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Conditional graphs generated by conditional schemas

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Abstract. In this paper we introduce the following concepts: conditional knowledge piece, conditional binary relation, conditional schema and conditional graph. We prove that every conditional schema generates one and only one conditional graph. A conditional knowledge piece includes sentences of the form if-then. A conditional schema can represent a conditional knowledge piece. The reasoning mechanism in a conditional schema is based on the paths in the corresponding conditional graph. This mechanism is described in a forthcoming paper. All the concepts presented in this paper are exemplified.

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1. Introduction

The rule-based reasoning was used successfully by various deductive systems. The inference engine of such systems combines the rules to obtain a final conclusion. Two main combination methods can be used: the backward method and the forward method ([5], [6], [7]). Several appreciated systems, such as MYCIN, JESS and GURU, were developed based on these ideas ([1], [2], [3], [4]).

An inference engine for a rule-based reasoning uses a given set R of rules of the form IF-THEN. In general a rule is defined by a proper syntax of the reasoning system. Various fields can be specified by the knowledge engineer: the name of a rule, the priority, the cost, an access code etc. Every time these fields are the same.

In this paper we try to initiate a research line concerning the representation of conditional knowledge. We propose a structure named conditional schema, that can represent this kind of knowledge. In order to prepare the reasoning mechanism for this structure, the concept of conditional graph is introduced. We prove that a conditional schema generates one and only one conditional graph. Intuitively a conditional graph is a directed labeled graph with additional information on the arcs. The reasoning mechanism for a conditional schema is a path-driven reasoning in the corresponding conditional graph.

The structure of this article is the following. In Section 2 we define what we mean by a conditional knowledge piece and we exemplify the basic concepts by an intuitive presentation. In the same section we define the concept of conditional binary relation and we develop the algebra of these relations. Our representation method uses some rules to help the reasoning process. The syntax of these rules is given in Section 3. We introduce the definition of the conditional schema in Section 4. In Section 5 we present the manner in which a conditional graph can be constructed based on the components of a conditional schema. The last section summarizes the theoretical

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concepts of this article and describes the future directions for our research in the domain of conditional knowledge representation and reasoning.

2. Conditional binary relations

By a *knowledge piece* KP we understand a text in a natural language that contains a description of some objects and the relations between them. In general nothing about the reasoning rules is specified in KP. We shall suppose that the relations between objects are binary ones, that is, each relation is a subset of some Cartesian product of two sets. The information specified in a knowledge piece is named *initial knowledge*. The start point of the research concerning the representation of the conditional knowledge can be considered the paper [8], where the most concepts are treated by an intuitive manner. In this paper we develop the ideas presented in [8].

An inference engine can combine the entities of the initial knowledge in order to obtain new properties for the objects, that are not specified in KP.

Definition 2.1. *A* **conditional knowledge piece** *consists of some initial knowledge and a set of sentences of the form if-then, each sentence representing a rule that can be used by the reasoning process.*

In general, from a knowledge piece KP some directed graph can be obtained. The nodes of this graph are the objects of KP and the arcs are given by the binary relations specified in KP.

A binary relation ρ on the set X is a subset $\rho \subseteq X \times X$ and a binary relation from X to Y is a subset of $X \times Y$. The binary relations of a knowledge piece KP are obtained by extracting from KP all the ordered pairs of objects satisfying some property. For example, if *Peter is John's brother* and *Mike is George's brother* are two sentences of KP then, the following binary relation is obtained:

$$is\_brother = \{(Peter, John), (Mike, George)\}$$

Let us now consider the following sentences:

If Bob lives in a fish bowl then it is a fish.

If Peter obtains the best score then he is a winner.

In order to describe this situation we consider the representation:

$((Bob, fish), p_1)$,

$((Peter, winner), p_2)$

where p_1 is the condition *Bob lives in a fish bowl* and p_2 represents the condition *Peter obtains the best score*. In this way, for the set:

$$X = \{Bob, fish, Peter, winner\}$$

we obtain the binary relation:

$$\rho = \{(Bob, fish), (Peter, winner)\}$$

and if we denote $P = \{p_1, p_2\}$ then the following relation from $X \times X$ to P is obtained:

$$\theta = \{((Bob, fish), p_1), ((Peter, winner), p_2)\}$$

We shall say that θ is a *conditional binary relation*. In the intuitive meaning we have:

$(Bob, fish)$ belongs to ρ

$(Bob, fish)$ belongs to θ if p_1 is *true*

In other words, by the conditional relation θ we represent the following rules:

If p_1 then $(Bob, fish) \in \theta$

If p_2 then $(Peter, winner) \in \theta$

In order to understand the rôle of the conditional binary relations in the reasoning mechanism we present in an intuitive manner this mechanism. Let us consider the following knowledge piece: *Peter is a winner if Peter obtains the best score. Peter is a teacher if Peter is graduated in higher teaching. John is a winner. Every winner obtains a diploma. Every teacher obtains a special prize if he is under 35 years old.* Let us denote by X the set of all objects described by the knowledge piece:

$$X = \{Peter, John, winner, teacher, diploma, special\_prize\}$$

We consider the following mappings:

$p_3 : X \longrightarrow \{true, false\}$

$p_3(x) = \begin{cases} true & \text{if } \ x \text{ obtains the best score} \\ false & \text{otherwise} \end{cases}$

$p_4 : X \longrightarrow \{true, false\}$

$p_4(x) = \begin{cases} true & \text{if } \ x \text{ is graduated in higher teaching} \\ false & \text{otherwise} \end{cases}$

$p_5 : X \longrightarrow \{true, false\}$

$p_5(x) = \begin{cases} true & \text{if } \ x \text{ is under 35 years old} \\ false & \text{otherwise} \end{cases}$

Let us consider the following conditional binary relations:

$\rho_1 = \{((Peter, winner), p_3(Peter)), ((John, winner), T),$
$((Peter, teacher), p_4(Peter))\}$

$\rho_2 = \{((winner, diploma), T), ((teacher, special\_prize), p_5(teacher))\}$

It is not difficult to observe that the semantics of the fact that $((x, y), p(x)) \in \rho_1$ is "*x is a y if p(x)*". This is a property for all pairs of ρ_1. In a similar manner, each tuple $((x, y), p(x)) \in \rho_2$ has the meaning "*every x obtains a y if p(x)*".

A natural reasoning mechanism that uses the above knowledge piece obtains the following conclusions:

- "*John obtains a diploma*" because "*John is a winner*" and "*Every winner obtains a diploma*".
- "*Peter obtains a diploma if Peter obtains the best score*" because "*Peter is a winner if Peter obtains the best score*" and "*Every winner obtains a diploma*".
- "*Peter obtains a special prize if he is under 35 years old and if Peter is graduated in higher teaching*" because "*Peter is a teacher if Peter is graduated in higher teaching*" and "*Peter is under 35 years old*".

The above reasoning can be modeled by means of a binary algebraic operation defined for conditional relations. This is a partial operation and for the example of above we obtain:

$$\rho_1 \circ \rho_2 = \{((Peter, diploma), p_3(Peter)), ((John, diploma), T),$$
$$((Peter, special\_prize), p_5(Peter) \wedge p_4(Peter))\}$$

In the remainder of this section we formalize all the theoretical aspects.

Definition 2.2. *Let us consider a set X of objects and a set P such that every element $p \in P$ is a mapping $p : X \longrightarrow \{true, false\}$. An (X, P)-**conditional binary relation** (shortly, a conditional relation) is a finite set of elements of the form $((x, y), p(x))$, where $x, y \in X$.*

Definition 2.3. *If $p : X \longrightarrow \{true, false\}$ and $q : X \longrightarrow \{true, false\}$ are two arbitrary mappings we define the mapping $p \wedge q : X \longrightarrow \{true, false\}$ as follows:*

$$p \wedge q(x) = true$$

if and only if $p(x) = true$ and $q(x) = true$.

Proposition 2.1. *The structure* $(\{true, false\}^X, \wedge)$ *is a semigroup.*

Proof. The binary operation $\wedge$ is associative because $(p \wedge q) \wedge r = p \wedge (q \wedge r)$. Really, the following sentences are equivalent:

- $(p \wedge q) \wedge r(x) = true$
- $p(x) = q(x) = r(x) = true$
- $p \wedge (q \wedge r)(x) = true$

■

Remark 2.1. *In what follows we suppose that* $P \subseteq \{true, false\}^X$ *is a closed set with respect to the operation* $\wedge$*.*

Remark 2.2. *Every* **binary relation** *can be considered a* **conditional binary relation**. *Really, the above relation* ρ *can be written:*

$$\rho = \{((Bob, fish), T), ((Peter, winner), T)\}$$

where T *is the mapping* $T(x) = true$ *for all* $x \in X$*.*

Remark 2.3. *In order to have a single representation for both types of relations we shall consider that both are conditional relations with the difference that the classical relations are unconditionally true.*

Definition 2.4. *We define the following binary operation on the set of all (X,P)-conditional relations:*

$$\rho_1 \circ \rho_2 = \{((x, z), p_1 \wedge p_2(x)) \mid \exists y : ((x, y), p_1(x)) \in \rho_1, ((y, z), p_2(y)) \in \rho_2\}$$

Proposition 2.2. *The operation* $\circ$ *is associative:* $(\rho_1 \circ \rho_2) \circ \rho_3 = \rho_1 \circ (\rho_2 \circ \rho_3)$ *for every (X,P)-conditional relations* ρ_1, ρ_2, ρ_3*.*

Proof. Really, if $((x, y), p_1 \wedge p_2(x)) \in (\rho_1 \circ \rho_2) \circ \rho_3$ then there are $((x, z_1), p_1(x)) \in \rho_1 \circ \rho_2$ and $((z_1, y), p_3(z_1)) \in \rho_3$. From $((x, z_1), p_1(x)) \in \rho_1 \circ \rho_2$ we deduce that there are $m \in X$ and $p_{11}, p_{12} \in P$ such that $p_1(x) = p_{11} \wedge p_{12}(x)$, $((x, m), p_{11}(x)) \in \rho_1$ and $((m, z_1), p_{12}(m)) \in \rho_2$. From $((m, z_1), p_{12}(m)) \in \rho_2$ and $((z_1, y), p_3(z_1)) \in \rho_3$ we deduce that $((m, y), p_{12} \wedge p_3(m)) \in \rho_2 \circ \rho_3$. Similarly, from $((x, m), p_{11}(x)) \in \rho_1$ and $((m, y), p_{12} \wedge p_3(m)) \in \rho_2 \circ \rho_3$ we deduce that $((x, y), p_{11} \wedge (p_{12} \wedge p_3(x))) \in \rho_1 \circ (\rho_2 \circ \rho_3)$. But $((x, y), p_1 \wedge p_2(x)) = ((x, y), (p_{11} \wedge p_{12}) \wedge p_3(x))$ and $(p_{11} \wedge p_{12}) \wedge p_3(x) = p_{11} \wedge (p_{12} \wedge p_3(x))$. Thus $((x, y), p_1 \wedge p_2(x)) \in \rho_1 \circ (\rho_2 \circ \rho_3)$ and the inclusion $(\rho_1 \circ \rho_2) \circ \rho_3 \subseteq \rho_1 \circ (\rho_2 \circ \rho_3)$ is proved. The converse inclusion is proved in a similar manner. ■

Remark 2.4. *Frequently, the condition* $p \in P$ *can be expressed in terms of initial knowledge about the used objects. The initial knowledge for some object will be specified in the following form:*

$$(attribute\_name, value)$$

that is, an attribute for the specified object and its corresponding value.

3. The syntax of rules and their aims

As we mentioned in the previous sections, the reasoning process uses a set of rules. In this section we define the syntax of these rules and we describe the aim of these entities.

In comparison with other methods that use a rule-based reasoning, in our method the rules are extracted from the knowledge piece, a path-driven reasoning is used and the rules help the reasoning process by assigning to some "semaphores" the values "on" or "off". All the concepts are exemplified on the following knowledge piece, named $KP2$:

Meitner, Astoria, Pax, Transilvania House, Medina and Alina are names of hotels or pensions of 2 or 3 stars as follows. Meitner and Astoria are hotels of 3 stars located in Predeal and Iaşi. Pax is a hotel of 2 stars in Cluj Napoca.

Transilvania House and Medina are pensions of 3 stars while Alina has 2 stars. These pensions are located in Predeal, Iaşi and Sinaia respectively.

A tourist considers that a hotel of 2 or 3 stars offers good conditions if the double room's surface is greater than 14 square meters, each room has telephone or access to internet and the hotel has its own parking.

For a pension, good conditions imply having access to a sauna, pool or hydro-massage.

A double room in a hotel of 2 or 3 stars has good price if it does not exceed 200 USD, while for a pension of the same class the limit is 150 USD.

The accommodation of these hotels and pensions consist of:

Meitner Hotel: double room - price: 170 USD, surface: 19-33 sqm, internet, free guarded parking, garden

Astoria Hotel: double room - price: 275 USD, surface: 15 sqm., phone and net, pay parking

Pax Hotel: double room - price: 160 USD, surface: 12,5 sqm, phone

Transilvania House Pension: double room - price: 180 USD, pool, sauna, jacuzzi

Alina Pension: double room - price: 80 USD

Medina Pension: double room - price: 140 USD, hydro-massage

From the above text we extract the following objects:

- **individual objects**: *Meitner, Astoria, Pax, Transilvania House, Alina, Medina*
- **abstract objects**: *hotel, pension, good conditions, good prices*

We consider the following attributes for the individual objects:

- *price*: to represent the price of a double room
- $surface$: to represent the surface of the hotel's double room
- *communication* (*comm.*): indicates if there is a telephone and/or access to Internet in the hotel's rooms
- *parking*: indicates if the hotel has its own parking and of what kind
- *garden*: indicates if the hotel has its own garden
- *leisure*: indicates the leisure activities offered at the pension.

In order to avoid a possible confusion we shall use the following notation:

$$a \Leftrightarrow \{(x_1, y_1), \ldots, (x_n, y_n)\}$$

to specify that the classical binary relation $\{(x_1, y_1), \ldots, (x_n, y_n)\}$ is represented by the symbol a. A similar notation will be used for conditional binary relations.

The following relations are extracted from $KP2$:

$is\_a \Leftrightarrow \{((Meitner, hotel), T), ((Astoria, hotel), T), ((Pax, hotel), T), ((Transilvania\ House, pension), T), ((Alina, pension), T), ((Medina, pension), T)\}$

$offers \Leftrightarrow \{((hotel, good\ conditions), p(hotel)), ((pension, good\ conditions), q(pension))\}$

$has \Leftrightarrow \{((hotel, good\ prices), r(hotel)), ((pension, good\ prices), s(pension))\}$

where p, q, r, s are symbols that specify some conditions, as will be explained below.

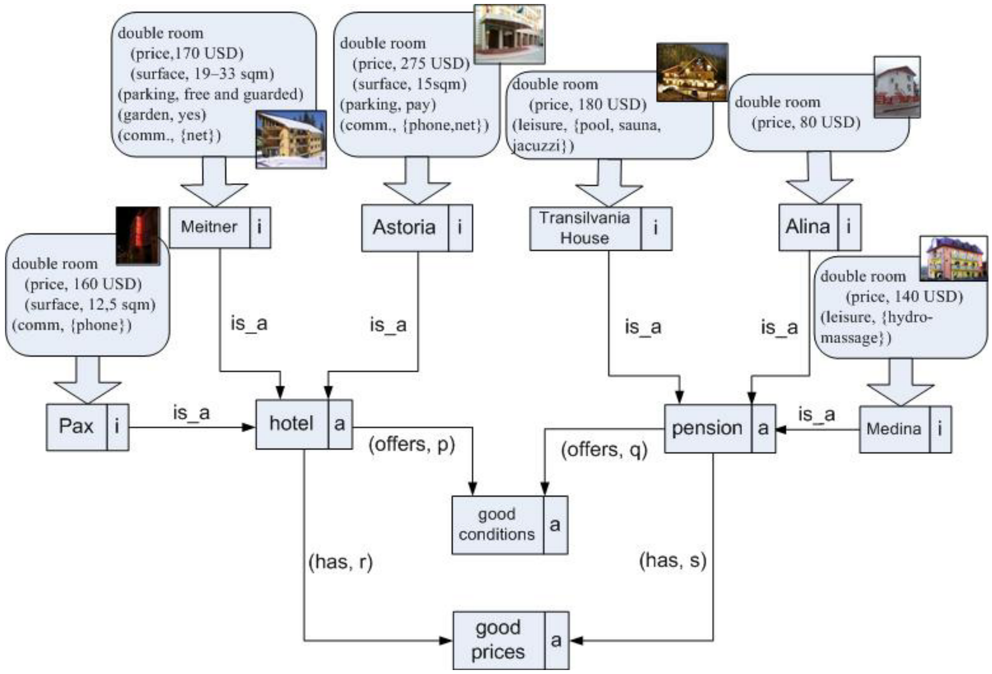

FIGURE 1. Graphical representation of $KP2$

For an object x if $(attr, value)$ represents an initial knowledge about it, then we shall denote:

$$V_x(attr) = value$$

in order to specify that $value$ is referred to x for the property $attr$. Using these notations for $KP2$ we have:

- $V_{Meitner}(price) = 170USD$, $V_{Meitner}(surface) = 19{-}33sqm$, $V_{Meitner}(parking) = free\ and\ guarded$, $V_{Meitner}(communication) = \{net\}$, $V_{Meitner}(garden) = yes$
- $V_{Astoria}(price) = 275USD$, $V_{Astoria}(surface) = 15sqm$, $V_{Astoria}(parking) = pay$, $V_{Astoria}(communication) = \{phone, net\}$, $V_{Astoria}(garden) = no$
- $V_{Pax}(price) = 160USD$, $V_{Pax}(surface) = 12.5sqm$, $V_{Pax}(parking) = no$, $V_{Pax}(communication) = \{phone\}$, $V_{Pax}(garden) = no$
- $V_{Transilvania\ House}(price) = 180USD$, $V_{Transilvania\ House}(leisure) = \{pool, sauna, jacuzzi\}$
- $V_{Alina}(price) = 80USD$, $V_{Alina}(leisure) = \emptyset$
- $V_{Medina}(price) = 140USD$, $V_{Medina}(leisure) = \{hydro\text{-}massage\}$

Remark 3.1. *As we can easily observe, the notation $V_x(attr)$ designates an attribute value for the* **individual object** *x.*

The conditions are denoted by the symbols p, q, r, s. They are named *conditional symbols* and they are represented by mappings defined on the objects set. We denote by C_s the set of these symbols. Thus, for $KP2$ we have $C_s = \{p, q, r, s\}$. Some logical condition is attached to every element of C_s.

Intuitively, for each symbol $t \in C_s$ and for an arbitrary object x we say that *"t is on"* for x if the condition attached to t is satisfied for x and *"t is off"* otherwise. We shall write $t(x) = true$ or $t(x) = false$.

The conditions attached to each symbol of C_s can be transposed in IF-THEN-ELSE rules using the above notations. Thus, for $KP2$ we obtain the following rules:

$R_1(x) : IF\ V_x(surface) \geq 15sqm\ \wedge V_x(communication) \cap \{phone, net\} \neq \emptyset\ \wedge$
$V_x(parking) \neq no\ THEN\ p(x) = true\ ELSE\ p(x) = false$

$R_2(x) : IF\ V_x(leisure) \cap \{sauna, pool, hydro - massage\} \neq \emptyset\ THEN\ q(x) = true\ ELSE\ \ q(x) = false$

$R_3(x) : IF\ V_x(price) \leq 200USD\ THEN\ r(x) = true\ ELSE\ r(x) = false$

$R_4(x) : IF\ V_x(price) \leq 150USD\ THEN\ s(x) = true\ ELSE\ s(x) = false$

where R_j is the name of the rule and $R_j(x)$ denotes the fact that the rule R_j is applied for the object x, $j = \overline{1,4}$.

It remains to formalize all these aspects and to define the reasoning based on this formalism. This is the aim of the next section.

Remark 3.2. *The graphical representation given in Figure 1 specifies a conditional schema. We shall refer to this figure later and we shall give all details concerning this representation after the formal treatment of this structure. We observe that the entities of the form $(attribute, value)$ corresponding to some individual object are introduced into a rounded rectangle that is linked by object.*

Also some pictures are attached to the representation, without having a meaning for the mechanism we want to introduced.

Remark 3.3. *The main task of the rules is to assign values for conditional symbols. These entities can be considered as "semaphores" that allow to use some parts of a path in the graphical representation of a knowledge piece.*

4. The components of a conditional schema

Let us summarize the components introduced in the previous section, components that will be used to define the concept of *conditional schema*:

- Ob is the set of the individual and abstract objects names extracted from the knowledge piece such that
  - $Ob = Ob_{ind} \cup Ob_{abstr}$, where Ob_{ind} gives all the individual objects names and Ob_{abstr} represents the set of all abstract objects names;
  - $Ob_{ind} \cap Ob_{abstr} = \emptyset$
- C_s is the set of the conditional symbols, for every $t \in C_s$ we have $t : Ob \longrightarrow \{true, false\}$
- E_r is the set of symbols for conditional binary relations; a conditional binary relation is a subset of $(Ob \times Ob) \times (C_s \cup \{T\})$
- A is the set of attribute names for the objects of Ob;
- $I = \{i, a\}$, where i is used to designate individual objects and a is used to specify abstract objects.

Thus, every element x of the set Ob_{ind} will be designated in what follows by (x, i). Similarly, an element $x \in Ob_{abstr}$ is designated by (x, a).

Remark 4.1. *If $r \in E_r$ then the conditional relation designated by r is denoted by $Rel_c(r)$.*

In what concerns the conditional symbols, the following notation is used:

$$((n, w_1), (m, w_2)) \in_c Rel_c(r) \tag{1}$$

to relieve the fact that the element $(((n, w_1), (m, w_2)), p(n))$ belongs to $Rel_c(r)$ for some $p \in C_s$ and we denote

$$p(n) = Cond_r((n, w_1), (m, w_2)) \tag{2}$$

For the knowledge piece $KP2$ we obtain:

- $Ob = \{Meitner, Astoria, Pax, Transilvania\ House, Alina, Medina, hotel, pension, good\ conditions, good\ prices\}$
- $C_s = \{p, q, r, s\}$
- $E_r = \{is\_a, offers, has\}$
- $A = \{price, surface, communication, parking, garden, leisure\}$
- $I = \{i, a\}$, where i is used to designate the individual objects and a is used to designate the abstract objects.

According to our notational convention from Remark 4.1 and the conditional membership from (1) we can write:

$((Pax, i), (hotel, a)) \in_c Rel_c(is\_a)$
$((hotel, a), (good\ conditions, a)) \in_c Rel_c(offers)$

Now we can introduce the concept of conditional schema as in the next definition.

Definition 4.1. *A* **conditional schema** *is a tuple* $(Ob, C_s, E_r, A, V, B_{cr}, h, f)$*, where*

- *Ob is a set of the objects' names; this set is divided into two subsets Ob_{ind} and Ob_{abstr} such that $Ob = Ob_{ind} \cup Ob_{abstr}$ and $Ob_{ind} \cap Ob_{abstr} = \emptyset$;*
- *C_s is a finite set of symbols named* **conditional symbols***;*
- *E_r is a finite set of symbols used to designate conditional binary relations over Ob;*
- *A is a set of* **attribute name** *for the elements of Ob;*
- *V is a set of values for the elements of A;*
- *$B_{cr} \subseteq 2^{((Ob \times I) \times (Ob \times I)) \times (C_s \cup \{T\})}$ is the set of the conditional binary relations;*
- *$h : E_r \to B_{cr}$ is a mapping that assigns a conditional binary relation for every symbol of E_r;*
- *$f : Ob_{ind} \to 2^{A \times V}$ is a mapping that assigns initial knowledge to the individual objects of Ob_{ind}.*

Remark 4.2. *The mapping h is used to establish a connection between an element $r \in E_r$ and the relation $h(r) \in B_{cr}$ designated by r. Obviously for every $r \in E_r$ we have $h(r) = Rel_c(r)$, therefore*

$$r \Leftrightarrow h(r)$$

For example, in the case of $KP2$ we have:

$$\begin{aligned} h(is\_a) = \{ & (((Pax, i), (hotel, a)), T), \ (((Meitner, i), (hotel, a)), T), \\ & (((Astoria, i), (hotel, a)), T), \\ & (((Transilvania\ House, i), (pension, a)), T), \\ & (((Alina, i), (pension, a)), T), (((Medina, i), (pension, a)), T)\} \end{aligned}$$

Remark 4.3. *Any pair $(attr, val) \in f(x)$, where $x \in Ob_{ind}$, specifies the value val of the attribute $attr$ for the object x.*

The following notation is used for a set $M \subseteq X_1 \times \ldots X_k$ and $i \in \{1, \ldots, k\}$:

$$pr_i M = \{x \in X_i \mid \exists (x_1, \ldots x_{i-1}, x, x_{i+1}, \ldots, x_k) \in M\}$$

Definition 4.2. *An abstract object $x \in Ob_{abstr}$ is a* **useless** *object if $(x, a) \notin pr_1 Q \cup pr_2 Q$, where $Q = pr_1(\bigcup_{m \in E_r} h(m))$.*

As we shall see such elements can not be used in the reasoning process. In the remainder of this chapter we consider that every conditional schema does not contain useless objects.

5. Conditional graphs

A conditional schema can be graphically represented as a particular labeled directed graph. The representation is named conditional graph and this concept is presented in the next definition. We relieve the fact that this structure is a useful one to formalize the reasoning process in a conditional schema.

Definition 5.1. *Let $\mathcal{S} = (Ob, C_s, E_r, A, V, B_{cr}, h, f)$ be a conditional schema. A* **conditional graph** *generated by $\mathcal{S}$ is a pair $G_{\mathcal{S}} = (X \cup Z, \Gamma_X \cup \Gamma_Z)$, where*

- *$X \subseteq Ob \times I$ is the set of nodes such that $pr_1 X = Ob$ and $x = (n, w) \in X$ if and only if there are $r \in E_r$ and $y \in Ob \times I$ such that $(x, y) \in_c h(r)$ or $(y, x) \in_c h(r)$.*
- *$\Gamma_X \subseteq X \times E_r \times X$ and $((n, w_1), r, (m, w_2)) \in \Gamma_X$ if and only if $((n, w_1), (m, w_2)) \in_c h(r)$.*
- *$Z = \{f(x) \mid x \in Ob_{ind}\}$ and $\Gamma_Z = \{(f(x), x) \mid x \in Ob_{ind}\}$*

For example, in the graph of $KP2$ we have:

$$((Pax, i), is\_a, (hotel, a)) \in \Gamma_X$$

because $((Pax, i), (hotel, a)) \in_c h(is\_a)$

Remark 5.1. *A distinctive aspect in a conditional graph is given by the fact that:*

- *There are two kinds of nodes, given by X and Z.*
- *There are two kinds of arcs specified by Γ_X and Γ_Z respectively. An arc from Γ_X is named* **arc of first category** *and an arc from Γ_Z is an* **arc of second category***.*

As we prove in the next proposition just one conditional graph can be generated by a given conditional schema.

Proposition 5.1. *A conditional schema generates one and only one conditional graph.*

Proof. Consider a conditional schema $\mathcal{S} = (Ob, C_s, E_r, A, V, B_{cr}, h, f)$. Directly from Definition 5.1 we can build a conditional graph if we proceed as follows:

- Obtain Z and Γ_Z;
- Take $W = pr_1(\bigcup_{r \in E_r} h(r))$ and $X = pr_1 W \cup pr_2 W$.
- Take $\Gamma_X = \bigcup_{r \in E_r} \{((n, w_1), r, (m, w_2)) \mid ((n, w_1), (m, w_2)) \in_c h(r)\}$.

Let us verify that $pr_1 X = Ob$. From Definition 4.1 we observe that if $r \in E_r$ then

$$pr_1 h(r) \subseteq (Ob \times I) \times (Ob \times I)$$

therefore

$$pr_i(pr_1 h(r)) \subseteq Ob \times I$$

for $i \in \{1, 2\}$. It follows that

$$pr_1 W \cup pr_2 W \subseteq Ob \times I$$

therefore $X \subseteq (Ob \times I)$.

Conversely, suppose $x \in Ob$. It results the following two possibilities for x $(x, i) \in Ob_{ind} \times I$ or $(x, a) \in Ob_{abstr} \times I$. Because $\mathcal{S}$ does not contain isolate objects we deduce that $\exists r \in E_r$ and $\exists y \in Ob \times I$: $((x, w), y) \in_c h(r)$ or $(y, (x, w)) \in_c h(r)$ which implies $(x, w) \in (pr_1 W \cup pr_2 W)$, for $w \in I$. But $(pr_1 W \cup pr_2 W) = X$, therefore $x \in pr_1 X$.

Suppose that $G^1_{\mathcal{S}} = (X_1 \cup Z, \Gamma_{X_1} \cup \Gamma_Z)$ and $G^2_{\mathcal{S}} = (X_2 \cup Z, \Gamma_{X_2} \cup \Gamma_Z)$ are two conditional graphs generated by $\mathcal{S}$. The following sentences are equivalent:

(1) $x \in X_1$
(2) There are $r \in E_r$ and $y \in Ob \times I$ such that $(x, y) \in_c h(r)$ or $(y, x) \in_c h(r)$
(3) $x \in X_2$

therefore $X_1 = X_2$. In a similar manner the following sentences are equivalent:

(1) $((n, w_1), r, (m, w_2)) \in \Gamma_{X_1}$,
(2) $((n, w_1), (m, w_2)) \in_c h(r)$, $((n, w_1), (m, w_2)) \in X_1 \times X_1$
(3) $((n, w_1), (m, w_2)) \in_c h(r)$, $((n, w_1), (m, w_2)) \in X_2 \times X_2$
(4) $((n, w_1), r, (m, w_2)) \in \Gamma_{X_2}$

therefore $\Gamma_{X_1} = \Gamma_{X_2}$. ■

Proposition 5.2. *If $G_{\mathcal{S}} = (X \cup Z, \Gamma_X \cup \Gamma_Z)$ is the conditional graph generated by $\mathcal{S} = (Ob, C_s, E_r, A, V, B_{cr}, h, f)$ then $pr_2\Gamma_X = E_r$.*

Proof. Directly from Definition 5.1 we have $\Gamma_X \subseteq X \times E_r \times X$, therefore $pr_2\Gamma_X \subseteq E_r$. To verify the converse inclusion we take an arbitrary element $r \in E_r$. We have $h(r) \neq \emptyset$ because the empty relation is not used in knowledge representation. Thus there is an element $((n, w_1), (m, w_2)) \in_c h(r)$. Using again Definition 5.1 we deduce that $((n, w_1), r, (m, w_2)) \in \Gamma_X$ and so $r \in pr_2\Gamma_X$. ■

The graphical representation of a conditional schema $\mathcal{S}$ is obtained as follows:

- Consider the conditional graph $G_{\mathcal{S}} = (X \cup Z, \Gamma_X \cup \Gamma_Z)$ generated by $\mathcal{S}$;
- Each element of X is represented by a rectangle;
- We draw an arc from the node $x \in X$ to $y \in X$ and we put the label $r \in E_r$ on this arc if and only if $(x, r, y) \in \Gamma_X$;
- For each individual node $x \in Ob_{ind} \times I$ we append the additional information given by $f(x)$. This information is collected in a rounded rectangle linked to x as in Figure 2.

Applying for $KP2$ this method we obtain the graphical representation from Figure 1.

The concept of conditional graph helps us to model the reasoning in a conditional schema. Moreover, this concept is a useful one to specify some features of a conditional schema.

For example, in Figure 2 we represented the conditional graph of a simple conditional schema. The corresponding conditional schema contains the following components:

(1) $Ob = Ob_{ind} = \{Meitner\}$;
(2) $C_s = E_r = B_{cr} = \emptyset$;
(3) $A = \{price, surface, parking, garden, communication\}$
(4) The mapping h is undefined because $E_r = \emptyset$;
(5) $V = \{170USD, 19\text{-}33sqm, free\ and\ guarded, yes, \{net\}\}$;
(6) $f(Peter) = \{(price, 170USD), (surface, 19\text{-}33sqm),$
$(parking, \textit{free and guarded}), (garden, yes), (communication, \{net\})\}$.

6. Future work

In this paper we formalized a new structure named conditional schema. This structure can be used to represent conditional knowledge pieces. We introduced also the concept of conditional graph and we proved that a conditional schema generates just one conditional graph. In a forthcoming paper we define the reasoning mechanism in a conditional schema and this mechanism is described by means of the corresponding

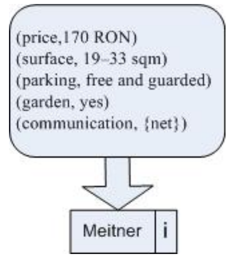

FIGURE 2. A simple conditional graph

conditional graph. A possible future research line is the extraction of the rules from the text description of a knowledge piece.

References

[1] Buchanan, B., Shortliffe, E.: Rule-Based Expert Systems:The MYCIN Experiments of the Stanford Heuristic Programming Project, Addison-Wesley, MA (1984)

[2] GURU: Integrated Components Manual, Version 3, MDBS - The Knowledge Management Company

[3] GURU: KGL Programmer's Reference, Version 3, MDBS - The Knowledge Management Company

[4] Hill E. F.: Jess in Action, Rule-Based Systems in Java, Manning Publications Co. (2003)

[5] Russell, S., Norvig, P.: Artificial Intelligence: A Modern Approach (2nd Edition), Prentice Hall, ISBN: 0137903952 (2003)

[6] Schnupp, P., Bernhard, W. L., Nguyen, C.T.: Expert Systems Lab Course, Springer-Verlag, ISBN 10: 0387505709, ISBN 13: 9780387505701 (1989)

[7] Siegel P.: Expert Systems, A non-Programmer's Guide to Development and Applications, TAB BOOKS Inc. (1986)

[8] Tandareanu N., Ghindeanu M.: Towards a Mathematical Modelling of Conditional Knowledge, 3nd Romanian Conference on Artificial Intelligence and Digital Communications Proceedings, ISBN 973-8419-71-9, Craiova, 103:82-96 (2003)

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