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Entire large solutions for logistic-type equations

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Abstract. We are discussing the existence of large solutions in $\mathbb{R}^N$ for $\Delta u = e^{-|x|^a} u^\alpha f(u)$ and $\Delta u = u + e^{-|x|^a} u^\alpha f(u)$. We prove that even though both equations $\Delta u = u$ and $\Delta u = e^{-|x|^a} u^\alpha f(u)$ have positive entire large solutions, the equation $\Delta u = u + e^{-|x|^a} u^\alpha f(u)$ does not have such solutions for $a = 1$, where $N \geq 3$, $\alpha > 2$ and f denotes a function satisfying hypotheses $f \in C^1([0, \infty))$, $f' \geq 0$, $f \geq 1$. In addition, for $a > 1$ sufficiently large, this last equation might still have positive entire large solutions.

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1. Introduction

We consider the following class of semilinear elliptic equations

$$\begin{cases} \Delta u = u + e^{-|x|^a} u^\alpha f(u) & \text{in } \mathbb{R}^N, \\ u \geq 0, \ u \not\equiv 0 & \text{in } \mathbb{R}^N, \end{cases} \tag{1}$$

where $N \geq 3$, $a \geq 1$, $\alpha > 1$ and f is under the assumptions

$$f \in C^1([0, \infty)), \quad f' \geq 0, \quad f \geq 1. \tag{2}$$

Definition 1.1. *(i) A positive solution u of an elliptic equation on $\Omega \not\equiv \mathbb{R}^N$ satisfying the condition*

$$u(x) \to \infty, \qquad \text{as } \ dist(x, \partial\Omega) \to 0$$

is called a large (blow-up, explosive) solution of that equation.

(ii) A positive solution u of an elliptic equation on $\mathbb{R}^N$ satisfying the condition

$$u(x) \to \infty, \qquad \text{as } \ |x| \to \infty$$

is called a positive entire large solution of that equation.

In this paper we are concerned with the existence and nonexistence of positive entire large solutions for (1). There is a vast literature on elliptic problems that have solutions which blow up. Starting with the pioneering papers [2], [13], [7], [12], problems related to large solutions have a long history, arise naturally from a number of different areas and are studied by many authors and in many contexts.

In 1916, in [2], Bieberbach studied the equation $\Delta u = e^u$ in the plane and in 1943, in [13], Rademacher studied the same equation in the space. Later on, singular value problems of this type were studied under the general form $\Delta u = f(u)$ in N-dimensional domains. A special attention was paid to the equations of the form

$$\Delta u = p(x) u^\gamma \tag{3}$$

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which was studied in bounded and in unbounded domains.

We are interested here only in studying the existence of positive entire large solutions, thus we will insist on the results obtained in unbounded domains. In [4], Cheng and Ni proved that (3) has a unique entire large solution in $\mathbb{R}^N$ provided that function p is positive and smooth, $\gamma > 1$ and that there exists $m > 2$ such that $|x|^m p(x)$ is bounded for large $|x|$. In [1], Bandle and Marcus showed the existence and uniqueness of a positive entire large solution for the more general equation

$$\Delta u = g(x, u),$$

which includes the case $g(x, u) = p(x)u^\gamma$ where $\gamma > 1$ and the function $p(x)$ is positive and continuous such that p and $1/p$ are bounded. Also, the study of superlinear case where $\gamma > 1$ is included in the more recent work [5], as we will see in Section 2. Nonexistence results of large positive solutions for (3) with $\gamma > 1$ were given in [11], [10] and [3]. For the sublinear case, where $0 < \gamma \leq 1$, Lair and Wood stated in [9] that for radial p with $\int_0^\infty rp(r)dr = \infty$ equation (3) has a positive entire large solution.

Let us denote

$$M_p(r) \equiv \max_{|x|=r} p(x) \tag{4}$$

and

$$m_p(r) \equiv \min_{|x|=r} p(x).$$

In fact, the characterization of the existence and the nonexistence of positive entire large solutions for (3) is the following. For the superlinear case where $\gamma > 1$ equation (3) has such solutions if p satisfies

$$\int_0^\infty rM_p(r)dr < \infty$$

and it will not generally have positive entire large solutions if

$$\int_0^\infty rm_p(r)dr = \infty.$$

The sublinear case, where $0 < \gamma \leq 1$, behaves generally in the opposite manner. For more information on problems with large solutions we refer to [14] or to the recent book [6].

In the present paper we are concerned with the intriguing situation when

$$\Delta u = q_1(x)u^{\gamma_1} \qquad \text{and} \qquad \Delta u = q_2(x)u^{\gamma_2}$$

both have positive entire large solutions while

$$\Delta u = q_1(x)u^{\gamma_1} + q_2(x)u^{\gamma_2}$$

does not have. This is the case for

$$\Delta u = u + e^{-|x|}u^\alpha, \tag{5}$$

where $\alpha > 2$ (see [8]). We will show here that a similar situation takes place for our equation (1) when $a = 1$ and $\alpha > 2$. Even more interesting, for a larger, (1) may have such solutions. We note here that equation (1) with $a = 1$ is more general then (5) since conditions (2) imposed on f are quite permissive. Therefore f can be chosen to be an appropriate polynomial function or logarithmical, exponential etc.

2. Preliminary results

In order to avoid repetition, we state here that everywhere below $N \geq 3$, $a \geq 1$, $\alpha > 1$ and f denotes a function satisfying (2). Also, throughout this paper, C will denote a universal positive constant, depending on different parameters, whose value may change from line to line.

We consider the following semilinear elliptic equation:

$$\begin{cases} \Delta u = p(|x|)g(u) & \text{in } \mathbb{R}^N, \\ u \geq 0, \ u \not\equiv 0 & \text{in } \mathbb{R}^N, \end{cases} \tag{6}$$

where $p > 0$.

We recall an important result (see [5]).

Theorem 2.1. *Assume that the nonlinearity g verifies*

- ***(g1)*** $g \in C^1([0, \infty))$, $g' \geq 0$, $g(0) = 0$ *and* $g > 0$ *on* $(0, \infty)$ *and the Keller-Osserman condition (see* [7] *and* [12]*)*
- ***(g2)*** $\int_1^\infty [2G(t)]^{-1/2} dt < \infty$ *where* $G(t) = \int_0^t g(s)ds$.

Also, assume that the function $p \in C^{0,\mu}_{\mathrm{loc}}(\mathbb{R}^N)$ $(0 < \mu < 1)$ *verifies* $p > 0$ *and*

- ***(p)*** $\int_0^\infty rp(r)dr < \infty$.

Then (6) *has a positive entire large solution.*

With the aid of the above theorem we can prove the following lemma.

Lemma 2.1. *Equation*

$$\begin{cases} \Delta u = e^{-|x|^a} u^\alpha f(u) & \text{in } \mathbb{R}^N, \\ u \geq 0, \ u \not\equiv 0 & \text{in } \mathbb{R}^N. \end{cases} \tag{7}$$

has a positive entire large solution.

Proof. We will show that we are under the hypotheses of Theorem 2.1.

First we consider the function g defined by $g(u) := u^\alpha f(u)$. We will show that g verifies conditions (g1) and (g2).

Due to the fact that f is satisfying (2), is easy to see that g is satisfying (g1). It remains to see if (g2) is also fulfilled.

Since $f \geq 1$,

$$G(t) = \int_0^t g(s)ds = \int_0^t s^\alpha f(s)ds \geq \int_0^t s^\alpha ds = \frac{t^{\alpha+1}}{\alpha+1}.$$

There exists t_0 (e.g. $t_0 := \alpha + 1$) such that for all $t > t_0 > 1$,

$$2G(t) \geq \frac{2}{\alpha+1} t^{\alpha+1} \geq 1.$$

Hence, for $t > t_0 > 1$,

$$[2G(t)]^{-1/2} \leq \left[\frac{2}{\alpha+1} t^{\alpha+1}\right]^{-1/2}$$

which implies

$$\int_{t_0}^\infty [2G(t)]^{-1/2} dt \leq \int_{t_0}^\infty \left[\frac{2}{\alpha+1} t^{\alpha+1}\right]^{-1/2} dt.$$

We obtain

$$\begin{aligned}\int_1^\infty [2G(t)]^{-1/2}dt &= \int_1^{t_0}[2G(t)]^{-1/2}dt + \int_{t_0}^\infty [2G(t)]^{-1/2}dt\\ &\le C + C\int_{t_0}^\infty t^{-(\alpha+1)/2}dt\\ &\le C + C\cdot t^{(-\alpha+1)/2}\big|_{t_0}^\infty .\end{aligned}$$

But $\alpha > 0$, therefore $\lim_{t\to\infty} t^{(-\alpha+1)/2} = 0$ and consequently

$$\int_1^\infty [2G(t)]^{-1/2}dt < \infty.$$

Now it only remains to show that

$$p_1(r) := e^{-r^a}$$

satisfies (p) for every $a \ge 1$. We have

$$\int_0^\infty rp_1(r)dr = \int_0^\infty re^{-r^a}dr = \int_0^1 re^{-r^a}dr + \int_1^\infty re^{-r^a}dr \le C + \int_1^\infty re^{-r^a}dr.$$

Since $a \ge 1$, then $-r^a \le -r$, for any $r \ge 1$. Hence

$$\int_0^\infty rp_1(r)dr \le C + \int_1^\infty re^{-r}dr = C - \int_1^\infty r\left(e^{-r}\right)' dr = C - re^{-r}\big|_1^\infty - e^{-r}\big|_1^\infty < \infty.$$

Therefore equation (7) has a positive entire large solution.

□

3. Main result

We first consider the particular case of equation (1) when $a = 1$ and $\alpha > 2$, namely

$$\begin{cases} \Delta u = u + e^{-|x|}u^\alpha f(u) & \text{in } \mathbb{R}^N,\\ u \ge 0,\ u \not\equiv 0 & \text{in } \mathbb{R}^N. \end{cases} \tag{8}$$

Theorem 3.1. *Although both equations*

$$\begin{cases} \Delta u = u & \text{in } \mathbb{R}^N,\\ u \ge 0,\ u \not\equiv 0 & \text{in } \mathbb{R}^N, \end{cases} \tag{9}$$

and

$$\begin{cases} \Delta u = e^{-|x|}u^\alpha f(u) & \text{in } \mathbb{R}^N,\\ u \ge 0,\ u \not\equiv 0 & \text{in } \mathbb{R}^N, \end{cases} \tag{10}$$

have positive entire large solutions, equation (8) *has no such solutions.*

Proof. The fact that (9) has positive entire large solutions is well known, while (10) is a particular case of (7) for $a = 1$. Consequently, by Lemma 2.1, this second equation also has positive entire large solutions.

We focus on showing that (8) does not have such solutions. The proof basically follows the same ideas as in the proof for (5), see [8]. Arguing by contradiction, we assume that there exists a positive entire large solution w of (8). Then, we can assume that there exists a radial solution u such that u satisfies the integral equation

$$u(r) = u_0 + \int_0^r t^{1-N}\int_0^t s^{N-1}[u(s) + e^{-s}u^\alpha(s)f(u(s))]dsdt, \tag{11}$$

where $0 < u_0 =: u(0) < w(0)$. We note that if (11) did not have a positive solution valid for all $r > 0$, its solution u, since it is an increasing function, would blow up at some $R > 0$ letting u to be a positive large solution of $\Delta u = u + e^{-|x|}u^\alpha f(u)$ on the ball $|x| \le R$, therefore $u \ge w$ on $|x| \le R$ and $u_0 < w(0)$ which is a contradiction. Since we have established that u satisfies (11) and it is clear that $e^{-s}u^\alpha(s)f(u(s)) \ge 0$,

$$u(r) \ge u_0 + \int_0^r t^{1-N} \int_0^t s^{N-1}u(s)dsdt.$$

We substitute $u(r) \ge u_0$ into the right side and we obtain

$$u(r) \ge u_0 \left(1 + \frac{r^2}{1!2^1 N}\right).$$

We substitute this new expression into the right side and we obtain

$$u(r) \ge u_0 \left(1 + \frac{r^2}{1!2^1 N} + \frac{r^4}{2!2^2 N(N+2)}\right).$$

Continuing to substitute every new expression obtained into the right side we arrive at

$$u(r) \ge u_0 \sum_{k=0}^{\infty} \frac{r^{2k}}{k!2^k N(N+2) \dots (N+2k-2)}.$$

Rewriting, we get

$$u(r) \ge u_0 \Gamma(N/2) \sum_{k=0}^{\infty} \frac{1}{k!\Gamma(\frac{N}{2}+k)} \left(\frac{r}{2}\right)^{2k}$$

hence

$$u(r) \ge C r^{1-N/2} I_{N/2-1}(r),$$

where $I_{N/2-1}$ is the modified Bessel function with index $N/2-1$. It is a known fact that for r large enough there exists C such that

$$I_{N/2-1}(r) \ge C e^r r^{-1/2}.$$

Combining the last two relations we obtain

$$u(r) \ge C e^r r^{(1-N)/2} \qquad \text{for large } r,$$

thus there exists $\varepsilon > 0$ small enough such that

$$u(r) \ge \varepsilon[1 + e^{(1-\varepsilon)r}] \qquad \text{for all } r \ge 0. \tag{12}$$

We will choose $\varepsilon > 0$ small enough so that $\alpha - \frac{1}{1-\varepsilon} > 1$.

For $\beta := \alpha - \frac{1}{1-\varepsilon} > 1$ and $c_0 := \varepsilon^{1/(1-\varepsilon)}$, let v_n be a nonnegative solution to the problem

$$\begin{cases} \Delta v_n = c_0 v_n^\beta & \text{in } \Omega_n, \\ v_n(x) \to \infty & \text{as } x \to \partial\Omega_n, \end{cases}$$

where by Ω_n we understand the ball $|x| < n$. Since $\overline{\Omega}_n \subset \Omega_{n+1}$ we can apply, for each $n \ge 1$, the maximum principle in order to find that $v_n \ge v_{n+1}$ in Ω_n. The nonnegative sequence $(v_n)_n$ is monotonically decreasing and thus converges to a function v on $\mathbb{R}^N$ with

$$\Delta v = c_0 v^\beta. \tag{13}$$

If we show that $u \le v$, it follows that v is a positive entire large solution of (13) and we obtain the desired contradiction since (13) has no such solution (see [7] and [12]). Therefore, when we will show that $u \le v$, the proof of our theorem will be complete.

To obtain $u \le v$ we will show that $u \le v_n$ in Ω_n, for all n, since $\mathbb{R}^N = \bigcup_{n=1}^\infty \Omega_n$. For that we will use again the method of reduction to absurdity. Suppose that there exists a n_0 such that $\max_{\overline{\Omega}_{n_0}}[u(x) - v_{n_0}(x)] > 0$. Since this maximum cannot occur on $\partial\Omega_{n_0}$, we deduce that there exists $x_0 \in \Omega_{n_0}$ where it does occur. Keeping in mind (12) and that $f \geq 1$, at this point x_0 we have

$$\begin{aligned}
0 &\geq \Delta(u - v_{n_0}) = u + e^{-|x_0|}u^\alpha f(u) - c_0 v_{n_0}^\beta \\
&\geq u + e^{-|x_0|}u^\alpha - c_0 v_{n_0}^\beta \\
&\geq u + e^{-|x_0|}u^{1/(1-\varepsilon)}u^\beta - c_0 v_{n_0}^\beta \\
&\geq u + e^{-|x_0|}\left[\varepsilon\left(1 + e^{(1-\varepsilon)|x_0|}\right)\right]^{1/(1-\varepsilon)} u^\beta - c_0 v_{n_0}^\beta \\
&\geq u + c_0 u^\beta - c_0 v_{n_0}^\beta > 0,
\end{aligned}$$

which is a contradiction.

□

4. An open problem

As we have seen in the last section, equation (1) has no positive entire large solutions for $a = 1$ and $\alpha > 2$. But what happens when a is bigger? One way to approach this matter is by considering the problem

$$\begin{cases} \Delta u = e^{-|x|^a + \frac{\alpha+2N-5}{2(N-2)}|x|^b} u^\alpha f(u) & \text{in } \mathbb{R}^N, \\ u \geq 0,\ u \not\equiv 0 & \text{in } \mathbb{R}^N. \end{cases} \tag{14}$$

We will demonstrate that equation (14) admits positive entire large solutions for $a > 3$ and $b \in (2, a-1)$. As in the proof of Lemma 2.1, we can prove that we are under hypotheses of Theorem 2.1 for $g(u) = u^\alpha f(u)$. Therefore it remains to show that

$$p_2(r) := e^{-r^a + \frac{\alpha+2N-5}{2(N-2)} r^2}$$

satisfies (p) for every $a > 3$ and $b \in (2, a-1)$.

For $r > \frac{\alpha+2N-5}{2(N-2)} + 1 > 1$, $a > 3$ and $b \in (2, a-1)$ it is true that $r^{a-b} > \frac{\alpha+2N-5}{2(N-2)} + 1$, hence

$$r^a > \frac{\alpha+2N-5}{2(N-2)} r^b + r^b > \frac{\alpha+2N-5}{2(N-2)} r^b + r.$$

We immediately deduce that

$$\begin{aligned}
\int_0^\infty r p_2(r) dr &= \int_0^\infty r e^{-r^a + \frac{\alpha+2N-5}{2(N-2)} r^b} dr \\
&= \int_0^{\frac{\alpha+2N-5}{2(N-2)}+1} r e^{-r^a + \frac{\alpha+2N-5}{2(N-2)} r^b} dr + \\
&+ \int_{\frac{\alpha+2N-5}{2(N-2)}+1}^\infty r e^{-r^a + \frac{\alpha+2N-5}{2(N-2)} r^b} dr \\
&\leq C + \int_{\frac{\alpha+2N-5}{2(N-2)}+1}^\infty r e^{-r} dr < \infty.
\end{aligned}$$

Based on Theorem 2.1, for $a > 3$ and $b \in (2, a-1)$, we can consider a positive radial entire large solution k of (14),

$$k(r) = k_0 + \int_0^r t^{1-N} \int_0^t s^{N-1} e^{-s^a + \frac{\alpha+2N-5}{2(N-2)} s^b} k^\alpha(s) f(k(s)) ds dt,$$

for some positive k_0 and a positive radial entire large solution h of (9),

$$h(r) = 1 + \int_0^r t^{1-N} \int_0^t s^{N-1} h(s) ds dt.$$

In order to show that (1) has positive entire large solutions for $a > 3$ and $b \in (2, a-1)$, we need to show that the integral equation

$$z(r) = z_0 + \int_0^r t^{1-N} \int_0^t s^{N-1} \left[z(s) + e^{-s^a} z^\alpha(s) f(z(s)) \right] ds dt, \tag{15}$$

has, for a well chosen z_0, a positive solution valid for all $r \geq 0$. It is clear that such a solution z, if exists, will of necessity be large because

$$z(r) \geq z_0 + \int_0^r t^{1-N} \int_0^t s^{N-1} z(s) ds dt \geq z_0 + z_0 \int_0^r t^{1-N} \int_0^t s^{N-1} ds dt \to \infty,$$

when $r \to \infty$. Thus it is enough to prove that equation (15) has a valid solution for all nonnegative r. If we let $z_0 \in (0, k_0)$, we deduce that a solution to (15) exists on some interval. It remains the question: is this interval $[0, \infty)$?

A problem of this type was treated by Lair in his recent work [8], and we refer to the problem

$$\Delta u = M_p(|x|) u + M_q(|x|) u^\beta, \tag{16}$$

where M_p, M_q are given as in the formula (4), $\beta > 1$, q satisfies

$$\int_0^\infty r M_q(r) e^{(\beta-1)(N-2) \int_0^r s M_p(s) ds} dr < \infty$$

and p satisfies

$$\int_0^\infty r M_p(r) dr = \infty.$$

The existence of a positive entire large solution to (16) was proved by showing that the integral equation

$$w(r) = w_0 + \int_0^r t^{1-N} \int_0^t s^{N-1} \left[M_p(s) w(s) + M_q(s) w^\beta(s) \right] ds dt$$

has, for an appropriately chosen positive value for w_0, a positive solution valid for all nonnegative r. In order to obtain such a result, Lair established that

$$w(r) \leq w_1(r) w_2(r) \qquad \text{for all } r \geq 0,$$

where w_1, w_2 are positive radial entire large solutions respectively to

$$\Delta u = M_p(r) u \quad \text{and} \quad \Delta u = M_q(r) e^{(\beta-1)(N-2) \int_0^r s M_p(s) ds} u^\beta.$$

In our case we should prove that $z(r) \leq h(r) k(r)$ on $[0, \infty)$. Following the steps from [8] we define R as

$$R := \sup\{ r_0 \in (0, \infty) | \quad z(r) \leq h(r) k(r) \quad \text{for all } r \in [0, r_0] \}.$$

If $R = \infty$, we are done. Arguing by contradiction, we suppose $R < \infty$. We have

$$\begin{aligned} z(R) &= z_0 + \int_0^R t^{1-N} \int_0^t s^{N-1} \left[z(s) + e^{-s^a} z^\alpha(s) f(z(s)) \right] ds dt \\ &< k_0 + \int_0^R t^{1-N} \int_0^t s^{N-1} \left[h(s) k(s) + e^{-s^a} h^\alpha(s) k^\alpha(s) f(z(s)) \right] ds dt. \end{aligned} \tag{17}$$

On the other hand,

$$\Delta(hk) \geq h \Delta k + k \Delta h.$$

We know that k and h are, respectively, positive radial entire large solutions of (14) and (9),

$$\Delta(hk) \geq he^{-|x|^a+\frac{\alpha+2N-5}{2(N-2)}|x|^b}k^\alpha f(k)+kh,$$

which, in radial form, produces

$$\left[r^{N-1}(kh)'\right]' \geq r^{N-1}\left[he^{-|x|^a+\frac{\alpha+2N-5}{2(N-2)}|x|^b}k^\alpha f(k)+kh\right].$$

Integrating, we come to

$$h(r)k(r) \geq k_0+\int_0^r t^{1-N}\int_0^t s^{N-1}\left[h(s)e^{-s^a+\frac{\alpha+2N-5}{2(N-2)}s^b}k^\alpha(s)f(k(s))+k(s)h(s)\right]dsdt, \tag{18}$$

for all $r \geq 0$. To obtain the desired contradiction, we would like to have

$$h^{\alpha-1}f(z) < e^{\frac{\alpha+2N-5}{2(N-2)}r^b}f(k), \tag{19}$$

for all $r \in [0,R]$. That is because if relation (19) holds, then, by (17) and (18), we have

$$z(R) < h(R)k(R)$$

and, consequently, there exists $\varepsilon > 0$ such that $z(r) < h(r)k(r)$ on $[0, R+\varepsilon]$. This contradicts the definition of R.

Let us focus on (19). We take into account that

$$h(r) \leq 1+\frac{1}{N-2}\int_0^r sh(s)ds, \quad \forall r \quad \text{(see relation (9) in [9])}$$

and by applying Grönwall's inequality we obtain

$$h(r) \leq e^{\int_0^r \frac{s}{N-2}ds},$$

therefore

$$h^{\alpha-1}(r) \leq e^{\frac{r^2(\alpha-1)}{2(N-2)}},$$

for $r > 1$. Thus for $r > 1$ and $b > 2$,

$$h^{\alpha-1}(r) < e^{\left[\frac{\alpha-1}{2(N-2)}+1\right]r^b} = e^{\frac{\alpha+2N-5}{2(N-2)}r^b}.$$

Unfortunately, this is not enough to prove relation (19) since f is increasing and $z < hk$ on $[0, R)$. For now, we let this matter as an open problem. Still, for b big enough, we believe that (19) could hold, giving us the contradiction mentioned above.

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