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ON A GENERALIZATION OF SLIGHT CONTINUITY

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Abstract. In 2002 Baker [1] introduced the notion of slightly precontinuous functions. In that paper, it is shown that the slightly precontinuous image of a preconnected space is connected. In this paper, we consider and study a weaker form of slightly precontinuity, precontinuity, α-continuity, α-irresoluteness and strongly α-irresoluteness called slightly δ-precontinuity. Furthermore, basic properties, preservation theorems and relationships of slightly δ-precontinuous functions are investigated.

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1. Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. General topologists have introduced and investigated many different generalizations of continuous functions. The purpose of this paper is to study slightly δ-precontinuous functions and to obtain several characterizations and properties of slightly δ-precontinuous functions. Moreover, the relationships between slightly δ-precontinuous functions and graphs are also investigated.

2. Preliminaries

In this paper, spaces (X, τ) and (Y, υ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of (X, τ), $cl(A)$ and $int(A)$ represent the closure of A and the interior of A, respectively.

A subset A of a space X is said to be regular open (respectively regular closed) if $A = int(cl(A))$ (respectively $A = cl(int(A))$) [15].

The δ-interior [16] of a subset A of X is the union of all regular open sets of X contained in A is denoted by $\delta - int(A)$. A subset A is called δ-open [16] if $A = \delta - int(A)$, i. e., a set is δ-open if it is the union of regular open sets. The complement of δ-open set is called δ-closed. Alternatively, a set A of (X, τ) is called δ-closed [16] if $A = \delta - cl(A)$, where $\delta - cl(A) = \{x \in X : A \cap int(cl(U)) \neq \emptyset, U \in \tau$ and $x \in U\}$.

A subset A of a space X is said to be α-open [11] (resp. preopen [9], δ-preopen [12]) if $A \subset int(cl(int(A)))$ (resp. $A \subset int(cl(A))$, $A \subset int(\delta\text{-}cl(A))$). The family of all preopen (resp. δ-preopen) sets of X containing a point $x \in X$ is denoted by $PO(X, x)$ (resp. $\delta PO(X, x)$).

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The complement of a preopen set is said to be preclosed [5].

The complement of a δ-preopen set is said to be δ-preclosed. The intersection of all δ-preclosed sets of X containing A is called the δ-preclosure [12] of A and is denoted by δ-$pcl(A)$. The union of all δ-preopen sets of X contained A is called δ-preinterior of A and is denoted by δ-$pint(A)$ [12]. A subset U of X is called a δ-preneighborhood [12] of a point $x \in X$ if there exists a δ-preopen set V such that $x \in V \subset U$. Note that $\delta\text{-}pcl(A) = A \cup cl(\delta\text{-}int(A))$ and $\delta\text{-}pint(A) = A \cap int(\delta\text{-}cl(A))$.

The family of all δ-open (resp. δ-preopen, δ-preclosed, clopen, δ-preclopen) sets of X is denoted by $\delta O(X)$ (resp. $\delta PO(X)$, $\delta PC(X)$, $CO(X)$, $\delta PCO(X)$).

Definition 2.1. *A function $f : X \to Y$ is said to be*

(1) δ-almost continuous [12] if $f^{-1}(V)$ is δ-preopen set in X for each open set V of Y,

(2) slightly continuous [7] if $f^{-1}(V)$ is open set in X for each clopen set V of Y,

(3) precontinuous [2, 9] if $f^{-1}(V)$ is preopen set in X for each open set V of Y,

(4) α-continuous [10] if $f^{-1}(V)$ is α-open in X for every open set V of Y,

(5) α-irresolute [8] if $f^{-1}(V)$ is α-open set in X for each α-open set V of Y,

(6) strongly α-irresolute [6] if for each $x \in X$ and each α-open subset V of Y containing $f(x)$, there exists an open subset U of X containing x such that $f(U) \subset V$,

(7) slightly precontinuous [1] if $f^{-1}(V)$ is preopen set in X for each clopen set V of Y.

3. Slightly δ-precontinuous functions

In this section, the notion of slightly δ-precontinuous functions is introduced and characterizations and some relationships of slightly δ-precontinuous functions and basic properties of slightly δ-precontinuous functions are investigated and obtained.

Definition 3.1. *A function $f : X \to Y$ is called*

(1) slightly δ-precontinuous at a point $x \in X$ if for each clopen subset V in Y containing $f(x)$, there exists a δ-preopen subset U in X containing x such that $f(U) \subset V$,

(2) slightly δ-precontinuous if it has this property at each point of X.

Theorem 3.1. *Let (X, τ) and (Y, υ) be topological spaces. The following statements are equivalent for a function $f : X \to Y$:*

(1) f is slightly δ-precontinuous;

(2) for every clopen set $V \subset Y$, $f^{-1}(V)$ is δ-preopen;

(3) for every clopen set $V \subset Y$, $f^{-1}(V)$ is δ-preclosed;

(4) for every clopen set $V \subset Y$, $f^{-1}(V)$ is δ-preclopen.

Proof. (1) $\Rightarrow$ (2) : Let V be a clopen subset of Y and let $x \in f^{-1}(V)$. Since $f(x) \in V$, by (1), there exists a δ-preopen set U_x in X containing x such that $U_x \subset f^{-1}(V)$. We obtain that $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Thus, $f^{-1}(V)$ is δ-preopen.

(2) $\Rightarrow$ (3) : Let V be a clopen subset of Y. Then, $Y \backslash V$ is clopen. By (2), $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is δ-preopen. Thus, $f^{-1}(V)$ is δ-preclosed.

(3) $\Rightarrow$ (4) : It can be shown easily.

(4) $\Rightarrow$ (1) : Let V be a clopen subset in Y containing $f(x)$. By (4), $f^{-1}(V)$ is δ-preclopen. Take $U = f^{-1}(V)$. Then, $f(U) \subset V$. Hence, f is slightly δ-precontinuous. ∎

Remark 3.1. *The following diagram holds:*

$$\begin{array}{ccccc}
 & & \textit{slightly continuous} & & \\
 & & \Downarrow & & \\
\textit{precontinuous} & \Rightarrow & \textit{slightly precontinuous} & \Rightarrow & \textit{slightly } \delta\textit{-precontinuous} \\
\Uparrow & & & & \\
\alpha\textit{-continuous} & & & & \\
\Uparrow & & & & \\
\alpha\textit{-irresolute} & & & & \\
\Uparrow & & & & \\
\textit{strongly } \alpha\textit{-irresolute} & & & &
\end{array}$$

None of these implications is reversible.

Example 3.1. *Let $\mathbb{R}$ be the set of real numbers and τ be the countable extension topology on $\mathbb{R}$, i. e., the topology with subbase $\tau_1 \cup \tau_2$ where τ_1 is the Euclidean topology of $\mathbb{R}$ and τ_2 is the topology of countable complements of $\mathbb{R}$ and σ be the discrete topology of $\mathbb{R}$. Define a function $f : (\mathbb{R}, \tau) \to (\mathbb{R}, \sigma)$ as follows:*

$$f(x) = \begin{cases} 1 & \text{, if } x \text{ is rational,} \\ 2 & \text{, if } x \text{ is irrational.} \end{cases}$$

Then f is slightly δ-precontinuous but not slightly precontinuous, since $\{1\}$ is clopen in $(\mathbb{R}, \sigma)$ and $f^{-1}(\{1\}) = \mathbb{Q}$ where $\mathbb{Q}$ is the set of rationals is not preopen in $(\mathbb{R}, \tau)$.

The other implications are not reversible as shown in several papers [1, 2, 8-10].

Definition 3.2. *Let (X, τ) be a topological space. The collection of all regular open sets forms a base for a topology τ^*. It is called the semiregularization.*
In case when $\tau = \tau^*$, the space (X, τ) is called semi-regular [15].

Theorem 3.2. *Let $f : X \to Y$ be a function from a semi-regular topological space (X, τ) to a topological space (Y, υ). f is slightly δ-precontinuous if and only if f is slightly precontinuous.*

Proof. Obvious. ∎

Lemma 3.1. *Let A and X_0 be subsets of a space (X, τ). If $A \in \delta PO(X)$ and $X_0 \in \delta O(X)$, then $A \cap X_0 \in \delta PO(X_0)$ [12].*

Theorem 3.3. *If $f : X \to Y$ is slightly δ-precontinuous and $A \in \delta O(X)$, then the restriction $f\mid_A : A \to Y$ is slightly δ-precontinuous.*

Proof. Let V be a clopen subset of Y. We have $(f\mid_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is δ-preopen and A is δ-open, it follows from the previous lemma that $(f\mid_A)^{-1}(V)$ is δ-preopen in the relative topology of A. Thus, $f\mid_A$ is slightly δ-precontinuous. ∎

Lemma 3.2. *Let $A \subset X_0 \subset X$. If $X_0 \in \delta O(X)$ and $A \in \delta PO(X_0)$, then $A \in \delta PO(X)$ [12].*

Theorem 3.4. *Let $f : X \to Y$ be a function and $\{U_\alpha : \alpha \in I\}$ be a cover of X such that $U_\alpha \in \delta O(X)$ for each $\alpha \in I$. If $f\mid_{U_\alpha}$ is slightly δ-precontinuous for each $\alpha \in I$, then f is a slightly δ-precontinuous function.*

Proof. Suppose that V is any clopen set of Y. Since $f\mid_{U_\alpha}$ is slightly δ-precontinuous for each $\alpha \in I$, it follows that $(f\mid_{U_\alpha})^{-1}(V) \in \delta PO(U_\alpha)$. We have $f^{-1}(V) = \bigcup_{\alpha\in I}(f^{-1}(V) \cap U_\alpha) = \bigcup_{\alpha\in I}(f\mid_{U_\alpha})^{-1}(V)$. Then, as a direct consequence of the previous lemma we obtain that $f^{-1}(V) \in \delta PO(X)$ which means that f is slightly δ-precontinuous. ∎

Theorem 3.5. *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is slightly δ-precontinuous, then f is slightly δ-precontinuous.*

Proof. Let $V \in CO(Y)$, then $X\times V \in CO(X\times Y)$. Since g is slightly δ-precontinuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \delta PO(X)$. Thus, f is slightly δ-precontinuous. ∎

Definition 3.3. *A function $f : X \to Y$ is called δ-preirresolute [3] if for every δ-preopen subset G of Y, $f^{-1}(G)$ is δ-preopen in Y.*

Theorem 3.6. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*

(1) If f is δ-preirresolute and g is slightly δ-precontinuous, then $g \circ f : X \to Z$ is slightly δ-precontinuous.

(2) If f is δ-preirresolute and g is δ-almost continuous, then $g \circ f : X \to Z$ is slightly δ-precontinuous.

(3) If f is δ-preirresolute and g is slightly continuous, then $g\circ f : X \to Z$ is slightly δ-precontinuous.

Proof. (1) Let V be any clopen set in Z. Since g is slightly δ-precontinuous, $g^{-1}(V)$ is δ-preopen. Since f is δ-preirresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is δ-preopen. Therefore, $g \circ f$ is slightly δ-precontinuous.

(2) and (3) can be obtained similarly. ∎

Definition 3.4. *A function $f : X \to Y$ is called δ-preopen [3] if for every δ-preopen subset A of X, $f(A)$ is δ-preopen in Y.*

Theorem 3.7. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is δ-preopen and surjective and $g \circ f : X \to Z$ is slightly δ-precontinuous, then g is slightly δ-precontinuous.*

Proof. Let V be any clopen set in Z. Since $g \circ f$ is slightly δ-precontinuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is δ-preopen. Since f is δ-preopen, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is δ-preopen. Hence, g is slightly δ-precontinuous. ∎

Combining the previous two theorem, we obtain the following result.

Theorem 3.8. *Let $f : X \to Y$ be surjective, δ-preirresolute and δ-preopen and $g : Y \to Z$ be a function. Then $g \circ f : X \to Z$ is slightly δ-precontinuous if and only if g is slightly δ-precontinuous.*

Definition 3.5. *A filter base Λ is said to be δ-preconvergent to a point x in X [3] if for any $U \in \delta PO(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.*

Definition 3.6. *A filter base Λ is said to be co-convergent to a point x in X if for any $U \in CO(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.*

Theorem 3.9. *If a function $f : X \to Y$ is slightly δ-precontinuous, then for each point $x \in X$ and each filter base Λ in X which is δ-preconvergent to x, the filter base $f(\Lambda)$ is co-convergent to $f(x)$.*

Proof. Let $x \in X$ and Λ be any filter base in X which is δ-preconvergent to x. Since f is slightly δ-precontinuous, then for any $V \in CO(Y)$ containing $f(x)$, there exists a $U \in \delta PO(X)$ containing x such that $f(U) \subset V$. Since Λ is δ-preconvergent to x, there exists a $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Lambda)$ is co-convergent to $f(x)$. ∎

Theorem 3.10. *Let $f : X \to Y$ be a function and $x \in X$. If there exists $U \in \delta O(X)$ such that $x \in U$ and the restriction of f to U is a slightly δ-precontinuous function at x, then f is slightly δ-precontinuous at x.*

Proof. Suppose that $F \in CO(Y)$ containing $f(x)$. Since $f\mid_U$ is slightly δ-precontinuous at x, there exists $V \in \delta PO(U)$ containing x such that $f(V) = (f\mid_U)(V) \subset F$. Since $U \in \delta O(X)$ containing x, it follows from Lemma 3.2 that $V \in \delta PO(X)$ containing x. This shows clearly that f is slightly δ-precontinuous at x. ∎

Definition 3.7. *A space X is said to be δ-pre-T_1 [3] if for each pair of distinct points x and y of X, there exist δ-preopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.*

Definition 3.8. *A space X is said to be clopen T_1 [4] if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.*

Remark 3.2. *The following implications are hold for a topological space X:*
(1) clopen $T_1 \Rightarrow T_1$,
(2) $T_1 \Rightarrow \delta$-pre-T_1.
None of these implications is reversible.

Example 3.2. *([4]) Let $\mathbb{R}$ be the real numbers with the finite complements topology τ. Then $(\mathbb{R}, \tau)$ is T_1 but not clopen T_1.*

Example 3.3. *Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$. Then (X, τ) is δ-pre-T_1 but not T_1.*

Theorem 3.11. *If $f : X \to Y$ is slightly δ-precontinuous injection and Y is clopen T_1, then X is δ-pre-T_1.*

Proof. Suppose that Y is clopen T_1. For any distict points x and y in X, there exist V, $W \in CO(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is slightly δ-precontinuous, $f^{-1}(V)$ and $f^{-1}(W)$ are δ-preopen subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is δ-pre-T_1. ∎

Definition 3.9. *A space X is said to be δ-pre-T_2 (δ-pre-Hausdorff) [3] if for each pair of distinct points x and y in X, there exist disjoint δ-preopen sets U and V in X such that $x \in U$ and $y \in V$.*

Definition 3.10. *A space X is said to be clopen T_2 (clopen Hausdorff) [4] if for each pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X such that $x \in U$ and $y \in V$.*

Remark 3.3. *The notions of clopen T_2 and clopen T_1 are equivalent.*

Theorem 3.12. *If $f : X \to Y$ is a slightly δ-precontinuous injection and Y is clopen T_1, then X is δ-pre-T_2.*

Proof. For any pair of distict points x and y in X, there exist disjoint clopen sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is slightly δ-precontinuous, $f^{-1}(U)$ and $f^{-1}(V)$ is δ-preopen in X containing x and y, respectively. We have $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. This shows that X is δ-pre-T_2. ∎

Lemma 3.3. *Let A be a subset of a space (X, τ). Then $A \in \delta PO(X)$ if and only if $A \cap U \in \delta PO(X)$ for each regular open (δ-open) set U of X [12].*

Definition 3.11. *A function $f : X \to Y$ is complete slightly continuous if $f^{-1}(V)$ is regular open set in X for each clopen set V of Y.*

Theorem 3.13. *If $f : X \to Y$ is complete slightly continuous function and $g : X \to Y$ is slightly δ-precontinuous function and Y is clopen Hausdorff, then $E = \{x \in X : f(x) = g(x)\}$ is δ-preclosed in X.*

Proof. If $x \in X \backslash E$, then it follows that $f(x) \neq g(x)$. Since Y is clopen Hausdorff, there exist $V \in CO(Y)$ and $W \in CO(Y)$ containing $f(x)$ and $g(x)$, respectively, such that $V \cap W = \emptyset$. Since f is complete slightly continuous and g is slightly δ-precontinuous, then $f^{-1}(V)$ is regular open and $g^{-1}(W)$ is δ-preopen in X with $x \in f^{-1}(V)$ and $x \in g^{-1}(W)$. Set $O = f^{-1}(V) \cap g^{-1}(W)$. Then, by the previous lemma, O is δ-preopen. Therefore $f(O) \cap g(O) = \emptyset$ and it follows that $x \notin \delta - pcl(E)$. This shows that E is δ-preclosed in X. ∎

4. Several properties

In this section, several properties of slightly δ-precontinuous functions are investigated.

Definition 4.1. *A space X is said to be mildly compact [13] (respectively δ-pre-compact [3]) if every clopen cover (resp. δ-preopen cover) of X has a finite subcover.*

A subset A of a space X is said to be mildly compact (respectively δ-pre-compact) relative to X if every cover of A by clopen (resp. δ-preopen) sets of X has a finite subcover.

A subset A of a space X is said to be mildly compact (respectively δ-pre-compact) if the subspace A is mildly compact (resp. δ-pre-compact).

Theorem 4.1. *If a function $f : X \to Y$ is slightly δ-precontinuous and K is δ-pre-compact relative to X, then $f(K)$ is mildly compact in Y.*

Proof. Let $\{H_\alpha : \alpha \in I\}$ be any cover of $f(K)$ by clopen sets of the subspace $f(K)$. For each $\alpha \in I$, there exists a clopen set K_α of Y such that $H_\alpha = K_\alpha \cap f(K)$. For each $x \in K$, there exists $\alpha_x \in I$ such that $f(x) \in K_{\alpha_x}$ and there exists $U_x \in \delta PO(X)$ containing x such that $f(U_x) \subset K_{\alpha_x}$. Since the family $\{U_x : x \in K\}$ is a cover of K by δ-preopen sets of K, there exists a finite subset K_0 of K such that $K \subset \{U_x : x \in K_0\}$. Therefore, we obtain $f(K) \subset \bigcup\{f(U_x) : x \in K_0\}$ which is a subset of $\bigcup\{K_{\alpha_x} : x \in K_0\}$. Thus $f(K) = \bigcup\{H_{\alpha_x} : x \in K_0\}$ and hence $f(K)$ is mildly compact. ∎

Corollary 4.1. *If $f : X \to Y$ is slightly δ-precontinuous surjection and X is δ-pre-compact, then Y is mildly compact.*

Definition 4.2. *A space X said to be*

(1) mildly countably compact [13] if every clopen countably cover of X has a finite subcover.

(2) mildly Lindelof [13] if every cover of X by clopen sets has a countable subcover.

(3) countably δ-pre-compact [3] if every δ-preopen countably cover of X has a finite subcover.

(4) δ-pre-Lindelof [3] if every δ-preopen cover of X has a countable subcover.

Theorem 4.2. *Let $f : X \to Y$ be a slightly δ-precontinuous surjection. Then the following statements hold:*

(1) if X is δ-pre-Lindelof, then Y is mildly Lindelof.

(2) if X is countably δ-pre-compact, then Y is mildly countably compact.

Proof. We prove (1), the proof of (2) being entirely analogous.

Let $\{V_\alpha : \alpha \in I\}$ be any clopen cover of Y. Since f is slightly δ-precontinuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a δ-preopen cover of X. Since X is δ-pre-Lindelof, there exists a countable subset I_0 of I such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \bigcup\{V_\alpha : \alpha \in I_0\}$ and Y is mildly Lindelof. ∎

Definition 4.3. *A space X said to be*

(1) δ-preclosed-compact [3] if every δ-preclosed cover of X has a finite subcover.

(2) countably δ-preclosed-compact [3] if every countable cover of X by δ-preclosed sets has a finite subcover.

(3) δ-preclosed-Lindelof [3] if every cover of X by δ-preclosed sets has a countable subcover.

Theorem 4.3. *Let $f : X \to Y$ be a slightly δ-precontinuous surjection. Then the following statements hold:*

(1) if X is δ-preclosed-compact, then Y is mildly compact.

(1) if X is δ-preclosed-Lindelof, then Y is mildly Lindelof.

(2) if X is countably δ-preclosed-compact, then Y is mildly countably compact.

Proof. It can be obtained similarly as the previous theorem. ∎

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 4.4. *A graph $G(f)$ of a function $f : X \to Y$ is said to be strongly δ-pre-co-closed if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in \delta PCO(X)$ containing x and $V \in CO(Y)$ containing y such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 4.1. *A graph $G(f)$ of a function $f : X \to Y$ is strongly δ-pre-co-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in \delta PCO(X)$ containing x and $V \in CO(Y)$ containing y such that $f(U) \cap V = \emptyset$.*

Theorem 4.4. *If $f : X \to Y$ is slightly δ-precontinuous and Y is clopen T_1, then $G(f)$ is strongly δ-pre-co-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$, then $f(x) \neq y$ and there exists a clopen set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is slightly δ-precontinuous, then $f^{-1}(V) \in \delta PCO(X)$ containing x. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Therefore, we obtain $f(U) \cap (Y\backslash V) = \emptyset$ and $Y\backslash V \in CO(Y)$ containing y. This shows that $G(f)$ is strongly δ-pre-co-closed in $X \times Y$. ∎

Theorem 4.5. *Let $f : X \to Y$ has a strongly δ-pre-co-closed graph $G(f)$. If f is injective, then X is δ-pre-T_1.*

Proof. Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y)\backslash G(f)$. By definition of strongly δ-pre-co-closed graph, there exist a δ-preclopen set U of X and $V \in CO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \emptyset$; hence $U \cap f^{-1}(V) = \emptyset$. Therefore, we have $y \notin U$. This implies that X is δ-pre-T_1. ∎

Theorem 4.6. *Let $f : X \to Y$ has a strongly δ-pre-co-closed graph $G(f)$. If f is surjective δ-preopen function, then Y is δ-pre-T_2.*

Proof. Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y)\backslash G(f)$. By strongly δ-pre-co-closedness of graph $G(f)$, there exist a δ-preclopen set U of X and $V \in CO(Y)$ such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \emptyset$. Then, we have $f(U) \cap V = \emptyset$. Since f is δ-preopen, then $f(U)$ is δ-preopen such that $f(x) = y_1 \in f(U)$. This implies that Y is δ-pre-T_2. ∎

Definition 4.5. *A space X is called δ-preconnected [3] provided that X is not the union of two disjoint nonempty δ-preopen sets.*

Theorem 4.7. *If $f : X \to Y$ is slightly δ-precontinuous surjective function and X is δ-preconnected space, then Y is connected space.*

Proof. Suppose that Y is not connected space. Then there exists nonempty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since f is slightly δ-precontinuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are δ-preclosed and δ-preopen in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not δ-preconnected. This is a contradiction. By contradiction, Y is connected. ∎

Definition 4.6. *A topological space X is called hyperconnected [14] if every open set is dense.*

Remark 4.1. *The following example shows that slightly δ-precontinuous surjection do not necessarily preserve hyperconnectedness.*

Example 4.1. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}\}$ and $\sigma = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly δ-precontinuous surjective. (X, τ) is hyperconnected. But (X, σ) is not hyperconnected.*

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