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# On Fuzzy P-Continuous Functions

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Abstract. In this paper, by using fuzzy property $P$, characterizations and properties of some types of fuzzy continuous functions including fuzzy continuous, fuzzy almost continuous, fuzzy c-continuous, fuzzy almost c-continuous, fuzzy c*-continuous, fuzzy s-continuous, fuzzy almost s-continuous, fuzzy $\ell$-continuous, fuzzy almost $\ell$-continuous functions are obtained.

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## 1. Introduction

It is well known that several types of fuzzy continuous functions are introduced in literature. The aim of this paper, by using property $P$, is to obtain characterizations and properties of some types of continuous functions including fuzzy continuous, fuzzy almost continuous, fuzzy c-continuous, fuzzy almost c-continuous, fuzzy c*-continuous, fuzzy s-continuous, fuzzy almost s-continuous, fuzzy $\ell$-continuous, fuzzy almost $\ell$-continuous functions.

Fuzzy sets on a universe $X$ will be denoted by Greek letters as $\mu$, $\rho$, $\eta$, etc. Fuzzy point will be denoted by $x_\varepsilon$, $y_\nu$, etc. For any fuzzy point $x_\varepsilon$ and any fuzzy set $\mu$, we write $x_\varepsilon \in \mu$ iff $\varepsilon \leq \mu(x)$. A fuzzy set $x_\varepsilon$ is called quasi-coincident with a fuzzy set $\rho$, denoted by $x_\varepsilon q \rho$, iff $\varepsilon + \rho(x) > 1$. A fuzzy set $\mu$ is called quasi-coincident with a fuzzy set $\rho$, denoted by $\mu q \rho$, iff there exists a $x \in X$ such that $\mu(x) + \rho(x) > 1$. In this paper we use the concept of a fuzzy topological space as introduced by [2]. By $int(\mu)$, $cl(\mu)$ and $co(\mu)$, we mean the interior of $\mu$, the closure of $\mu$ and complement of $\mu$.

Let $f : X \to Y$ a fuzzy function from a fuzzy topological space $X$ to a fuzzy topological space $Y$. Then the function $g : X \to X \times Y$ defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ is called the graph function of $f$ and it will be denoted by $grf$ [1].

## 2. Fuzzy P-continuous Functions

**Definition 2.1.** *Let $(X, \tau)$ be a fuzzy topological space and let $\mu \leq X$ be a fuzzy set. Then it is said that*

*i-) $\mu$ is a fuzzy $P$-set if $\mu$ possesses fuzzy property $P$,*

*ii-) $\mu$ has fuzzy $P$-complement if $co(\mu)$ possesses fuzzy property $P$.*

**Definition 2.2.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$. Then $f$ is called fuzzy $P$-continuous if for each $x_\varepsilon \in X$ and for each fuzzy set $\mu$ containing $f(x_\varepsilon)$ and having $P$-complement, there exists an open fuzzy set $\rho$ containing $x_\varepsilon$ such that $f(\rho) \leq \mu$.*

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It will be shown in Theorem 1 that the above definition is equivalent the following definition.

**Definition 2.3.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$. Then $f$ is said to be fuzzy $P$-continuous if for each $x_\varepsilon \in X$ and for each fuzzy set $\mu$ having $P$-complement such that $f(x_\varepsilon)q\mu$, there exists an open fuzzy set $\rho$ such that $x_\varepsilon q\rho$ and $f(\rho) \leq \mu$.*

The following table give us the list of some types of fuzzy $P$-continuous functions with property $P$.

The definitions of fuzzy c-continuous, fuzzy almost c-continuous, fuzzy c$^*$-continuous, fuzzy $\ell$-continuous and fuzzy almost $\ell$-continuous function are considered for fuzzy setting from [3], [4], [8], [6] and [7], respectively.

The definitions of fuzzy continuous, fuzzy almost continuous and fuzzy s-continuous function are considered from [2], [1] and [5], respectively.

| Fuzzy $P$-set | Fuzzy $P$-contunuity |
|---|---|
| 1. f. closed | f. continuity |
| 2. f. regular closed | f. almost continuity |
| 3. f. closed compact | f. c-continuity |
| 4. f. regular closed compact | f. almost c-continuity |
| 5. f. closed countable compact | f. c$^*$-continuity |
| 6. f. closed connected | f. s-continuity |
| 7. f. regular closed connected | f. almost s-continuity |
| 8. f. closed Lindelof | f. $\ell$-continuity |
| 9. f. regular closed Lindelof | f. almost $\ell$-continuity |

**Definition 2.4.** *Let $(X, \tau)$ be a fuzzy topological space and let $(x^\alpha_{\varepsilon_\alpha})$ be a net in $X$. $(x^\alpha_{\varepsilon_\alpha})$ is called fuzzy $P$-converges to $x_\varepsilon$ if for each fuzzy set $\mu$ containing $x_\varepsilon$ and having fuzzy $P$-complement, there exists an index $\alpha_0 \in J$ such that $x^\alpha_{\varepsilon_\alpha} \in \mu$ for all $\alpha \geq \alpha_0$. We will denote by $x^\alpha_{\varepsilon_\alpha} \xrightarrow{P} x_\varepsilon$.*

The following theorem give us some characterizations of fuzzy $P$-continuous function.

**Theorem 2.1.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$. Then the following statements are equivalent.*

*a-) $f$ is fuzzy $P$-continuous,*

*b-) $f^{-1}(\mu) \leq int(f^{-1}(\mu))$ for any fuzzy set $\mu \leq Y$ which has $P$-complement,*

*c-) $cl(f^{-1}(\beta)) \leq f^{-1}(\beta)$ for any fuzzy $P$-set $\beta \leq Y$,*

*d-) For any fuzzy set $\mu \leq Y$ which has $P$-complement and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then $x_\varepsilon q int(f^{-1}(\mu))$,*

*e-) For any fuzzy set $\mu \leq Y$ which has $P$-complement and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then there exists a fuzzy open set $\eta$ such that $x_\varepsilon q\eta$ and $f(\eta) \leq \mu$,*

*f-) For each $x_\varepsilon \in X$ and for each net $(x^\alpha_{\varepsilon_\alpha})$ in $X$, if $x^\alpha_{\varepsilon_\alpha} \to x_\varepsilon$, then $f(x^\alpha_{\varepsilon_\alpha}) \xrightarrow{P} f(x_\varepsilon)$.*

*Proof.* (a)⇒(b). Let $\mu \leq Y$ be a fuzzy set having $P$-complement and let $x_\varepsilon \in f^{-1}(\mu)$ be a fuzzy point. Then $f(x_\varepsilon) \in \mu$. By (a), there exists a fuzzy open set $\beta$ such that $x_\varepsilon \in \beta \leq X$ and $f(\beta) \leq \mu$. This implies that $\beta \leq f^{-1}(\mu)$ and $x_\varepsilon \in int(f^{-1}(\mu))$. Thus, $f^{-1}(\mu) \leq int(f^{-1}(\mu))$.

(b)⇒(a). Suppose that the fuzzy set $\mu \leq Y$ has $P$-complement and $f(x_\varepsilon) \in \mu$. Then $x_\varepsilon \in f^{-1}(\mu)$ and by (b), $x_\varepsilon \in f^{-1}(\mu) \leq int(f^{-1}(\mu))$. Take $\beta = int(f^{-1}(\mu))$. Thus, $f(\beta) \leq \mu$.

(b)⇔(c). Let $\beta \leq Y$ be a fuzzy $P$-set. Then the fuzzy set $co(\beta) \leq Y$ has $P$-complement. By (b), $f^{-1}(co(\beta)) \leq int(f^{-1}(co(\beta)))$ and then $co(f^{-1}(\beta)) \leq int(co(f^{-1}(\beta)))$ iff $co(f^{-1}(\beta)) \leq co(cl(f^{-1}(\beta)))$ iff $cl(f^{-1}(\beta)) \leq f^{-1}(\beta)$.

The reverse is similar.

(b)⇔(d). Suppose that the fuzzy set $\mu \leq Y$ has $P$-complement and $f(x_\varepsilon)q\mu$. Then $x_\varepsilon q f^{-1}(\mu)$ and by (b), $f^{-1}(\mu) \leq int(f^{-1}(\mu))$. Thus, $x_\varepsilon q int(f^{-1}(\mu))$ and hence, (d) holds.

The reverse is obvious.

(d)⇒(e). Let $\mu \leq Y$ be any fuzzy set having $P$-complement and let $f(x_\varepsilon)q\mu$. Then $x_\varepsilon q int(f^{-1}(\mu))$. Take $\eta = int(f^{-1}(\mu))$. Thus, $f(\eta) = f(int(f^{-1}(\mu))) \leq f(f^{-1}(\mu)) \leq \mu$.

(e)⇒(d). Let $\mu \leq Y$ be any fuzzy set having $P$-complement and let $f(x_\varepsilon)q\mu$. By (e), there exists open fuzzy set $\eta$ such that $x_\varepsilon q\eta$ and $f(\eta) \leq \mu$. Thus, $\eta \leq f^{-1}(\mu)$ and then $x_\varepsilon q int(f^{-1}(\mu))$.

(a)⇒(f). Suppose that $\rho$ is a fuzzy set containing $f(x_\varepsilon)$ and having $P$-complement. Since $f$ is fuzzy $P$-continuous, then there exists a fuzzy open set $\mu$ containing $x_\varepsilon$ such that $f(\mu) \leq \rho$. Since $x^{\alpha}_{\varepsilon_\alpha} \to x_\varepsilon$, there exists an index $\alpha_0 \in J$ such that $x^{\alpha}_{\varepsilon_\alpha} \in \mu$ for all $\alpha \geq \alpha_0$. Thus, $f(x^{\alpha}_{\varepsilon_\alpha}) \in f(\mu) \leq \rho$ and $f(x^{\alpha}_{\varepsilon_\alpha}) \in \rho$ for all $\alpha \geq \alpha_0$. Hence, $f(x^{\alpha}_{\varepsilon_\alpha}) \xrightarrow{P} f(x_\varepsilon)$.

(f)⇒(a). Suppose that (a) is not true. There exists a point $x_\varepsilon$ and a fuzzy set $\rho$ containing $f(x_\varepsilon)$ and having $P$-complement such that $f(\mu) \nleq \rho$ for each $\mu \in \tau$ where $x_\varepsilon \in \mu$. Let $x_{\varepsilon_\mu} \in \mu$ and $f(x_{\varepsilon_\mu}) \notin \rho$ for each $\mu \in \tau$ where $x_\varepsilon \in \mu$. Then for the neighborhood net $(x_{\varepsilon_\mu})$, $x_{\varepsilon_\mu} \to x_\varepsilon$, but the net $(f(x_{\varepsilon_\mu}))$ is not $P$-converges to $x_\varepsilon$. This is a contradiction. Thus, $f$ is fuzzy $P$-continuous function. ∎

**Example 2.1.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$. If we consider fuzzy $P$-set as fuzzy closed compact set, then by Theorem 1, the following statements are equivalent.*

*a-) $f$ is fuzzy c-continuous,*

*b-) $f^{-1}(\mu) \leq int(f^{-1}(\mu))$ for any fuzzy open set $\mu \leq Y$ which has compact complement,*

*c-) $cl(f^{-1}(\beta)) \leq f^{-1}(\beta)$ for any fuzzy closed compact set $\beta \leq Y$,*

*d-) For any fuzzy open set $\mu \leq Y$ which has compact complement and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then $x_\varepsilon q int(f^{-1}(\mu))$,*

*e-) For any fuzzy open set $\mu \leq Y$ which has compact complement and for any $x_\varepsilon \in X$ if $f(x_\varepsilon)q\mu$, then there exists a fuzzy open set $\eta$ such that $x_\varepsilon q\eta$ and $f(\eta) \leq \mu$,*

*f-) For each $x_\varepsilon \in X$ and for each net $x^{\alpha}_{\varepsilon_\alpha}$ in $X$, if $x^{\alpha}_{\varepsilon_\alpha} \to x_\varepsilon$, then for each fuzzy open set $\mu$ containing $f(x_\varepsilon)$ and having compact complement, there exists an index $\alpha_0 \in J$ such that $f(x^{\alpha}_{\varepsilon_\alpha}) \in \mu$ for all $\alpha \geq \alpha_0$.*

**Theorem 2.2.** *Let $(X, \tau)$, $(Y, \upsilon)$, $(Z, \omega)$ be fuzzy topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be fuzzy functions. If $f : X \to Y$ is fuzzy continuous function and $g : Y \to Z$ is fuzzy $P$-continuous function, then $g \circ f : X \to Z$ is a fuzzy $P$-continuous function.*

*Proof.* Let $\mu \leq Z$ be any fuzzy set having $P$-complement and let $(g \circ f)(x_\varepsilon) \in \mu$. Then $g(f(x_\varepsilon)) \in \mu$. Since $g$ is fuzzy $P$-continuous function, then there exists an open fuzzy set $\rho$ containing $f(x_\varepsilon)$ such that $g(\rho) \leq \mu$. Since $f$ is fuzzy continuous function, then there exists an open fuzzy set $\beta$ containing $x_\varepsilon$ such that $f(\beta) \leq \rho$. This implies that $(g \circ f)(\beta) = g(f(\beta)) \leq g(\rho) \leq \mu$. Thus, $g \circ f$ is a fuzzy $P$-continuous function. ∎

**Theorem 2.3.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$ and let $\mu \leq X$ be a fuzzy set. If $f$ is a fuzzy $P$-continuous function, then the restriction function $f \mid_\mu : \mu \to Y$ is a fuzzy $P$-continuous function.*

*Proof.* Suppose that $\beta \leq Y$ is a fuzzy set having $P$-complement. Let $x_\varepsilon \in \mu$ and let $f \mid_\mu (x_\varepsilon) \in \beta$. Since $f$ is fuzzy $P$-continuous, then there exists a fuzzy open set $x_\varepsilon \in \rho$ such that $f(\rho) \leq \beta$. We have $x_\varepsilon \in \rho \wedge \mu$ and $f(\rho \wedge \mu) = f \mid_\mu (\rho \wedge \mu) \leq \beta$. Thus, the restriction function $f \mid_\mu$is a fuzzy $P$-continuous. ∎

**Theorem 2.4.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$. Let $\{\gamma_\alpha : \alpha \in \Phi\}$ be an open cover of $X$. If the restriction function $f_\alpha = f_{\gamma_\alpha}$ is a fuzzy $P$-continuous function for each $\alpha \in \Phi$, then $f$ is a fuzzy $P$-continuous function.*

*Proof.* Let $\mu \leq Y$ be a fuzzy set having $P$-complement. Since $f_\alpha$ is fuzzy $P$-continuous for each $\alpha$, by Theorem 1, $f_\alpha^{-1}(\mu) \leq int_{\gamma_\alpha}(f_\alpha^{-1}(\mu))$ and then $f^{-1}(\mu) \wedge \gamma_\alpha \leq int_{\gamma_\alpha}(f^{-1}(\mu) \wedge \gamma_\alpha) = int_X(f^{-1}(\mu)) \wedge \gamma_\alpha$. Since $\{\gamma_\alpha : \alpha \in \Phi\}$ is an open cover of $X$, then $f^{-1}(\mu) \leq int_X(f^{-1}(\mu))$. Thus, by Theorem 1, $f$ is a fuzzy $P$-continuous function. ∎

**Theorem 2.5.** *Suppose that a finete product of fuzzy $P$-sets is a fuzzy $P$-set. Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$ and let $X$ be a fuzzy $P$-set. If the graph function of $f$ is fuzzy $P$-continuous, then $f$ is fuzzy $P$-continuous function.*

*Proof.* Let $x_\varepsilon \in X$, $f(x_\varepsilon) \in \upsilon$ and let $\upsilon$ be a fuzzy set having $P$-complement. Then $grf(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon)) \in X \times \upsilon$ and $X \times \upsilon$ is a fuzzy set having $P$-complement. Since $grf$ is fuzzy $P$-continuous, then there exists an open fuzzy set $\beta$ containing $x_\varepsilon$ such that $grf(\beta) \leq X \times \upsilon$. We have $f(\beta) \leq \upsilon$. Thus, $f$ is fuzzy $P$-continuous function. ∎

**Theorem 2.6.** *Suppose that for each $\alpha \in J$, $(X_\alpha, \tau_\alpha)$ and $(X, \tau)$ are fuzzy topological spaces and $X_\alpha$ possesses property $P$ for each $\alpha \in J$. Let $f : (X, \tau) \to (\prod_{\alpha \in J} X_\alpha, \tau')$ be a fuzzy function from $X$ to the product space $\prod_{\alpha \in J} X_\alpha$ and let product of fuzzy $P$-sets be a fuzzy $P$-set. If $f$ is fuzzy $P$-continuous function, then each $p_\alpha \circ f$ is fuzzy $P$-continuous function where $p_\alpha$ is the projection function for each $\alpha \in J$.*

*Proof.* Let $\mu$ be any fuzzy set having $P$-complement and let $(p_\alpha \circ f)(x_\varepsilon) \in \mu$. Then $p_\alpha(f(x_\varepsilon)) \in \mu$. We have $f(x_\varepsilon) \in p_\alpha^{-1}(\mu) = \mu \times \prod_{\beta \neq \alpha} X_\beta$. Since product of fuzzy $P$-sets is fuzzy $P$-set and $X_\alpha$ possesses property $P$ for each $\alpha$, then $\mu \times \prod_{\beta \neq \alpha} X_\beta$ has $P$-complement. This implies that there exists an open fuzzy set $\rho$ containing $x_\varepsilon$ such that $f(\rho) \leq \mu \times \prod_{\beta \neq \alpha} X_\beta$. We obtain $p_\alpha(f(\rho)) = (p_\alpha \circ f)(\rho) \leq p_\alpha(\mu \times \prod_{\beta \neq \alpha} X_\beta) = \mu$. Thus, $p_\alpha \circ f$ is fuzzy $P$-continuous function for each $\alpha \in J$. ∎

**Theorem 2.7.** *Suppose that $(X_\alpha, \tau_\alpha)$, $(Y_\alpha, \upsilon_\alpha)$ are fuzzy topological spaces and $Y_\alpha$ possesses property $P$ for each $\alpha \in J$ and the product of fuzzy $P$-sets is a fuzzy $P$-set. Let $f_\alpha : X_\alpha \to Y_\alpha$ be a fuzzy function for each $\alpha \in J$ and let the fuzzy function $f : \prod\limits_{\alpha\in J} X_\alpha \to \prod\limits_{\alpha\in J} Y_\alpha$ be defined by $f((x_\alpha)) = (f_\alpha(x_\alpha))$ from the product space $\prod\limits_{\alpha\in J} X_\alpha$ to the product space $\prod\limits_{\alpha\in J} Y_\alpha$. If $f$ is fuzzy $P$-continuous function, then each $f_\alpha$ is fuzzy $P$-continuous function.*

*Proof.* Let $p_\alpha : \prod\limits_{\alpha\in J} X_\alpha \to X_\alpha$ and $q_\alpha : \prod\limits_{\alpha\in J} Y_\alpha \to Y_\alpha$ be the projection functions. We have $q_\alpha \circ f = f_\alpha \circ p_\alpha$ for each $\alpha \in J$.

Let $x_\alpha \in X_\alpha$ and let $\mu_\alpha \le Y_\alpha$ be a fuzzy set containing $f_\alpha(x_\alpha)$ and having $P$-complement. Take any point $x_\varepsilon \in p_\alpha^{-1}(x_\alpha)$. Since each $Y_\alpha$ is fuzzy $P$-set and the product of fuzzy $P$-sets is a fuzzy $P$-set, then $q_\alpha^{-1}(\mu_\alpha) = \mu_\alpha \times \prod\limits_{\beta\ne\alpha} Y_\beta \le \prod\limits_{\alpha\in J} Y_\alpha$ is a fuzzy set having $P$-complement. This implies that there exists an open fuzzy set $\beta$ containing $x_\varepsilon$ such that $f(\beta) \le q_\alpha^{-1}(\mu_\alpha)$. We obtain $q_\alpha(f(\beta)) \le \mu_\alpha$ and $(f_\alpha \circ p_\alpha)(\beta) = (q_\alpha \circ f)(\beta) \le \mu_\alpha$. Since $p_\alpha(\beta)$ is an open fuzzy set containing $x_\alpha$, then $f_\alpha$ is $P$-continuous fuzzy function. ∎

**Theorem 2.8.** *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $(X, \tau)$ to a fuzzy topological space $(Y, \upsilon)$. Suppose that $Y$ have a base of neigbourhood such that complement of each fuzzy set of the base of neighbourhood is finite unions of $P$-sets. If $f$ is fuzzy $P$-continuous function, then $f$ is fuzzy continuous.*

*Proof.* Let $x_\varepsilon \in X$ and let $\mu$ be a fuzzy open set containing $f(x_\varepsilon)$. Then there exists a neighbourhood $\beta$ of $f(x_\varepsilon)$ such that $\beta \le \mu$ and $co(\beta) = \bigvee\limits_{i=1}^{n} \eta_i$ where each $\eta_i$ is a $P$-set. We have $\beta = \bigwedge\limits_{i=1}^{n} co(\eta_i)$. Then $f(x_\varepsilon) \in co(\eta_i)$ for each $i = 1, 2, ..., n$. Since $f$ is $P$-continuous fuzzy function, then there exists an open set $\rho_i$ containing $x_\varepsilon$ such that $f(\rho_i) \le co(\eta_i)$. Take $\rho = \bigwedge\limits_{i=1}^{n} \rho_i$. Then $f(\rho) \le \bigwedge\limits_{i=1}^{n} f(\rho_i) \le \bigwedge\limits_{i=1}^{n} co(\eta_i) = \beta \le \mu$. Thus, $f$ is continuous fuzzy function. ∎

**Theorem 2.9.** *Suppose that $(X_1, \tau_1)$, $(X_2, \tau_2)$, $(Y_1, \upsilon_1)$ and $(Y_2, \upsilon_2)$ are fuzzy topological spaces and $f_1 : X_1 \to Y_1$, $f_2 : X_2 \to Y_2$ are fuzzy functions and suppose that $\eta \times \beta$ has fuzzy $P$-complement iff $\eta$ and $\beta$ have fuzy $P$-complements for any fuzzy sets $\eta \le Y_1$, $\beta \le Y_2$. Let $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ be a fuzzy function which is defined by $(f_1 \times f_2)(x_\varepsilon, y_\nu) = (f_1(x_\varepsilon), f_2(y_\nu))$. Then $f_1 \times f_2$ is fuzzy $P$-continuous function iff $f_1$ and $f_2$ are fuzzy $P$-continuous functions.*

*Proof.* It is know that $(\mu^* \times \beta^*)(x_\varepsilon, y_\nu) = \min\{(\mu^*(x), \beta^*(y)\}$ for any fuzzy sets $\mu^*$, $\beta^*$ and for any fuzzy points $x_\varepsilon$, $y_\nu$. Let $\mu \times \beta \le Y_1 \times Y_2$ be a fuzzy set having $P$-complement. We have $(f_1 \times f_2)^{-1}(\mu \times \beta) = f_1^{-1}(\mu) \times f_2^{-1}(\beta)$. By Theorem 1, the proof is obtained. ∎

**Theorem 2.10.** *Suppose that $(X, \tau)$, $(Y, \upsilon)$, $(Z, \omega)$ are fuzzy topological spaces and $f_1 : X \to Y$, $f_2 : X \to Z$ are fuzzy functions and suppose that $\eta \times \beta$ has fuzzy $P$-complement iff $\eta$ and $\beta$ have fuzy $P$-complements for any fuzzy sets $\eta \le Y$, $\beta \le Z$. Let $f_1 \times f_2 : X \to Y \times Z$ be a fuzzy function which is defined by $(f_1 \times f_2)(x_\varepsilon) = (f_1(x_\varepsilon), f_2(x_\varepsilon))$. Then $f_1 \times f_2$ is fuzzy $P$-continuous function iff $f_1$ and $f_2$ are fuzzy $P$-continuous functions.*

*Proof.* (⇒:) Let $x_\varepsilon \in X$ and let $\mu \leq Y$, $\beta \leq Z$ be fuzzy sets having $P$-complement such that $f_1(x_\varepsilon) \in \mu$ and $f_2(x_\varepsilon) \in \beta$. We have $(f_1 \times f_2)(x_\varepsilon) = (f_1(x_\varepsilon), f_2(x_\varepsilon)) \in \mu \times \beta$ and $\mu \times \beta$ has $P$-complement. Since the fuzzy function $f_1 \times f_2$ is fuzzy $P$-continuous, then there exists an open fuzzy set $\rho$ containing $x_\varepsilon$ such that $(f_1 \times f_2)(\rho) \leq \mu \times \beta$. We have $(f_1 \times f_2)(\rho) = f_1(\rho) \times f_2(\rho) \leq \mu \times \beta$ and $f_1(\rho) \leq \mu$, $f_2(\rho) \leq \beta$. Thus, $f_1$ and $f_2$ are fuzzy $P$-continuous functions.

(⇐:) Let $x_\varepsilon \in X$ and let $\mu \times \rho \leq Y \times Z$ be a fuzzy set having $P$-complement such that $(f_1 \times f_2)(x_\varepsilon) \in \mu \times \rho$. We have $(f_1 \times f_2)(x_\varepsilon) = (f_1(x_\varepsilon), f_2(x_\varepsilon)) \in \mu \times \rho$ and $f_1(x_\varepsilon) \in \mu$, $f_2(x_\varepsilon) \in \rho$. Since $f_1$ and $f_2$ are fuzzy $P$-continuous functions, then there exists fuzzy open sets $\eta$ containing $x_\varepsilon$ and $\beta$ containing $x_\varepsilon$ such that $f_1(\eta) \leq \mu$ and $f_2(\beta) \leq \rho$. Take $x_\varepsilon \in \eta \wedge \beta$. Thus, $(f_1 \times f_2)(\eta \wedge \beta) \leq \mu \times \rho$ and hence $f_1 \times f_2$ is fuzzy $P$-continuous function. ∎

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