\longrightarrow\mathbb{R}$. The conformable fractional derivative of the function ξ with order ϖ, is defined as

$$\mathcal{T}^{\varpi}(\xi)(\varsigma)=\frac{d^{\varpi}}{d\varsigma^{\varpi}}\xi(\varsigma)=\lim_{\varepsilon\to 0}\frac{\xi(\varsigma+\varepsilon\varsigma^{1-\varpi})-\xi(\varsigma)}{\varepsilon}.$$

Moreover, if the function $\xi(\varsigma)$ admits the conformable fractional derivative with order ϖ in the set $(0,+\infty)$, we call that $\xi(\varsigma)$ is $\varpi-$differentiable in the set $(0,+\infty)$. If $\xi(\varsigma)$ is $\varpi-$differentiable in the set $(0,+\infty)$ and $\lim_{\varsigma\to 0^{+}}\mathcal{T}^{\varpi}\left(\xi\right)\left(\varsigma\right)$ exists, then we define

$$\mathcal{T}^{\varpi}\left(\xi\right)\left(0\right)=\lim_{\varsigma\to 0^{+}}\mathcal{T}^{\varpi}\left(\xi\right)\left(\varsigma\right).$$

Theorem 2.1. [5] *Let $0<\varpi\leq 1$ and $\xi,\vartheta:[0,+\infty)\longrightarrow\mathbb{R}$ be $\varpi-$differentiable at $\varsigma>0$. Then*

1) $\mathcal{T}^{\varpi}(\lambda\xi+\mu\vartheta)(\varsigma)=\lambda\mathcal{T}^{\varpi}\left(\xi\right)\left(\varsigma\right)+\mu\mathcal{T}^{\varpi}\left(\vartheta\right)\left(\varsigma\right)$, $\forall\lambda,\mu\in\mathbb{R}$.

2) $\mathcal{T}^{\varpi}(C)=0$, $\forall C\in\mathbb{R}$.

3) $\mathcal{T}^{\varpi}(\varsigma^{q}) = q\varsigma^{q-\varpi}, \forall\ q \in \mathbb{R}$.
4) $\mathcal{T}^{\varpi}(\xi\vartheta)(\varsigma) = \xi(\varsigma)\mathcal{T}^{\varpi}(\vartheta)(\varsigma) + \vartheta(\varsigma)\mathcal{T}^{\varpi}(\xi)(\varsigma)$.
5) $\mathcal{T}^{\varpi}(\xi/\vartheta)(\varsigma) = (\vartheta(\varsigma)\mathcal{T}^{\varpi}(\xi)(\varsigma) - \xi(\varsigma)\mathcal{T}^{\varpi}(\vartheta)(\varsigma))/\vartheta^{2}(\varsigma), \vartheta(\varsigma) \neq 0$.
6) If ξ *is* $\varpi-$*differentiable, then* $\mathcal{T}^{\varpi}(\xi)(\varsigma) = \varsigma^{1-\varpi}\frac{d}{d\varsigma}\xi(\varsigma)$.

Theorem 2.2. [5] *Let* $0 < \varpi \leq 1$ *and* $\xi, \vartheta : [0, +\infty) \longrightarrow \mathbb{R}$ *be* $\varpi-$*differentiable functions. Let* $\phi(\varsigma) = \xi(\vartheta(\varsigma))$, *then* $\phi(\varsigma)$ *is* $\varpi-$*differentiable and for all* ς *with* $\varsigma \neq 0$ *and* $\vartheta(\varsigma) \neq 0$, *we have*

$$\mathcal{T}^{\varpi}(\phi)(\varsigma) = \mathcal{T}^{\varpi}(\xi)(\vartheta(\varsigma)).\mathcal{T}^{\varpi}(\vartheta)(\varsigma).\vartheta(\varsigma)^{\varpi-1}.$$

Definition 2.2. [1] Let $n-1 < \varpi \leq n$ with $n \in \mathbb{N}^{*}$ and $\partial^{k}\xi/\partial\varsigma^{k}$ and $\partial^{k}\xi/\partial\varrho^{k}$ defined on $\mathbb{R}\times[0, +\infty)$ for $k = 1, 2, ..., n-1$. Then, the conformable time-fractional derivative of the function ξ with order ϖ, is defined as

$$\mathcal{T}_{\varsigma}^{\varpi}(\xi)(\varrho, \varsigma) = \frac{\partial^{\varpi}}{\partial\varsigma^{\varpi}}\xi(\varrho, \varsigma) = \lim_{\varepsilon\to 0}\frac{\xi_{\varsigma}^{(n-1)}(\varrho, \varsigma + \varepsilon\varsigma^{n-\varpi}) - \xi_{\varsigma}^{(n-1)}(\varrho, \varsigma)}{\varepsilon}.$$

Definition 2.3. [13] Let $0 < \varpi \leq 1$ and $\xi : [0, +\infty) \longrightarrow \mathbb{R}$ be a real value function. Thus, the conformable fractional Khalouta transform of order ϖ is defined on the set

$$\mathcal{S}_{\varpi} = \left\{\xi(\varsigma) : \exists K, \vartheta_{1}, \vartheta_{2} > 0, |\xi(\varsigma)| < K\exp\left(\varpi\vartheta_{j}\left|\varsigma^{\varpi}\right|\right),\ \text{if } \varsigma^{\varpi} \in (-1)^{j}\times[0, \infty)\right\},$$

by the following integral

$$\mathbb{KH}_{\varpi}[\xi(\varsigma)] = \mathcal{K}_{\varpi}(s, \gamma, \eta) = \frac{s}{\gamma\eta}\int_{0}^{\infty}\exp\left(-\frac{s\varsigma^{\varpi}}{\gamma\eta\varpi}\right)\xi(\varsigma)\varsigma^{\varpi-1}d\varsigma.$$

where $s > 0, \gamma > 0$ and $\eta > 0$ are the Khalouta transform variables.

Theorem 2.3. [13] *Assuming that* λ, μ *and* $\nu \in \mathbb{R}$ *and* $0 < \varpi \leq 1$, *we have the following properties*

1) Linearity property.

$$\mathbb{KH}_{\varpi}[\lambda\xi(\varsigma) \pm \mu\zeta(\varsigma)] = \lambda\mathbb{KH}_{\varpi}[\xi(\varsigma)] \pm \mu\mathbb{KH}_{\varpi}[\zeta(\varsigma)].$$

2) Convolution property.

$$\mathbb{KH}_{\varpi}[(\xi * \zeta)(\varsigma)] = \frac{\gamma\eta}{s}\mathbb{KH}_{\varpi}[\xi(\varsigma)]\,\mathbb{KH}_{\varpi}[\zeta(\varsigma)],$$

where $\mathbb{KH}_{\varpi}[(\xi * \zeta)(\varsigma)]$ *is the conformable fractional Khalouta convolution of the functions* $\xi(\varsigma)$ *and* $\zeta(\varsigma)$.

3) The conformable fractional Khalouta transforms of the usual functions.

$$\begin{aligned}
\mathbb{KH}_{\varpi}[\mu] &= \mu, \\
\mathbb{KH}_{\varpi}[\varsigma^{\nu}] &= \left(\frac{\varpi\gamma\eta}{s}\right)^{\frac{\nu}{\varpi}}\Gamma\left(\frac{\nu}{\varpi}+1\right), \\
\mathbb{KH}_{\varpi}\left[\frac{\varsigma^{n\varpi}}{\varpi^{n}}\right] &= \left(\frac{\gamma\eta}{s}\right)^{n}\Gamma(n+1), n \geq 0.
\end{aligned}$$

Theorem 2.4. [13] *Let* $\xi : [0, +\infty) \longrightarrow \mathbb{R}$ *be* $\varpi-$*differentiable function and* $0 < \varpi \leq 1$, *then the conformable fractional Khalouta transform of the conformable fractional derivative is given by*

$$\mathbb{KH}_{\varpi}[\mathcal{T}^{\varpi}(\xi)(\varsigma)] = \frac{s}{\gamma\eta}\mathbb{KH}_{\varpi}[\xi(\varsigma)] - \frac{s}{\gamma\eta}\xi(0).$$

3. The conformable fractional Khalouta decomosition method (CFKHDM)

This part presents the process of the CFKHDM for deriving exact solution of the conformable time-fractional Newell-Whitehead-Segel equation.

Theorem 3.1. *Consider the conformable time-fractional Newell- Whitehead-Segel equation (1) which is subject to the equation (2).*

Then, by the CFKHDM, the solution of equations (1)-(2) is given as an infinite series that converges rapidly to the exact solution, in other words

$$\xi(\varrho,\varsigma)=\sum_{n=0}^{\infty}\xi_n(\varrho,\varsigma). \tag{3}$$

Proof. To arrive at the desired result, let

$$\mathcal{N}(\xi)(\varrho,\varsigma)=\xi^p(\varrho,\varsigma), \tag{4}$$

to represent the nonlinear term in equation (1), and then our equation can be rewritten as

$$\mathcal{T}_{\varsigma}^{\varpi}(\xi)(\varrho,\varsigma)=a\xi_{\varrho\varrho}(\varrho,\varsigma)+b\xi(\varrho,\varsigma)+c\mathcal{N}(\xi)(\varrho,\varsigma). \tag{5}$$

Now, by implementing the conformable fractional Khalouta transform on equation (5) and using Theorem 2.3, we get

$$\mathbb{KH}_{\varpi}\left[\mathcal{T}_{\varsigma}^{\varpi}(\xi)(\varrho,\varsigma)\right]=a\mathbb{KH}_{\varpi}\left[\xi_{\varrho\varrho}(\varrho,\varsigma)\right]+b\mathbb{KH}_{\varpi}\left[\xi(\varrho,\varsigma)\right]+c\mathbb{KH}_{\varpi}\left[\mathcal{N}(\xi)(\varrho,\varsigma)\right].$$

By utilizing Theorem 2.4, we get

$$\mathbb{KH}_{\varpi}\left[\xi(\varrho,\varsigma)\right]=\xi(\varrho,0)+\frac{\gamma\eta}{s}\left(a\mathbb{KH}_{\varpi}\left[\xi_{\varrho\varrho}(\varrho,\varsigma)\right]+b\mathbb{KH}_{\varpi}\left[\xi(\varrho,\varsigma)\right]+c\mathbb{KH}_{\varpi}\left[\mathcal{N}(\xi)(\varrho,\varsigma)\right]\right). \tag{6}$$

By applying the inverse conformable fractional Khalouta transform, equation (6) becomes

$$\xi(\varrho,\varsigma)=\xi_0(\varrho)+\mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(a\mathbb{KH}_{\varpi}\left[\xi_{\varrho\varrho}(\varrho,\varsigma)\right]+b\mathbb{KH}_{\varpi}\left[\xi(\varrho,\varsigma)\right]+c\mathbb{KH}_{\varpi}\left[\mathcal{N}(\xi)(\varrho,\varsigma)\right]\right)\right]. \tag{7}$$

Now, according to the Adomian decomposition method [3], we can acquire the solution $\xi(\varrho,\varsigma)$ of equation (5) as follows

$$\xi(\varrho,\varsigma)=\sum_{n=0}^{\infty}\xi_n(\varrho,\varsigma),$$

and the nonlinear operator $\mathcal{N}(\xi)(\varrho,\varsigma)$ is decomposed as

$$\mathcal{N}(\xi)(\varrho,\varsigma)=\sum_{n=0}^{\infty}\mathcal{A}_n(\xi), \tag{8}$$

where

$$\mathcal{A}_n(\xi)=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\mathcal{N}\left(\sum_{i=0}^{n}\lambda^i\xi_i\right)\right]_{\lambda=0},\ n\geq 0, \tag{9}$$

where $\mathcal{A}_n(\xi)$ represents the Adomian polynomials and denotes the nonlinear term (4).

From relation (9), the values of the few nonlinear terms are given as

$$\begin{aligned} A_0(\xi) &= \xi_0^p, \\ A_1(\xi) &= p\xi_0^{p-1}\xi_1, \\ A_2(\xi) &= p\xi_0^{p-1}\xi_2 + \frac{p(p-1)}{2!}\xi_0^{p-2}\xi_1^2, \\ A_3(\xi) &= p\xi_0^{p-1}\xi_3 + p(p-1)\xi_0^{p-2}\xi_1\xi_2 + \frac{p(p-1)(p-2)}{3!}\xi_0^{p-3}\xi_1^3. \end{aligned} \tag{10}$$

By inserting equations (3) and (8) into equation (7), we attain the following

$$\sum_{n=0}^{\infty}\xi_n(\varrho,\varsigma) = \xi_0(\varrho) + \mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(\begin{array}{c} a\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\xi_{n\varrho\varrho}(\varrho,\varsigma)\right] \\ +b\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\xi_n(\varrho,\varsigma)\right] \\ +c\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\mathcal{A}_n(\xi)\right]\end{array}\right)\right]. \tag{11}$$

By equating both sides of quation (11), we arrive at

$$\begin{aligned} \xi_0(\varrho,\varsigma) &= \xi_0(\varrho), \\ \xi_1(\varrho,\varsigma) &= \mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(a\mathbb{KH}_{\varpi}\left[\xi_{0\varrho\varrho}(\varrho,\varsigma)\right] + b\mathbb{KH}_{\varpi}\left[\xi_0(\varrho,\varsigma)\right] + c\mathbb{KH}_{\varpi}\left[\mathcal{A}_0(\xi)\right]\right)\right], \\ \xi_2(\varrho,\varsigma) &= \mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(a\mathbb{KH}_{\varpi}\left[\xi_{1\varrho\varrho}(\varrho,\varsigma)\right] + b\mathbb{KH}_{\varpi}\left[\xi_1(\varrho,\varsigma)\right] + c\mathbb{KH}_{\varpi}\left[\mathcal{A}_1(\xi)\right]\right)\right], \\ \xi_3(\varrho,\varsigma) &= \mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(a\mathbb{KH}_{\varpi}\left[\xi_{2\varrho\varrho}(\varrho,\varsigma)\right] + b\mathbb{KH}_{\varpi}\left[\xi_2(\varrho,\varsigma)\right] + c\mathbb{KH}_{\varpi}\left[\mathcal{A}_2(\xi)\right]\right)\right], \\ &\vdots \\ \xi_n(\varrho,\varsigma) &= \mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(\begin{array}{c} a\mathbb{KH}_{\varpi}\left[\xi_{(n-1)\varrho\varrho}(\varrho,\varsigma)\right] + b\mathbb{KH}_{\varpi}\left[\xi_{(n-1)}(\varrho,\varsigma)\right] \\ +c\mathbb{KH}_{\varpi}\left[\mathcal{A}_{(n-1)}(\xi)\right]\end{array}\right)\right]. \end{aligned}$$

Finally, the CFKHDM-solution $\xi(\varrho,\varsigma)$ is approximated as follows

$$\xi(\varrho,\varsigma) = \xi_0(\varrho,\varsigma) + \xi_1(\varrho,\varsigma) + \xi_2(\varrho,\varsigma) + ... = \sum_{n=0}^{\infty}\xi_n(\varrho,\varsigma).$$

□

The following theorem clarifies and governs the convergence condition of the series solution (3).

Theorem 3.2. *Let $\xi(\varrho,\varsigma) \in \mathcal{H}$ and $0 < \varpi \leq 1$ where $\mathcal{H}$ denotes the Hilbert space and assume that $\xi(\varrho,\varsigma)$ is an exact solution to equation (1). The derived results $\sum\limits_{n=0}^{\infty}\xi_n(\varrho,\varsigma)$ converge to $\xi(\varrho,\varsigma)$, if there exists $\varkappa \in (0,1)$ such that $\|\xi_n(\varrho,\varsigma)\| \leq \varkappa\|\xi_{n-1}(\varrho,\varsigma)\|, \forall n \in \mathbb{N}^+$.*

Proof. Consider a sequence of $\sum\limits_{n=0}^{\infty} \xi_n(\varrho,\varsigma)$

$$\begin{aligned}
\mathcal{S}_0(\varrho,\varsigma) &= \xi_0(\varrho,\varsigma), \\
\mathcal{S}_1(\varrho,\varsigma) &= \xi_0(\varrho,\varsigma)+\xi_1(\varrho,\varsigma), \\
\mathcal{S}_2(\varrho,\varsigma) &= \xi_0(\varrho,\varsigma)+\xi_1(\varrho,\varsigma)+\xi_2(\varrho,\varsigma), \\
&\vdots \\
\mathcal{S}_n(\varrho,\varsigma) &= \xi_0(\varrho,\varsigma)+\xi_1(\varrho,\varsigma)+\xi_2(\varrho,\varsigma)+...+\xi_n(\varrho,\varsigma).
\end{aligned}$$

It is necessary to show that $\sum\limits_{n=0}^{\infty} \xi_n(\varrho,\varsigma)$ gives a "Cauchy sequence" based on the obtained results.

Furthermore, consider that

$$\begin{aligned}
\left\|\mathcal{S}_{n+1}(\varrho,\varsigma)-\mathcal{S}_n(\varrho,\varsigma)\right\| &\leq \left\|\xi_{n+1}(\varrho,\varsigma)\right\| \leq \varkappa \left\|\xi_n(\varrho,\varsigma)\right\| \\
&\leq \varkappa^2 \left\|\xi_{n-1}(\varrho,\varsigma)\right\| \leq ... \leq \varkappa^{n+1} \left\|\xi_0(\varrho,\varsigma)\right\|. \qquad (12)
\end{aligned}$$

For every $n,m \in \mathbb{N}$ such that $n \geq m$, using (12) and triangle inequality successively, one has

$$\begin{aligned}
\left\|\mathcal{S}_n(\varrho,\varsigma)-\mathcal{S}_m(\varrho,\varsigma)\right\| &= \left\| \begin{array}{c} \mathcal{S}_n(\varrho,\varsigma)-\mathcal{S}_{n-1}(\varrho,\varsigma)+\mathcal{S}_{n-1}(\varrho,\varsigma)-\mathcal{S}_{n-2}(\varrho,\varsigma) \\ +...+\mathcal{S}_{m+1}(\varrho,\varsigma)-\mathcal{S}_m(\varrho,\varsigma) \end{array} \right\| \\
&\leq \left\|\mathcal{S}_n(\varrho,\varsigma)-\mathcal{S}_{n-1}(\varrho,\varsigma)\right\| + \left\|\mathcal{S}_{n-1}(\varrho,\varsigma)-\mathcal{S}_{n-2}(\varrho,\varsigma)\right\| \\
&\quad +...+\left\|\mathcal{S}_{m+1}(\varrho,\varsigma)-\mathcal{S}_m(\varrho,\varsigma)\right\| \\
&\leq \varkappa^n \left\|\xi_0(\varrho,\varsigma)\right\| + \varkappa^{n-1} \left\|\xi_0(\varrho,\varsigma)\right\| + ... + \varkappa^{m+1} \left\|\xi_0(\varrho,\varsigma)\right\| \\
&= \varkappa^{m+1} \left(1+\varkappa+...+\varkappa^{n-m-1}\right) \left\|\xi_0(\varrho,\varsigma)\right\| \\
&\leq \varkappa^{m+1} \left(\frac{1-\varkappa^{n-m}}{1-\varkappa}\right) \left\|\xi_0(\varrho,\varsigma)\right\|. \qquad (13)
\end{aligned}$$

Since $0<\varkappa<1$, we have $1-\varkappa^{n-m}<1$, therefore, the inequality equation (13) can be reduced to

$$\left\|\mathcal{S}_n(\varrho,\varsigma)-\mathcal{S}_m(\varrho,\varsigma)\right\| \leq \frac{\varkappa^{m+1}}{1-\varkappa} \left\|\xi_0(\varrho,\varsigma)\right\|.$$

So

$$\left\|\mathcal{S}_n(\varrho,\varsigma)-\mathcal{S}_m(\varrho,\varsigma)\right\| \to 0 \text{ as } n,m \to \infty,$$

as $\xi_0(\varrho,\varsigma)$ is bounded.

Therefore, $\sum\limits_{n=0}^{\infty} \xi_n(\varrho,\varsigma)$ produces a "Cauchy sequence" in $\mathcal{H}$. This shows that $\sum\limits_{n=0}^{\infty} \xi_n(\varrho,\varsigma)$ gives a convergence sequence along the $\lim\limits_{n\to\infty} \xi_n(\varrho,\varsigma)=\xi(\varrho,\varsigma)$ for $\xi_n(\varrho,\varsigma)\in\mathcal{H}$. □

Corollary 3.3. *If the series $\sum\limits_{n=0}^{\infty} \xi_n(\varrho,\varsigma)$ converges then it is an exact solution of the conformable time-fractional Newell-Whitehead-Segel equation (1).*

Theorem 3.4. *Let $\xi(\varrho,\varsigma)$ be the approximate solution of the truncated finite series $\sum\limits_{l=0}^{N}\xi_l(\varrho,\varsigma)$, then the maximum absolute error is*

$$\left\|\xi(\varrho,\varsigma)-\sum_{l=0}^{N}\xi_l(\varrho,\varsigma)\right\|\leq\frac{\varkappa^{N+1}}{1-\varkappa}\left\|\xi_0(\varrho,\varsigma)\right\|.$$

Proof. Let the series $\mathcal{S}_N(\varrho,\varsigma)=\sum\limits_{l=0}^{N}\xi_l(\varrho,\varsigma)$ be finite, according to Theorem 3.2 and (12), we have

$$\left\|\mathcal{S}_k(\varrho,\varsigma)-\mathcal{S}_N(\varrho,\varsigma)\right\|\leq\frac{\varkappa^{N+1}}{1-\varkappa}\left\|\xi_0(\varrho,\varsigma)\right\|.$$

But we assume that $\mathcal{S}_k(\varrho,\varsigma)=\sum\limits_{l=0}^{k}\xi_l(\varrho,\varsigma)$ and since $k\to+\infty$, we obtain $\mathcal{S}_k(\varrho,\varsigma)\to\xi(\varrho,\varsigma)$, so the inequality equation (3) can be rewritten as

$$\left\|\xi(\varrho,\varsigma)-\mathcal{S}_N(\varrho,\varsigma)\right\|=\left\|\xi(\varrho,\varsigma)-\sum_{l=0}^{N}\xi_l(\varrho,\varsigma)\right\|\leq\frac{\varkappa^{N+1}}{1-\varkappa}\left\|\xi_0(\varrho,\varsigma)\right\|.$$

□

4. The conformable fractional Taylor series method (CFTSM)

This part offers the process of the CFTSM for construing exact solution of the fractional Newell-Whitehead-Segel equation in conformable time-fractional derivative.

Theorem 4.1. *Let the conformable time-fractional Newell-Whitehead-Segel equation (1) which is subject to the equation (2). From the CFTSM, the solution of equations (1)-(2) can be expressed as an infinite expansion as follows*

$$\xi(\varrho,\varsigma)=\sum_{k=0}^{\infty}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!},\tag{14}$$

where $0<\varpi\leq 1$ and $\xi_k(\varrho)$ are the coefficients functions of series (14).

Proof. To arrive at this result, we take the conformable time-fractional Newell-Whitehead-Segel equation (1) with equation (2).

Now, let us assume that the solution of equations (1) and (2) takes the following infinite expansion

$$\xi(\varrho,\varsigma)=\sum_{k=0}^{\infty}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}.\tag{15}$$

Thus, the n^{th}-CFTS approximate solution can be expressed as

$$\xi_n(\varrho,\varsigma)=\sum_{k=0}^{n}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}=\xi_0(\varrho)+\sum_{k=1}^{n}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}.\tag{16}$$

Applying the operator $\mathcal{T}_\varsigma^\varpi$ on equation (16), and using Theorem 2.1, we get the following formula

$$\mathcal{T}_\varsigma^\varpi\left(\xi_n\right)(\varrho,\varsigma)=\sum_{k=0}^{n-1}\xi_{k+1}(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}.\tag{17}$$

Substituting equations (16) and (17) into equation (1), we get the following iterative relation

$$0 = \sum_{k=0}^{n-1} \xi_{k+1}(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} - a \left(\sum_{k=0}^{n} \xi_k(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} \right)_{\varrho\varrho} - b \left(\sum_{k=0}^{n} \xi_k(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} \right) - c \left(\sum_{k=0}^{n} \xi_k(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} \right)^p .$$

To determine the form of the unknown coefficients $\xi_n(\varrho), n = 1, 2, 3, ..,$ we follow the same method to get the Taylor series coefficients so we need to solve the following equation

$$\mathcal{T}_{\varsigma}^{(n-1)\varpi} \{\Phi(\varrho, \varsigma, \varpi, n)\} \downarrow_{\varsigma=0} = 0,$$

where

$$\Phi(\varrho, \varsigma, \varpi, n) = \sum_{k=0}^{n-1} \xi_{k+1}(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} - a \left(\sum_{k=0}^{n} \xi_k(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} \right)_{\varrho\varrho} - b \left(\sum_{k=0}^{n} \xi_k(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} \right) - c \left(\sum_{k=0}^{n} \xi_k(\varrho) \frac{\varsigma^{k\varpi}}{\varpi^k k!} \right)^p , \tag{18}$$

and $\mathcal{T}_{\varsigma}^{(n-1)\varpi}$ is the sequential conformable time-fractional derivative operator given by

$$\mathcal{T}_{\varsigma}^{(n-1)\varpi} \{\Phi(\varrho, \varsigma, \varpi, n)\} \downarrow_{\varsigma=0} = \underbrace{\mathcal{T}_{\varsigma}^{\varpi}.\mathcal{T}_{\varsigma}^{\varpi}...\mathcal{T}_{\varsigma}^{\varpi} \{\Phi(\varrho, \varsigma, \varpi, n)\} \downarrow_{\varsigma=0}}_{(n-1)-times}$$

Now, to determine the form of the first unknown coefficient $\xi_1(\varrho)$, we substitute $n = 1$ in equation (18), we obtain

$$\Phi(\varrho, \varsigma, \varpi, 1) = \xi_1(\varrho) - a \left(\xi_0(\varrho) + \xi_1(\varrho) \frac{\varsigma^{\varpi}}{\varpi} \right)_{\varrho\varrho} - b \left(\xi_0(\varrho) + \xi_1(\varrho) \frac{\varsigma^{\varpi}}{\varpi} \right) - c \left(\xi_0(\varrho) + \xi_1(\varrho) \frac{\varsigma^{\varpi}}{\varpi} \right)^p .$$

Solving equation $\Phi(\varrho, 0, \varpi, 1) = 0$, gives

$$\xi_1(\varrho) = a (\xi_0(\varrho))_{\varrho\varrho} + b (\xi_0(\varrho)) + c (\xi_0(\varrho))^p .$$

To determine the form of the second unknown coefficient $\xi_2(\varrho)$, we substitute $n = 2$ in equation (18), we obtain

$$\Phi(\varrho, \varsigma, \varpi, 2) = \xi_1(\varrho) + \xi_2(\varrho) \frac{\varsigma^{\varpi}}{\varpi} - a \left(\xi_0(\varrho) + \xi_1(\varrho) \frac{\varsigma^{\varpi}}{\varpi} + \xi_2(\varrho) \frac{\varsigma^{2\varpi}}{\varpi^2 2!} \right)_{\varrho\varrho} - b \left(\xi_0(\varrho) + \xi_1(\varrho) \frac{\varsigma^{\varpi}}{\varpi} + \xi_2(\varrho) \frac{\varsigma^{2\varpi}}{\varpi^2 2!} \right) - c \left(\xi_0(\varrho) + \xi_1(\varrho) \frac{\varsigma^{\varpi}}{\varpi} + \xi_2(\varrho) \frac{\varsigma^{2\varpi}}{\varpi^2 2!} \right)^p .$$

From Theorems 2.1 and 2.2 and solving $\mathcal{T}_{\varsigma}^{\varpi} \{\Phi(\varrho, \varsigma, \varpi, 2)\} \downarrow_{\varsigma=0} = 0$, we get

$$\xi_2(\varrho) = a (\xi_1(\varrho))_{\varrho\varrho} + b (\xi_1(\varrho)) + cp (\xi_0(\varrho))^{p-1} (\xi_1(\varrho)) .$$

To determine the form of the third unknown coefficient $\xi_3(\varrho)$, we substitute $n=3$ in equation (18), we obtain

$$\begin{aligned}\Phi(\varrho,\varsigma,\varpi,3) &= \xi_1(\varrho)+\xi_2(\varrho)\frac{\varsigma^{\varpi}}{\varpi}+\xi_3(\varrho)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}\\ &\quad -a\left(\xi_0(\varrho)+\xi_1(\varrho)\frac{\varsigma^{\varpi}}{\varpi}+\xi_2(\varrho)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}+\xi_3(\varrho)\frac{\varsigma^{3\varpi}}{\varpi^3 3!}\right)_{\varrho\varrho}\\ &\quad -b\left(\xi_0(\varrho)+\xi_1(\varrho)\frac{\varsigma^{\varpi}}{\varpi}+\xi_2(\varrho)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}+\xi_3(\varrho)\frac{\varsigma^{3\varpi}}{\varpi^3 3!}\right)\\ &\quad -c\left(\xi_0(\varrho)+\xi_1(\varrho)\frac{\varsigma^{\varpi}}{\varpi}+\xi_2(\varrho)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}+\xi_3(\varrho)\frac{\varsigma^{3\varpi}}{\varpi^3 3!}\right)^p.\end{aligned}$$

From Theorems 2.1 and 2.2 and solving $\mathcal{T}_{\varsigma}^{2\varpi}\{\Phi(\varrho,\varsigma,\varpi,3)\}\downarrow_{\varsigma=0}=0$, we

$$\begin{aligned}\xi_3(\varrho) &= a\left(\xi_2(\varrho)\right)_{\varrho\varrho}+b\left(\xi_2(\varrho)\right)\\ &\quad +c\left(p(p-1)\left(\xi_0(\varrho)\right)^{p-2}\left(\xi_1(\varrho)\right)^2+p\left(\xi_0(\varrho)\right)^{p-1}\left(\xi_2(\varrho)\right)\right).\end{aligned}$$

To determine the form of the fourth unknown coefficient $\xi_4(\varrho)$, consider $\Phi(\varrho,\varsigma,\varpi,4)$ and solving

$$\mathcal{T}_{\varsigma}^{3\varpi}\{\Phi(\varrho,\varsigma,\varpi,4)\}\downarrow_{\varsigma=0}=0.$$

In total, to determine the form of the r^{th} unknown coefficient $\xi_r(\varrho)$, we solve

$$\mathcal{T}_{\varsigma}^{(r-1)\varpi}\{\Phi(\varrho,\varsigma,\varpi,r)\}\downarrow_{\varsigma=0}=0.$$

where

$$\begin{aligned}\Phi(\varrho,\varsigma,\varpi,r) &= \sum_{k=0}^{r-1}\xi_{k+1}(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}-a\left(\sum_{k=0}^{r}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}\right)_{\varrho\varrho}-b\left(\sum_{k=0}^{r}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}\right)\\ &\quad -c\left(\sum_{k=0}^{r}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}\right)^p.\end{aligned}$$

Finally, the CFTSM-solution $\xi(\varrho,\varsigma)$ of equations (1)-(2) can be expressed in an infinite expansion as follows

$$\begin{aligned}\xi(\varrho,\varsigma) &= \lim_{n\to\infty}\xi_n(\varrho,\varsigma)\\ &= \lim_{n\to\infty}\sum_{k=0}^{n}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}\\ &= \sum_{k=0}^{\infty}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!}.\end{aligned}$$

□

The following theorem explains and governs the condition for convergence of the expansion solution (15).

Theorem 4.2. *Let $\xi_n(\varrho,\varsigma)$ with $n\in\mathbb{N}$ and $\xi(\varrho,\varsigma)$ be recpectivly, the CFTS-approximate and exact solutions of equations (1)-(2). If $\exists\ \theta: 0<\theta<1$, such that*

$$\left\|\xi_{n+1}(\varrho,\varsigma)\right\|\leq\theta\left\|\xi_n(\varrho,\varsigma)\right\|,\forall(\varrho,\varsigma)\in\mathbb{R}\times\mathbb{R}^+, \tag{19}$$

with

$$\|\xi_0(\varrho)\| < \infty, \forall \varrho \in \mathbb{R}.$$

Then, $\xi_n(\varrho,\varsigma)$ *converges to* $\xi(\varrho,\varsigma)$ *as soon as* $n \to \infty$.

Proof. From the inequality (19), we have

$$\begin{aligned} \|\xi_{n+1}(\varrho,\varsigma)\| &\leq \theta \|\xi_n(\varrho,\varsigma)\| \leq \theta^2 \|\xi_{n-1}(\varrho,\varsigma)\| \leq \theta^3 \|\xi_{n-2}(\varrho,\varsigma)\| \\ &\leq ... \leq \theta^{n+1} \|\xi_0(\varrho)\|. \end{aligned}$$

Therefore, we obtain

$$\sum_{m=n+1}^{\infty} \|\xi_m(\varrho,\varsigma)\| \leq \sum_{m=n+1}^{\infty} \theta^m \|\xi_0(\varrho)\| = \|\xi_0(\varrho)\| \sum_{m=n+1}^{\infty} \theta^m.$$

And so, we have

$$\begin{aligned} \|\xi(\varrho,\varsigma) - \xi_n(\varrho,\varsigma)\| &= \left\| \sum_{m=n+1}^{\infty} \xi_m(\varrho,\varsigma) \right\| \\ &\leq \sum_{m=n+1}^{\infty} \|\xi_m(\varrho,\varsigma)\| \\ &\leq \|\xi_0(\varrho)\| \sum_{m=n+1}^{\infty} \theta^m \\ &= \frac{\theta^{n+1}}{1-\theta} \|\xi_0(\varrho)\| \underset{n\to\infty}{\to} 0. \end{aligned}$$

□

5. Numerical tests

This part examines the validity of the CFKHDM and CFTSM for three numerical tests of the conformable time-fractional Newell-Whitehead-Segel equation.

Test 5.1. Let the conformable time-fractional Newell-Whitehead-Segel equation

$$\mathcal{T}_{\varsigma}^{\varpi}(\xi)(\varrho,\varsigma) = \xi_{\varrho\varrho}(\varrho,\varsigma) - 2\xi(\varrho,\varsigma), \tag{20}$$

subject to

$$\xi(\varrho,0) = \xi_0(\varrho) = e^{\varrho}, \tag{21}$$

where $0 < \varpi \leq 1$.

Case 1: CFKHDM-solution

Using the methodology of the CFKHDM as explained in Part 3, the series solution of (20)-(21) can be represented by the following form

$$\begin{aligned} \xi(\varrho,\varsigma) &= \sum_{n=0}^{\infty} \xi_n(\varrho,\varsigma) \qquad (22) \\ &= e^{\varrho} + \mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(\mathbb{KH}_{\varpi}\left[\sum_{n=0}^{\infty} \xi_{n\varrho\varrho}(\varrho,\varsigma)\right] - 2\mathbb{KH}_{\varpi}\left[\sum_{n=0}^{\infty} \xi_n(\varrho,\varsigma)\right]\right)\right]. \end{aligned}$$

From relation (22), the successive terms are determined as follows

$$\begin{aligned}
\xi_0(\varrho,\varsigma) &= e^{\varrho},\\
\xi_1(\varrho,\varsigma) &= -e^{\varrho}\frac{\varsigma^{\varpi}}{\varpi},\\
\xi_2(\varrho,\varsigma) &= e^{\varrho}\frac{\varsigma^{2\varpi}}{\varpi^2 2!},\\
\xi_3(\varrho,\varsigma) &= -e^{\varrho}\frac{\varsigma^{3\varpi}}{\varpi^3 3!},\\
\xi_4(\varrho,\varsigma) &= e^{\varrho}\frac{\varsigma^{4\varpi}}{\varpi^4 4!},\\
\xi_5(\varrho,\varsigma) &= -e^{\varrho}\frac{\varsigma^{5\varpi}}{\varpi^5 5!},\\
&\vdots
\end{aligned}$$

As a result, the CFKHDM-solution $\xi(\varrho,\varsigma)$ of equations (20)-(21) can be found as

$$\begin{aligned}
\xi(\varrho,\varsigma) &= e^{\varrho} - e^{\varrho}\frac{\varsigma^{\varpi}}{\varpi} + e^{\varrho}\frac{\varsigma^{2\varpi}}{\varpi^2 2!} - e^{\varrho}\frac{\varsigma^{3\varpi}}{\varpi^3 3!} + e^{\varrho}\frac{\varsigma^{4\varpi}}{\varpi^4 4!} - \frac{\varsigma^{5\varpi}}{\varpi^5 5!} + \dots\\
&= e^{\varrho}\left(1 - \frac{\varsigma^{\varpi}}{\varpi} + \frac{\varsigma^{2\varpi}}{\varpi^2 2!} - \frac{\varsigma^{3\varpi}}{\varpi^3 3!} + \frac{\varsigma^{4\varpi}}{\varpi^4 4!} - \frac{\varsigma^{5\varpi}}{\varpi^5 5!} + \dots\right)\\
&= e^{\varrho}\sum_{k=0}^{\infty}\frac{(-1)^k\varsigma^{k\varpi}}{\varpi^k k!}\\
&= e^{\varrho - \frac{\varsigma^{\varpi}}{\varpi}}.
\end{aligned}$$

When $\varpi = 1$, we obtain a closed form solution of equations (20)-(21) in the following form

$$\xi(\varrho,\varsigma) = e^{\varrho-\varsigma},$$

which is the exact solution of our equation available in the literature [14].

Case 2: CFTSM-solution

By employing the same procedure of the CFTSM described in Part 4, we obtain the n^{th}-CFTS approximate solution of equations (20)-(21) as the following

$$\xi_n(\varrho,\varsigma) = \xi_0(\varrho) + \sum_{k=1}^{n}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!},$$

and the forms for the unknown coefficients $\xi_k(\varrho), k = 1,2,3,\dots$ are

$$\xi_k(\varrho) = (-1)^k e^{\varrho}, \text{ for all } k = 1,2,3,\dots$$

Hence, equation (5.1) can take the following form

$$\begin{aligned}
\xi_n(\varrho,\varsigma) &= e^{\varrho} - e^{\varrho}\frac{\varsigma^{\varpi}}{\varpi} + e^{\varrho}\frac{\varsigma^{2\varpi}}{\varpi^2 2!} - e^{\varrho}\frac{\varsigma^{3\varpi}}{\varpi^3 3!} + e^{\varrho}\frac{\varsigma^{4\varpi}}{\varpi^4 4!} - e^{\varrho}\frac{\varsigma^{5\varpi}}{\varpi^5 5!} + \dots + e^{\varrho}\frac{\varsigma^{n\varpi}}{\varpi^n n!}\\
&= e^{\varrho}\left(1 - \frac{\varsigma^{\varpi}}{\varpi} + \frac{\varsigma^{2\varpi}}{\varpi^2 2!} - \frac{\varsigma^{3\varpi}}{\varpi^3 3!} + \frac{\varsigma^{4\varpi}}{\varpi^4 4!} - \frac{\varsigma^{5\varpi}}{\varpi^5 5!} + \dots + \frac{\varsigma^{n\varpi}}{\varpi^n n!}\right)\\
&= \sum_{k=0}^{n} e^{\varrho}\frac{(-1)^k\varsigma^{k\varpi}}{\varpi^k k!}.
\end{aligned}$$

Therefore, the CFTSM-solution $\xi(\varrho,\varsigma)$ of equations (20)-(21) can be expressed in an infinite expansion as follows

$$\xi(\varrho,\varsigma)=\lim_{n\to\infty}\xi_n(\varrho,\varsigma)=\sum_{k=0}^{\infty}e^{\varrho}\frac{(-1)^k\varsigma^{k\varpi}}{\varpi^k k!}. \tag{23}$$

For $\varpi=1$, the CFTSM-solution (23) converge rapidly to

$$\xi(\varrho,\varsigma)=e^{\varrho-\varsigma},$$

which is the exact solution obtained by the natural reduced differential transform method (NRDTM) [14].

Test 5.2. Let the conformable time-fractional Newell-Whitehead-Segel equation

$$\mathcal{T}_{\varsigma}^{\varpi}(\xi)(\varrho,\varsigma)=\xi_{\varrho\varrho}(\varrho,\varsigma)+2\xi(\varrho,\varsigma)-3\xi^2(\varrho,\varsigma), \tag{24}$$

subject to

$$\xi(\varrho,0)=\xi_0(\varrho)=\lambda, \tag{25}$$

where $0<\varpi\leq 1$ and $\lambda\in\mathbb{R}$.

Case 1: CFKHDM-solution

Using the methodology of the CFKHDM as explained in Part 3, the series solution of (24)-(25) can be represented by the following form

$$\begin{aligned}\xi(\varrho,\varsigma) &= \sum_{n=0}^{\infty}\xi_n(\varrho,\varsigma) \qquad (26)\\ &= \lambda+\mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(\begin{array}{c}\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\xi_{n\varrho\varrho}(\varrho,\varsigma)\right]+2\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\xi_n(\varrho,\varsigma)\right]\\ -3\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\mathcal{A}_n(\xi)\right]\end{array}\right)\right],\end{aligned}$$

where $\mathcal{A}_n(\xi)$ represents the Adomian polynomials and denotes the nonlinear term $\mathcal{N}(\xi)(\varrho,\varsigma)=\xi^2(\varrho,\varsigma)$.

From relation (10) and equation (26), the successive terms are determined as follows

$$\begin{aligned}\xi_0(\varrho,\varsigma) &= \lambda,\\ \xi_1(\varrho,\varsigma) &= \left(2\lambda-3\lambda^2\right)\frac{\varsigma^{\varpi}}{\varpi},\\ \xi_2(\varrho,\varsigma) &= (2-6\lambda)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{2\varpi}}{\varpi^2 2!},\\ \xi_3(\varrho,\varsigma) &= \left(4-36\lambda+54\lambda^2\right)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{3\varpi}}{\varpi^3 3!},\\ &\vdots\end{aligned}$$

As a result, the CFKHDM-solution $\xi(\varrho,\varsigma)$ of equations (24)-(25) can be found as

$$\begin{aligned}\xi(\varrho,\varsigma)=&\lambda+\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{\varpi}}{\varpi}+(2-6\lambda)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}\\ &+\left(4-36\lambda+54\lambda^2\right)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{3\varpi}}{\varpi^3 3!}+\ldots.\end{aligned}$$

When $\varpi = 1$, we obtain a closed form solution of equations (24)-(25) in the following form

$$\begin{aligned}\xi(\varrho,\varsigma) &= \lambda + \left(2\lambda - 3\lambda^2\right)\varsigma + (2-6\lambda)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^2}{2!} \\ &\quad + \left(4-36\lambda+54\lambda^2\right)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^3}{3!} + \dots \\ &= \frac{2\lambda e^{2\varsigma}}{2-3\lambda(1-e^{2\varsigma})},\end{aligned}$$

which is the exact solution of our equation available in the literature [14].

Case 2: CFTSM-solution

Employing the same procedure of the CFTSM described in Part 4, we obtain the n^{th}-CFTS approximate solution of equations (24)-(25) as the following

$$\xi_n(\varrho,\varsigma) = \xi_0(\varrho) + \sum_{k=1}^{n} \xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!},$$

and the forms for the unknown coefficients $\xi_k(\varrho), k = 1, 2, 3, \dots$ are

$$\begin{aligned}\xi_1(\varrho) &= 2\lambda - 3\lambda^2, \\ \xi_2(\varrho) &= (2-6\lambda)\left(2\lambda-3\lambda^2\right), \\ \xi_3(\varrho) &= \left(4-36\lambda+54\lambda^2\right)\left(2\lambda-3\lambda^2\right), \\ &\vdots\end{aligned}$$

Therefore, the CFTSM-solution $\xi(\varrho,\varsigma)$ of equations (24)-(25), can be expressed as

$$\xi(\varrho,\varsigma) = \lambda + \left(2\lambda-3\lambda^2\right)\frac{\varsigma^{\varpi}}{\varpi} + (2-6\lambda)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{2\varpi}}{\varpi^2 2!} \tag{27}$$

$$+ \left(4-36\lambda+54\lambda^2\right)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^{3\varpi}}{\varpi^3 3!} + \dots. \tag{28}$$

For $\varpi = 1$, the CFTSM-solution (27) becomes

$$\begin{aligned}\xi(\varrho,\varsigma) =& \lambda + \left(2\lambda-3\lambda^2\right)\varsigma + (2-6\lambda)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^2}{2!} \\ &+ \left(4-36\lambda+54\lambda^2\right)\left(2\lambda-3\lambda^2\right)\frac{\varsigma^3}{3!} + \dots\end{aligned}$$

which converge rapidly to

$$\xi(\varrho,\varsigma) = \frac{2\lambda e^{2\varsigma}}{2-3\lambda(1-e^{2\varsigma})}.$$

which is the exact solution obtained by the natural reduced differential transform method (NRDTM) [14].

Test 5.3. Let the conformable time-fractional Newell-Whitehead-Segel equation

$$\mathcal{T}_{\varsigma}^{\varpi}(\xi)(\varrho,\varsigma) = 5\xi_{xx}(\varrho,\varsigma) + 2\xi(\varrho,\varsigma) + \xi^2(\varrho,\varsigma), \tag{29}$$

subject to

$$\xi(\varrho,0) = \xi_0(\varrho) = \beta, \tag{30}$$

where $0 < \varpi \leq 1$ and $\beta \in \mathbb{R}$.

Case 1: CFKHDM-solution

Using the methodology of the CFKHDM as explained in Part 3, the series solution of (29)-(30) can be represented by the following form

$$\begin{aligned}\xi(\varrho,\varsigma) &= \sum_{n=0}^{\infty}\xi_n(\varrho,\varsigma) \qquad (31)\\ &= \beta+\mathbb{KH}_{\varpi}^{-1}\left[\frac{\gamma\eta}{s}\left(\begin{array}{c}5\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\xi_{n\varrho\varrho}(\varrho,\varsigma)\right]+2\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\xi_n(\varrho,\varsigma)\right]\\ +\mathbb{KH}_{\varpi}\left[\sum\limits_{n=0}^{\infty}\mathcal{A}_n(\xi)\right]\end{array}\right)\right],\end{aligned}$$

where $\mathcal{A}_n(\xi)$ represents the Adomian polynomials and denotes the nonlinear term $\mathcal{N}(\xi)(\varrho,\varsigma)=\xi^2(\varrho,\varsigma)$.

From relation (10) and equation (31), the successive terms are determined as follows

$$\begin{aligned}\xi_0(\varrho,\varsigma) &= \beta,\\ \xi_1(\varrho,\varsigma) &= \left(2\beta+\beta^2\right)\frac{\varsigma^{\varpi}}{\varpi},\\ \xi_2(\varrho,\varsigma) &= (2+2\beta)\left(2\beta+\beta^2\right)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}, \qquad (32)\\ \xi_3(\varrho,\varsigma) &= \left(4+12\beta+6\beta^2\right)\left(2\beta+\beta^2\right)\frac{\varsigma^{3\varpi}}{\varpi^3 3!},\\ &\vdots\end{aligned}$$

As a result, the CFKHDM-solution $\xi(\varrho,\varsigma)$ of equations (29)-(30) can be found as

$$\begin{aligned}\xi(\varrho,\varsigma) &= \beta+\left(2\beta+\beta^2\right)\frac{\varsigma^{\varpi}}{\varpi}+(2+2\beta)\left(2\beta+\beta^2\right)\frac{\varsigma^{2\varpi}}{\varpi^2 2!}\\ &\quad+\left(4+12\beta+6\beta^2\right)\left(2\beta+\beta^2\right)\frac{\varsigma^{3\varpi}}{\varpi^3 3!}+\ldots.\end{aligned}$$

When $\varpi=1$, we obtain a closed form solution of equations (29)-(30) in the following form

$$\begin{aligned}\xi(\varrho,\varsigma) &= \beta+\left(2\beta+\beta^2\right)\varsigma+(2+2\beta)\left(2\beta+\beta^2\right)\frac{\varsigma^2}{2!}\\ &\quad+\left(4+12\beta+6\beta^2\right)\left(2\beta+\beta^2\right)\frac{\varsigma^3}{3!}+\ldots\\ &= \frac{2\beta e^{2\varsigma}}{2+\beta(1-e^{2\varsigma})},\end{aligned}$$

which is the exact solution of our equation available in the literature [14].

Case 2: CFTSM-solution

Employing the same procedure of the CFTSM described in Part 4, we obtain the n^{th}-CFTS approximate solution of equations (29)-(30) as the following

$$\xi_n(\varrho,\varsigma)=\xi_0(\varrho)+\sum_{k=1}^{n}\xi_k(\varrho)\frac{\varsigma^{k\varpi}}{\varpi^k k!},$$

and the forms for the unknown coefficients $\xi_k(\varrho), k = 1, 2, 3, ...$ are

$$\begin{aligned}
\xi_1(\varrho) &= 2\beta + \beta^2, \\
\xi_2(\varrho) &= (2 + 2\beta)\left(2\beta + \beta^2\right) \\
\xi_3(\varrho) &= \left(4 + 12\beta + 6\beta^2\right)\left(2\beta + \beta^2\right), \\
&\vdots
\end{aligned}$$

Therefore, the CFTSM-solution $\xi(\varrho, \varsigma)$ of the equations (30)-(5.3), can be expressed as

$$\begin{aligned}
\xi(\varrho, \varsigma) &= \beta + \left(2\beta + \beta^2\right)\frac{\varsigma^{\varpi}}{\varpi} + (2 + 2\beta)\left(2\beta + \beta^2\right)\frac{\varsigma^{2\varpi}}{\varpi^2 2!} \\
&\quad + \left(4 + 12\beta + 6\beta^2\right)\left(2\beta + \beta^2\right)\frac{\varsigma^{3\varpi}}{\varpi^3 3!} + ...
\end{aligned} \tag{33}$$

For $\varpi = 1$, the CFTSM-solution (33) becomes

$$\xi(\varrho, \varsigma) = \beta + \left(2\beta + \beta^2\right)\varsigma + (2 + 2\beta)\left(2\beta + \beta^2\right)\frac{\varsigma^2}{2!} + \left(4 + 12\beta + 6\beta^2\right)\left(2\beta + \beta^2\right)\frac{\varsigma^3}{3!} + ...$$

which converge rapidly to

$$\xi(\varrho, \varsigma) = \frac{2\beta e^{2\varsigma}}{2 + \beta(1 - e^{2\varsigma})}.$$

which is the exact solution obtained by natural reduced differential transform method (NRDTM) [14].

6. Simulation and discussion

This part presents the comparison results in the form of graphs and tables, illustrate the efficiency, quality and strength, of the CFKHDM and the CFTSM.

Figures 1, 2 and 3 show a comparison between the CFKHDM-solution, CFTSM-solution at $\varpi = 1, 0.9, 0.8, 0.7$ and exact solution when $\varrho = 1$. From the figures, we can predict that our obtained solutions are very close to the analytical solutions. Moreover, the results obtained are consistent with the analytical solutions at $\varpi = 1$, which confirms the efficiency and accuracy of the current methods. In Tables 1, 2 and 3, we have calculated the numerical comparison of absolute errors for differences between the exact solutions and the 4−term approximate solution by CFKHDM and the 4^{th}−order approximate solution by CFTSM at $\varrho = 1$. The absolute errors obtained by CFKHDM are the same results obtained by CFTSM. It can be seen from this, that we get a very good approximation for some terms. However, additional terms can be calculated in order to obtain better accuracy for CFKHDM and CFTSM. Our numerical estimates show very good accuracy. Obviously, the total errors can be reduced by considering additional terms by the CFKHDM and CFTSM.

7. Conclusions

In this work, we compared the solution obtained by the conformable fractional Khalouta decomposition method (CFKHDM) and the conformable fractional Taylor series method

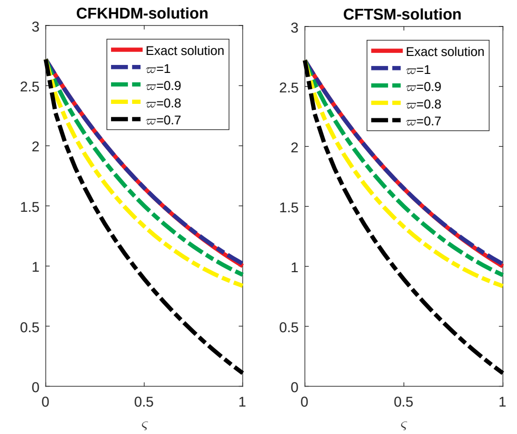

FIGURE 1. Comparison graph of CFKHDM-solution, CFTSM-solution, and exact solution of equation (20) at $\varpi = 1$.

ς	$\lvert \xi_{exact} - \xi_{CFKHDM} \rvert$	$\lvert \xi_{exact} - \xi_{CFTSM} \rvert$
0.01	3.7700×10^{-15}	3.7700×10^{-15}
0.03	2.7405×10^{-12}	2.7405×10^{-12}
0.05	5.8572×10^{-11}	5.8572×10^{-11}
0.07	4.3977×10^{-10}	4.3977×10^{-10}
0.09	1.9809×10^{-9}	1.9809×10^{-9}

TABLE 1. Comparison between CFKHDM-solution and CFTSM-solution in terms of absolute error for Test 5.1 at $\varpi = 1$.

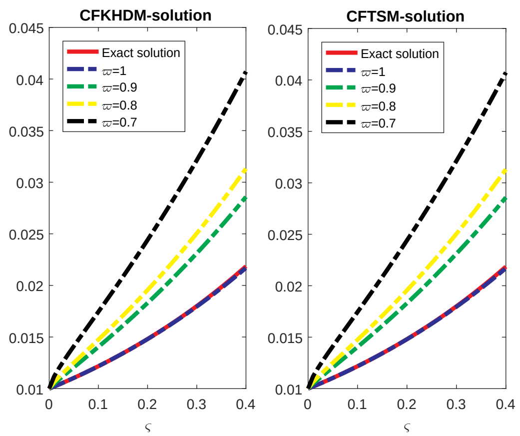

FIGURE 2. Comparison graph of CFKHDM-solution, CFTSM-solution, and exact solution of equation (24) at $\varpi = 1$.

ς	$\lvert\xi_{exact}-\xi_{CFKHDM}\rvert$	$\lvert\xi_{exact}-\xi_{CFTSM}\rvert$
0.01	5.2557×10^{-11}	5.2557×10^{-11}
0.03	4.2820×10^{-9}	4.2820×10^{-9}
0.05	3.3233×10^{-8}	3.3233×10^{-8}
0.07	1.2841×10^{-7}	1.2841×10^{-7}
0.09	3.5294×10^{-7}	3.5294×10^{-7}

TABLE 2. Comparison between CFKHDM-solution and CFTSM-solution in terms of absolute error for Test 5.2 at $\varpi=1$.

ς	$\lvert\xi_{exact}-\xi_{CFKHDM}\rvert$	$\lvert\xi_{exact}-\xi_{CFTSM}\rvert$
0.01	4.7160×10^{-10}	4.7160×10^{-10}
0.03	3.8618×10^{-8}	3.8618×10^{-8}
0.05	3.0127×10^{-7}	3.0127×10^{-7}
0.07	1.1703×10^{-6}	1.1703×10^{-6}
0.09	3.2340×10^{-6}	3.2340×10^{-6}

TABLE 3. Comparison between CFKHDM-solution and CFTSM-solution in terms of absolute error for Test 5.3 at $\varpi=1$.

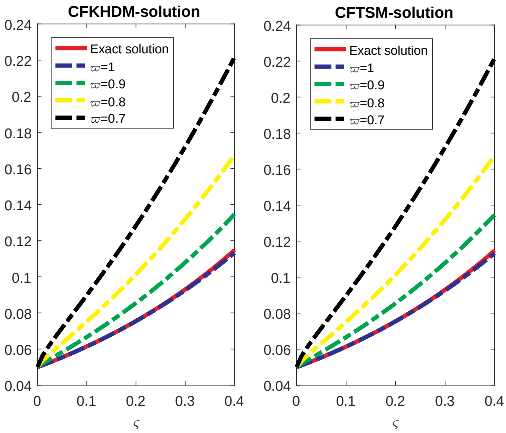

FIGURE 3. Comparison graph of CFKHDM-solution, CFTSM-solution, and exact solution of equation (29) at $\varpi=1$.

(CFTSM) for solving the conformable time-fractional Newell-Whitehead-Segel equation. These two methods are considered reliable and effective as they both provide approximate solutions with higher accuracy. The correlation between the outcome of the fourth term using CFKHDM and the fourth order within CFTSM shows remarkable agreement. However, a distinct advantage of CFTSM lies in its ability to solve nonlinear problems without using Adomian polynomials, which provides an additional advantage over the decomposition method. It is concluded that these methods are very powerful mathematical tools for solving different kinds of conformable fractional partial differenial equations. In the future, thess methods will take an important consideration for solving real-life problems.

References

[1] T. Abdeljawad, On conformable fractional calculus, *Journal of Computational and Applied Mathematics* **279** (2015), 57–66.

[2] O. Acan, O. Firat, Y. Keskin, Conformable variational iteration method, conformable fractional reduced differential transform method and conformable homotopy analysis method for non-linear fractional partial differential eq., *Waves Random Complex Media* **30** (2020), no. 2, 250–268.

[3] G. Adomian, A review of the decomposition method in applied mathematics, *Journal of Mathematical Analysis and Applications* **135** (1988), no. 2, 501–544.

[4] Z. Al-Zhour, N. Al-Mutairi, F. Alrawajeh, R. Alkhasawneh, New theoretical results and applications on conformable fractional Natural transform, *Ain Shams Engineering J.* **12** (2021), no. 1, 927–933.

[5] D.R. Anderson, D.J. Ulness, Newly defined conformable derivative, *Advances in Dynamical Systems and Applications* **10** (2015), no. 2, 109–137.

[6] A. Atangana, D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model, *Thermal Science* **20** (2016, no. 2, 763–769.

[7] M.E. Benattia, K. Belghaba, Shehu Conformable Fractional Transform, Theories and Applications, *Journal of Science and Engineering* **12** (2021), no. 1, 024–032.

[8] M.E. Benattia, K. Belghaba, Solution of Newel-Whitehead-Segel equation using conformable fractional Sumudu decomposition method, *J. of Science and Arts* **21** (2021), no. 2, 479–486.

[9] M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel, *Progress in Fractional Differentiation and Applications* **1** (2015), no. 2, 73–85.

[10] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.

[11] N. Kadkhoda, H. Jafari, An analytical approach to obtain exact solutions of some space-time conformable fractional differential equations, *Advances in Difference Equations* **2019** (2019).

[12] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, *Journal of Computational and Applied Mathematics* **264** (2014), 65–70.

[13] A. Khalouta, Conformable Fractional Khalouta Transform and Its Applications to Fractional Differential Equations, *Nonlinear Dynamics and Systems Theory* **24** (2024), no. 5, 495–504.

[14] A. Khalouta, The Existence and Uniqueness of Solution for Fractional Newel-Whitehead-Segel Equation within Caputo-Fabrizio Fractional Operator, *Applications and Applied Mathematics* **16** (2021), no. 2, 1932–9466.

[15] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Application of Fractional Differential equations*, Elsevier, Amsterdam, 2006.

[16] A.A. Kilbas, Hadamard type fractional calculus, *Journal of the Korean Mathematical Society* **38** (2001), no. 6, 1191–1204.

[17] M.I. Liaqat, A. Khan, A. Akgül, Adaptation on power series method with conformable operator for solving fractional order systems of nonlinear partial differential equations, *Chaos Solitons & Fractals* **157** (2022), https://doi.org/10.1016/j.chaos.2022.111984

[18] K.S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, Wiley: Hoboken, NJ, USA, 1993.

[19] M. Molaei, F.D. Saei, M. Javidi, Y. Mahmoudi, New analytical methods for solving a class of conformable fractional differential equations by fractional Laplace transform, *Computational Methods for Differential Equations* **10** (2022), no. 2, 396–407.

[20] S.S. Omorodion, Conformable Fractional Reduced Differential Transform Method for Solving Linear and Nonlinear Time-Fractional Swift-Hohenberg (S-H) Equation, *International Journal of Scientific Research in Mathematical and Statistical Sciences* **8** (2021), no. 6, 20–29.

[21] A. Zulfiqar, J. Ahmad, Comparative study of two techniques on some nonlinear problems based using conformable derivative, *Nonlinear Engineering* **9** (2020), 470-482.

(Ali Khalouta) Laboratory of Fundamental and Numerical Mathematics, Department of Mathematics, Faculty of Sciences, Setif 1 University–Ferhat ABBAS, Algeria
E-mail address: nadjibkh@yahoo.fr, ali.khalouta@univ-setif.dz