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On the forms of continuity for fuzzy functions

Erdal Ekici

Abstract. Joseph and Kwack introduced the notion of (θ, s)-continuous functions in order to investigate S-closed spaces. The aim of this paper is to introduce some forms of fuzzy functions related to S-closed spaces and to investigate properties of such fuzzy functions.

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1. Introduction

Joseph and Kwack [7] introduced (θ, s)-continuous functions in order to investigate S-closed spaces due to Thompson [13]. A function f is called (θ, s)-continuous if inverse image of each regular open set is closed. Moreover, Chang introduced fuzzy S-closed spaces in 1968. The purpose of this paper is to introduce forms of fuzzy almost contra-continuous functions and to investigate properties and relationships of fuzzy almost contra-β-continuous functions. Also, by using this paper, properties of fuzzy almost contra-continuous functions, fuzzy almost contra-precontinuous functions and fuzzy almost contra-semicontinuous functions can be obtained with similar way.

The class of fuzzy sets on a universe X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as μ, ρ, η, etc.

A family τ of fuzzy sets in X is called a fuzzy topology for X if and only if (1) $\emptyset$, $X \in \tau$, (2) $\mu \wedge \rho \in \tau$ whenever μ, $\rho \in \tau$, (3) If $\mu_i \in \tau$ for each $i \in I$, then $\vee\mu_i \in \tau$. Moreover, the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a fuzzy open set [9].

In this paper, X and Y are fuzzy topological spaces. Let μ be a fuzzy set in X. We denote the interior and the closure of a fuzzy set μ by $int(\mu)$ and $cl(\mu)$, respectively.

A fuzzy set μ in a space X is called fuzzy preopen [11] (resp. fuzzy semi-open [1]) if $\mu \leq int(cl(\mu))$ (resp. $\mu \leq cl(int(\mu))$). The complement of a fuzzy preopen (resp. fuzzy semi-open) set is said to be fuzzy preclosed (resp. fuzzy semi-closed).

A fuzzy set μ in a space X is called fuzzy β-open [8] or fuzzy semipreopen [12] if $\mu \leq cl(int(cl(\mu)))$. The complement of a fuzzy β-open set is said to be fuzzy β-closed.

Let μ be a fuzzy set in a fuzzy topological space X. The fuzzy β-closure and β-interior of μ are defined as $\wedge\{\eta : \mu \leq \rho, \rho \text{ is } \beta\text{-closed}\}$, $\vee\{\eta : \mu \geq \rho, \rho \text{ is } \beta\text{-open}\}$,and denoted by β-$cl(\mu)$ and β-$int(\mu)$, respectively.

A fuzzy set in X is called a fuzzy singleton if and only if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is ε $(0 < \varepsilon \leq 1)$ we denote this fuzzy singleton by x_ε, where the point x is called its support [9]. For any fuzzy singleton x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ if and only if $\varepsilon \leq \mu(x)$.

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A fuzzy singleton x_ε is called quasi-coincident with a fuzzy set ρ, denoted by $x_\varepsilon q\rho$, iff $\varepsilon + \rho(x) > 1$. A fuzzy set μ is called quasi-coincident with a fuzzy set ρ, denoted by $\mu q\rho$, if and only if there exists a $x \in X$ such that $\mu(x) + \rho(x) > 1$.

Let $f : X \to Y$ a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$ defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ is called the fuzzy graph function of f [1].

Recall that for a fuzzy function $f : X \to Y$, the subset $\{(x_\varepsilon, f(x_\varepsilon)) : x_\varepsilon \in X\} \leq X \times Y$ is called the fuzzy graph of f and is denoted by $G(f)$.

A fuzzy set μ of a fuzzy space X is said to be fuzzy regular open (respectively fuzzy regular closed) if $\mu = int(cl(\mu))$ (respectively $\mu = cl(int(\mu))$) [1].

2. Fuzzy almost contra-β-continuous functions

In this section, the notion of fuzzy almost contra-β-continuous functions is introduced.

Definition 2.1. *Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy almost contra-β-continuous if inverse image of each fuzzy regular open set in Y is fuzzy β-closed in X.*

Theorem 2.1. *For a fuzzy function $f : X \to Y$, the following statements are equivalent:*

(1) f is fuzzy almost contra-β-continuous,

(2) for every fuzzy regular closed set μ in Y, $f^{-1}(\mu)$ is fuzzy β-open,

(3) for any fuzzy regular closed set $\mu \leq Y$ and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$,

(4) for any fuzzy regular closed set $\mu \leq Y$ and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then there exists a fuzzy β-open set η such that $x_\varepsilon q\eta$ and $f(\eta) \leq \mu$,

(5) $f^{-1}(int(cl(\mu)))$ is fuzzy β-closed for every fuzzy open set,

(6) $f^{-1}(cl(int(\rho)))$ is fuzzy β-open for every fuzzy closed subset ρ,

(7) for each fuzzy singleton $x_\varepsilon \in X$ and each fuzzy regular closed set η in Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \leq \eta$.

Proof. (1) $\Leftrightarrow$ (2) : Let ρ be a fuzzy regular open set in Y. Then, $Y\backslash\rho$ is fuzzy regular closed. By (2), $f^{-1}(Y\backslash\rho) = X\backslash f^{-1}(\rho)$ is fuzzy β-open. Thus, $f^{-1}(\rho)$ is fuzzy β-closed.

Converse is similar.

(2) $\Leftrightarrow$ (3) : Let $\mu \leq Y$ be a fuzzy regular closed set and $f(x_\varepsilon)q\mu$. Then $x_\varepsilon q f^{-1}(\mu)$ and from (2), $f^{-1}(\mu) \leq \beta\text{-}int(f^{-1}(\mu))$. From here $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$. Thus, (3) holds.

The reverse is obvious.

(3) $\Rightarrow$ (4) : Let $\mu \leq Y$ be any fuzzy regular closed set and let $f(x_\varepsilon)q\mu$. Then $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$. Take $\eta = \beta\text{-}int(f^{-1}(\mu))$, then $f(\eta) = f(\beta\text{-}int(f^{-1}(\mu))) \leq f(f^{-1}(\mu)) \leq \mu$.

(4) $\Rightarrow$ (3) : Let $\mu \leq Y$ be any fuzzy regular closed set and let $f(x_\varepsilon)q\mu$. From (4), there exists fuzzy β-open set η such that $x_\varepsilon q\eta$ and $f(\eta) \leq \mu$. From here $\eta \leq f^{-1}(\mu)$ and then $x_\varepsilon q\beta\text{-}int(f^{-1}(\mu))$.

(1) $\Leftrightarrow$ (5) : Let μ be a fuzzy open set. Since $int(cl(\mu))$ is fuzzy regular open, then by (1), it follows that $f^{-1}(int(cl(\mu)))$ is β-closed.

The converse can be shown easily.

(2) $\Leftrightarrow$ (6) : It can be obtained similar as (1)$\Leftrightarrow$(5).
(2) $\Leftrightarrow$ (7) : Obvious. ∎

Theorem 2.2. *Let $f : X \to Y$ be a fuzzy function and let $g : X \to X \times Y$ be the fuzzy graph function of f, defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ for every $x_\varepsilon \in X$. If g is fuzzy almost contra-β-continuous, then f is fuzzy almost contra-β-continuous.*

Proof. Let η be a fuzzy regular closed set in Y, then $X \times \eta$ is a fuzzy regular closed set in $X \times Y$. Since g is fuzzy almost contra-β-continuous, then $f^{-1}(\eta) = g^{-1}(X \times \eta)$ is fuzzy β-open in X. Thus, f is fuzzy almost contra-β-continuous. ∎

Definition 2.2. *A fuzzy filter base Λ is said to be fuzzy β-convergent to a fuzzy singleton x_ε in X [3] if for any fuzzy β-open set η in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.*

Definition 2.3. *A fuzzy filter base Λ is said to be fuzzy rc-convergent to a fuzzy singleton x_ε in X if for any fuzzy regular closed set η in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.*

Theorem 2.3. *If a fuzzy function $f : X \to Y$ is fuzzy almost contra-β-continuous, then for each fuzzy singleton $x_\varepsilon \in X$ and each fuzzy filter base Λ in X β-converging to x_ε, the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\varepsilon)$.*

Proof. Let $x_\varepsilon \in X$ and Λ be any fuzzy filter base in X β-converging to x_ε. Since f is fuzzy almost contra-β-continuous, then for any fuzzy regular closed set λ in Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \leq \lambda$. Since Λ is fuzzy β-converging to x_ε, there exists a $\xi \in \Lambda$ such that $\xi \leq \mu$. This means that $f(\xi) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\varepsilon)$. ∎

Definition 2.4. *A space X is called fuzzy β-connected [3] if X is not the union of two disjoint nonempty fuzzy β-open sets.*

Definition 2.5. *A space X is called fuzzy connected [10] if X is not the union of two disjoint nonempty fuzzy open sets.*

Theorem 2.4. *If $f : X \to Y$ is a fuzzy almost contra-β-continuous surjection and X is fuzzy β-connected, then Y is fuzzy connected.*

Proof. Suppose that Y is not a fuzzy connected space. There exist nonempty disjoint fuzzy open sets η_1 and η_2 such that $Y = \eta_1 \vee \eta_2$. Therefore, η_1 and η_2 are fuzzy clopen in Y. Since f is fuzzy almost contra-β-continuous, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy β-open in X. Moreover, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are nonempty disjoint and $X = f^{-1}(\eta_1) \vee f^{-1}(\eta_2)$. This shows that X is not fuzzy β-connected. This contradicts that Y is not fuzzy connected assumed. Hence, Y is fuzzy connected. ∎

Definition 2.6. *A fuzzy space X is said to be fuzzy β-normal if every pair of nonempty disjoint fuzzy closed sets can be separeted by disjoint fuzzy β-open sets.*

Definition 2.7. *A fuzzy space X is said to be fuzzy strongly normal if for every pair of nonempty disjoint fuzzy closed sets μ and η there exist disjoint fuzzy open sets ρ and ξ such that $\mu \leq \rho$, $\eta \leq \xi$ and $cl(\rho) \wedge cl(\xi) = \varnothing$.*

Theorem 2.5. *If Y is fuzzy strongly normal and $f : X \to Y$ is fuzzy almost contra-β-continuous closed injection, then X is fuzzy β-normal.*

Proof. Let η and ρ be disjoint nonempty fuzzy closed sets of X. Since f is injective and closed, $f(\eta)$ and $f(\rho)$ are disjoint fuzzy closed sets. Since Y is fuzzy strongly normal, there exist fuzzy open sets μ and ξ such that $f(\eta) \leq \mu$ and $f(\rho) \leq \xi$ and $cl(\mu) \wedge cl(\xi) = \varnothing$. Then, since $cl(\mu)$ and $cl(\xi)$ are fuzzy regular closed and f is fuzzy almost contra-β-continuous, $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi))$ are fuzzy β-open set. Since $\eta \leq f^{-1}(cl(\mu))$, $\rho \leq f^{-1}(cl(\xi))$, and $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi))$ are disjoint, X is fuzzy β-normal. ∎

Definition 2.8. *A space X is said to be fuzzy weakly T_2 if each element of X is an intersection of fuzzy regular closed sets.*

Definition 2.9. *A space X is said to be fuzzy β-T_2 [3] if for each pair of distinct points x_ε and y_ν in X, there exist fizzy β-open set μ containing x_ε and fizzy β-open set η containing y_ν such that $\mu \wedge \eta = \emptyset$.*

Definition 2.10. *A space X is said to be fuzzy β-T_1 [3] if for each pair of distinct fuzzy singletons x_ε and y_ν in X, there exist fuzzy β-open sets μ and η containing x_ε and y_ν, respectively, such that $y_\nu \notin \mu$ and $x_\varepsilon \notin \eta$.*

Theorem 2.6. *If $f : X \to Y$ is a fuzzy almost contra-β-continuous injection and Y is fuzzy Urysohn, then X is fuzzy β-T_2.*

Proof. Suppose that Y is fuzzy Urysohn. By the injectivity of f, it follows that $f(x_\varepsilon) \neq f(y_\nu)$ for any distinct fuzzy singletons x_ε and y_ν in X. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\varepsilon) \in \eta$, $f(y_\nu) \in \rho$ and $cl(\eta) \wedge cl(\rho) = \emptyset$. Since f is fuzzy almost contra-β-continuous, there exist fuzzy β-open sets μ and ξ in X containing x_ε and y_ν, respectively, such that $f(\mu) \leq cl(\eta)$ and $f(\xi) \leq cl(\rho)$. Hence $\mu \wedge \xi = \emptyset$. This shows that X is fuzzy β-T_2. ∎

Theorem 2.7. *If $f : X \to Y$ is a fuzzy almost contra-β-continuous injection and Y is fuzzy weakly T_2, then X is fuzzy β-T_1.*

Proof. Suppose that Y is fuzzy weakly T_2. For any distinct points x_ε and y_ν in X, there exist fuzzy regular closed sets η, ρ in Y such that $f(x_\varepsilon) \in \eta$, $f(y_\nu) \notin \eta$, $f(x_\varepsilon) \notin \rho$ and $f(y_\nu) \in \rho$. Since f is fuzzy almost contra-β-continuous, by Theorem 2, $f^{-1}(\eta)$ and $f^{-1}(\rho)$ are fuzzy β-open subsets of X such that $x_\varepsilon \in f^{-1}(\eta)$, $y_\nu \notin f^{-1}(\eta)$, $x_\varepsilon \notin f^{-1}(\rho)$ and $y_\nu \in f^{-1}(\rho)$. This shows that X is fuzzy β-T_1. ∎

Theorem 2.8. *Let (X_i, τ_i) be fuzzy topological space for all $i \in I$ and I be finite. Suppose that $(\prod_{i \in I} X_i, \sigma)$ is a product space and $f : (X, \tau) \to (\prod_{i \in I} X_i, \sigma)$ is any fuzzy function. If f fuzzy almost contra-β-continuous, then $pr_i \circ f$ is fuzzy almost contra-β-continuous where pr_i is projection function for each $i \in I$.*

Proof. Let $x_\varepsilon \in X$ and $(pr_i \circ f)(x_\varepsilon) \in \rho_i$ and ρ_i be a fuzzy regular closed set in (X_i, τ_i). Then $f(x_\varepsilon) \in pr_i^{-1}(\rho_i) = \rho_i \times \prod_{j \neq i} X_j$ a fuzzy regular closed set in $(\prod_{i \in I} X_i, \sigma)$. Since f is fuzzy almost contra-β-continuous, there exists a fuzzy β-open set μ containing x_ε such that $f(\mu) \leq \rho_i \times \prod_{j \neq i} X_j = pr_i^{-1}(\rho_i)$ and hence $\mu \leq (pr_i \circ f)^{-1}(\rho_i)$ and we obtain that $pr_i \circ f$ is fuzzy almost contra-β-continuous for each $i \in I$. ∎

Definition 2.11. *The fuzzy graph $G(f)$ of a fuzzy function $f : X \to Y$ is said to be fuzzy strongly contra-β-closed if for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy β-open set μ in X containing x_ε and a fuzzy regular closed set η in Y containing y_ν such that $(\mu \times \eta) \wedge G(f) = \emptyset$.*

Lemma 2.1. *The following properties are equivalent for the fuzzy graph $G(f)$ of a fuzzy function f:*

(1) $G(f)$ is fuzzy strongly contra-β-closed;

(2) for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy β-open set μ in X containing x_ε and a fuzzy regular closed set η containing y_ν such that $f(\mu) \wedge \eta = \emptyset$.

Theorem 2.9. *If $f : X \to Y$ is fuzzy almost contra-β-continuous and Y is fuzzy Urysohn, $G(f)$ is fuzzy strongly contra-β-closed in $X \times Y$.*

Proof. Suppose that Y is fuzzy Urysohn. Let $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$. It follows that $f(x_\varepsilon) \neq y_\nu$. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\varepsilon) \in \eta$, $y_\nu \in \rho$ and $cl(\eta) \wedge cl(\rho) = \emptyset$. Since f is fuzzy almost contra-β-continuous, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \leq cl(\eta)$. Therefore, $f(\mu) \wedge cl(\rho) = \emptyset$ and $G(f)$ is fuzzy strongly contra-β-closed in $X \times Y$. ∎

Theorem 2.10. *Let $f : X \to Y$ have a fuzzy strongly contra-β-closed graph. If f is injective, then X is fuzzy β-T_1.*

Proof. Let x_ε and y_ν be any two distinct points of X. Then, we have $(x_\varepsilon, f(y_\nu)) \in (X \times Y)\backslash G(f)$. By Lemma 20, there exist a fuzzy β-open set μ in X containing x_ε and a fuzzy regular closed set ρ in Y containing $f(y_\nu)$ such that $f(\mu) \wedge \rho = \emptyset$; hence $\mu \wedge f^{-1}(\rho) = \emptyset$. Therefore, we have $y_\nu \notin \mu$. This implies that X is fuzzy β-T_1. ∎

3. The relationships

In this section, the relationships between fuzzy almost contra-β-continuous functions and the other forms are investigated.

Definition 3.1. *A function $f : X \to Y$ is called fuzzy weakly almost contra-β-continuous if for each $x \in X$ and each fuzzy regular closed set η of Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $int(f(\mu)) \leq \eta$.*

Definition 3.2. *A function $f : X \to Y$ is called fuzzy (β, s)-open if the image of each fuzzy β-open set is fuzzy semi-open.*

Theorem 3.1. *If a function $f : X \to Y$ is fuzzy weakly almost contra-β-continuous and fuzzy (β, s)-open, then f is fuzzy almost contra-β-continuous.*

Proof. Let $x_\varepsilon \in X$ and η be a fuzzy regular closed set containing $f(x_\varepsilon)$. Since f is fuzzy weakly almost contra-β-continuous, there exists a fuzzy β-open set μ in X containing x_ε such that $int(f(\mu)) \leq \eta$. Since f is fuzzy (β, s)-open, $f(\mu)$ is a semi-open set in Y and $f(\mu) \leq cl(int(f(\mu))) \leq \eta$. This shows that f is fuzzy almost contra-β-continuous. ∎

Definition 3.3. *Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be*

(1) fuzzy almost contra-precontinuous if inverse image of each fuzzy regular open set in Y is fuzzy preclosed in X,

(2) fuzzy almost contra-semicontinuous if inverse image of each fuzzy regular open set in Y is fuzzy semi-closed in X,

(3) fuzzy almost contra-continuous if inverse image of each fuzzy regular open set in Y is fuzzy closed in X.

Remark 3.1. *The following diagram hold for a fuzzy function $f : X \to Y$:*

fuzzy almost contra-continuous	$\Rightarrow$	*fuzzy almost contra-semicontinuous*
$\Downarrow$		$\Downarrow$
fuzzy almost contra-precontinuous	$\Rightarrow$	*fuzzy almost contra-β-continuous*

None of the above implications is reversible.

Example 3.1. *Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:*

$$\lambda(a) = 0,3 \quad \lambda(b) = 0,6$$
$$\mu(x) = 0,3 \quad \mu(y) = 0,5$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the fuzzy function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy almost contra-β-continuous but not fuzzy almost contra-semicontinuous.

Example 3.2. *Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:*

$$\lambda(a) = 0,5 \quad \lambda(b) = 0,3$$
$$\mu(x) = 0,5 \quad \mu(y) = 0,3$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the fuzzy function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy almost contra-semicontinuous but not fuzzy almost contra-continuous.

Example 3.3. *Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:*

$$\lambda(a) = 0,3 \quad \lambda(b) = 0,4$$
$$\mu(x) = 0,5 \quad \mu(y) = 0,5$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the fuzzy function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy almost contra-β-continuous but not fuzzy almost contra-precontinuous.

Example 3.4. *Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:*

$$\lambda(a) = 0,6 \quad \lambda(b) = 0,5$$
$$\mu(x) = 0,3 \quad \mu(y) = 0,5$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the fuzzy function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy almost contra-precontinuous but not fuzzy almost contra-continuous.

Definition 3.4. *A fuzzy space is said to be fuzzy P_Σ if for any fuzzy open set μ of X and each $x_\varepsilon \in \mu$, there exists fuzzy regular closed set ρ containing x_ε such that $x_\varepsilon \in \rho \leq \mu$.*

Definition 3.5. *A fuzzy function $f : X \to Y$ is said to be fuzzy β-continuous [12] if $f^{-1}(\mu)$ is fuzzy β-open in X for every fuzzy open set μ in Y.*

Theorem 3.2. *Let $f : X \to Y$ be a fuzzy function. Then, if f is fuzzy almost contra-β-continuous and Y is fuzzy P_Σ, then f is fuzzy β-continuous.*

Proof. Let μ be any fuzzy open set in Y. Since Y is fuzzy P_Σ, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \vee\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra-β-continuous, $f^{-1}(\rho)$ is fuzzy β-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is fuzzy β-open in X. Therefore, f is fuzzy almost contra-β-continuous. ∎

Definition 3.6. *A space is said to be fuzzy weakly P_Σ if for any fuzzy regular open set μ and each $x_\varepsilon \in \mu$, there exists a fuzzy regular closed set ρ containing x_ε such that $x_\varepsilon \in \rho \leq \mu$.*

Definition 3.7. *A fuzzy function $f : X \to Y$ is said to be fuzzy almost β-continuous at $x_\varepsilon \in X$ if for each fuzzy open set η containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ containing x_ε such that $f(\mu) \leq int(cl(\eta))$.*

Theorem 3.3. *Let $f : X \to Y$ be a fuzzy almost contra-β-continuous function. If Y is fuzzy weakly P_Σ, then f is fuzzy almost β-continuous.*

Proof. Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_Σ, there exists a family Ψ whose members are fuzzy regular closed set of Y such that $\mu = \vee\{\rho : \rho \in \Psi\}$. Since f is fuzzy almost contra-β-continuous, $f^{-1}(\rho)$ is fuzzy β-open in X for each $\rho \in \Psi$ and $f^{-1}(\mu)$ is fuzzy β-open in X. Hence, f is fuzzy almost β-continuous. ∎

Definition 3.8. *A fuzzy function $f : X \to Y$ is called fuzzy β-irresolute [4] if inverse image of each fuzzy β-open set is fuzzy β-open.*

Theorem 3.4. *Let X, Y, Z be fuzzy topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be fuzzy functions. If f is fuzzy β-irresolute and g is fuzzy almost contra-β-continuous, then $g \circ f : X \to Z$ is a fuzzy almost contra-β-continuous function.*

Proof. Let $\mu \leq Z$ be any fuzzy regular closed set and let $(g \circ f)(x_\varepsilon) \in \mu$. Then $g(f(x_\varepsilon)) \in \mu$. Since g is fuzzy almost contra-β-continuous function, it follows that there exists a fuzzy β-open set ρ containing $f(x_\varepsilon)$ such that $g(\rho) \leq \mu$. Since f is fuzzy β-irresolute function, it follows that there exists a fuzzy β-open set η containing x_ε such that $f(\eta) \leq \rho$. From here we obtain that $(g \circ f)(\eta) = g(f(\eta)) \leq g(\rho) \leq \mu$. Thus, we show that $g \circ f$ is a fuzzy almost contra-β-continuous function. ∎

Definition 3.9. *A fuzzy function $f : X \to Y$ is called fuzzy β-open [3] if image of each fuzzy β-open set is fuzzy β-open.*

Theorem 3.5. *If $f : X \to Y$ is a surjective fuzzy β-open function and $g : Y \to Z$ is a fuzzy function such that $g \circ f : X \to Z$ is fuzzy almost contra-β-continuous, then g is fuzzy almost contra-β-continuous.*

Proof. Suppose that x_ε is a fuzzy singleton in X. Let η be a regular closed set in Z containing $(g \circ f)(x_\varepsilon)$. Then there exists a fuzzy β-open set μ in X containing x_ε such that $g(f(\mu)) \leq \eta$. Since f is fuzzy β-open, $f(\mu)$ is a fuzzy β-open set in Y containing $f(x_\varepsilon)$ such that $g(f(\mu)) \leq \eta$. This implies that g is fuzzy almost contra-β-continuous. ∎

Corollary 3.1. *Let $f : X \to Y$ be a surjective fuzzy β-irresolute and fuzzy β-open function and let $g : Y \to Z$ be a fuzzy function. Then, $g \circ f : X \to Z$ is fuzzy almost contra-β-continuous if and only if g is fuzzy almost contra-β-continuous.*

Proof. It can be obtained from Theorem 39 and Theorem 41. ∎

Definition 3.10. *A space X is said to be fuzzy β-compact [6] (fuzzy S-closed [2]) if every fuzzy β-open (respectively fuzzy regular closed) cover of X has a finite subcover.*

Theorem 3.6. *The fuzzy almost contra-β-continuous images of fuzzy β-compact spaces are S-closed.*

Proof. Suppose that $f : X \to Y$ is a fuzzy almost contra-β-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular closed cover of Y. Since f is fuzzy almost contra-β-continuous, then $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy β-open cover of X and hence there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$. Therefore, we have $Y = \vee\{\eta_i : i \in I_0\}$ and Y is fuzzy S-closed. ∎

Definition 3.11. *A space X is said to be*

(1) fuzzy β-closed-compact [3] if every fuzzy β-closed cover of X has a finite subcover,

(4) fuzzy nearly compact [5] if every fuzzy regular open cover of X has a finite subcover.

Theorem 3.7. *The fuzzy almost contra-β-continuous images of fuzzy β-closed-compact spaces are fuzzy nearly compact.*

Proof. Suppose that $f : X \to Y$ is a fuzzy almost contra-β-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of Y. Since f is fuzzy almost contra-β-continuous, then $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy β-closed cover of X. Since X is fuzzy β-closed-compact, there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$. Thus, we have $Y = \vee\{\eta_i : i \in I_0\}$ and Y is fuzzy nearly compact. ∎

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(Erdal Ekici) Department of Mathematics, Canakkale Onsekiz Mart University, Terzioglu Campus, 17020 Canakkale/TURKEY
E-mail address: eekici@comu.edu.tr