Annals of University of Craiova, Math. Comp. Sci. Ser.
Volume 36(2), 2009, Pages 85–91
ISSN: 1223-6934

# Aircraft Influence on PIO Conditions. A Lateral-Directional Approaching

Claudia-Alice State

Abstract. We describe in this paper models of lateral-directional movement of an aircraft. Starting from the two decoupled motions (yaw and roll), we analyze the stability of the coupled motion (this movement is described in literature as lateral-directional (Roskam, [4]) p. 346). The systems are analysed at constant speed within in the normal flight envelope, with constant altitude. With the help of the simulations we put in evidence the fact that the systems are stable.
*Key words and phrases.* Lateral Directional, Stability, Simulation, Flight Control System, Flight Envelope.

## 1. Introduction

We processed informations obtained from the Aerodata Model ADMIRE [6] together with some results obtained by a team from the Straero Institute [1] (see the systems (1), (3) and (5)). We study lateral directional movement along the Ox axis (roll), along the axis Oz (yaw) and on the Ox and Oz axes combined. Similar with [3](Rahman and Whidborne) we analyze the lateral-directional flight control systems, but, in our case, using the ADMIRE database. Results of computer simulations using Simulink (which is a product of MathWorks [5]) are presented.
An aircraft is free to rotate around one, two, or all three axes simultaneously, which are perpendicular to each other (Fig. 1), and intersect at the plane's center of gravity. The vertical (yaw) axis which is perpendicular to the other two axes passes through the plane from top to bottom . The rotation about this axis is called yaw and the primary control of yaw is with the rudder. Rotation about the lateral (pitch) axis is controlled by the elevator, this rotation being similar to a seesaw. Longitudinal (roll) axis is an axis drawn through the body of the vehicle from tail to nose in the normal direction of flight.

## 2. Lateral-directional motion along the OX axis (Roll)

The rotation about the longitudinal axis which is called roll, is also known as bank. This particular movement can be expressed by the following elements:

- $\phi$ - which is an angle (the roll or bank angle);
- p - $\dot{\phi}$.

We start from the following system:

$$\begin{cases} \dot{p} = l_p p - l_{\delta_a} \delta_a \\ \dot{\phi} = p \end{cases} \tag{1}$$

where we have:


FIGURE 1. The possible maneuvers in flight, on the tree axes

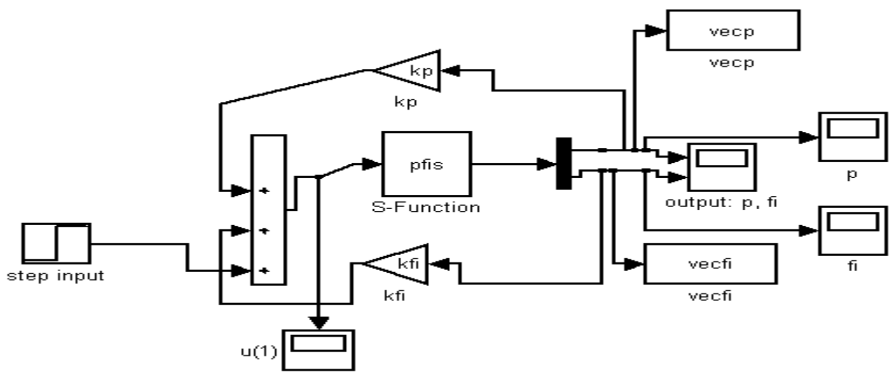


FIGURE 2. The *simulink*$^{TM}$ scheme of the system [1]

$$\delta_a = k_p p + k_\phi \phi \tag{2}$$

**Remark 2.1.** *$\delta_a$ is the command for the aileron.*

**Remark 2.2.** *$k_p$, $k_\phi$ are specific aerodynamics constants.*

**Remark 2.3.** *In Figure (2) the step unit enters in an additive manner, together with the input (2).*

**Remark 2.4.** *If we look at Figure (4), we notice that $\phi$ is stabilized around the value of 3.5 degrees.*

## 3. Lateral-directional motion along the OZ axis (Yaw)

Rotation about OZ axis, which is called yaw, changes the direction the aircraft's nose is pointing, left or right.
For this model of motion the state variables vector has the following elements:

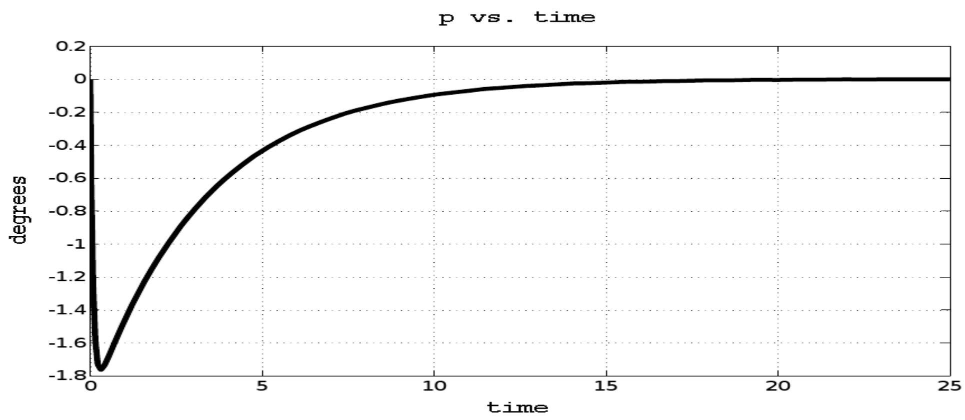


FIGURE 3. p vs. time (for the system (1))

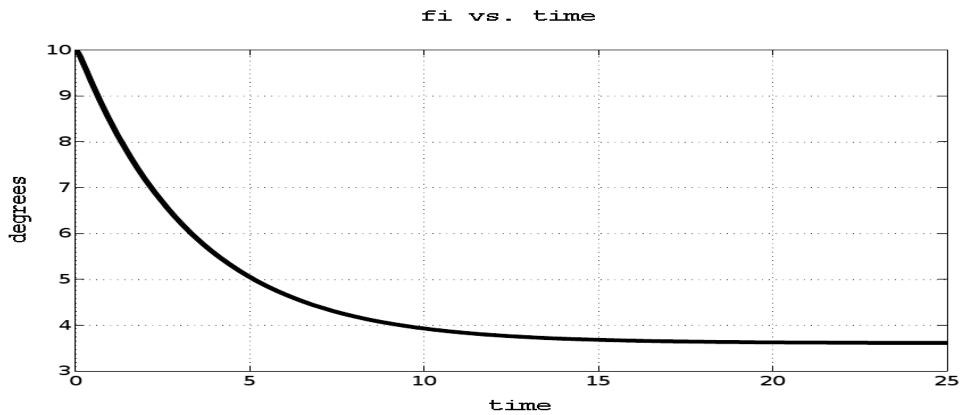


FIGURE 4. $\phi$ vs. time (for the system (1))

- $\beta$ - the sideslip angle
- $r = \dot{\Psi}$ - the derivative of yaw angle ($\Psi$)

We consider:

$$\begin{cases} \dot{\beta} = y_\beta \beta + y_{\delta_r} \delta_r - r \\ \dot{r} = n_\beta \beta + rX(\beta) + n_{\delta_r} \delta_r \end{cases} \tag{3}$$

where we have:

$$\delta_r = k_\beta \beta + k_r r \tag{4}$$

**Remark 3.1.** *$\delta_r$ is the command for the rudder.*

**Remark 3.2.** *$y_\beta$, $y_{\delta_r}$, $n_\beta$, $n_{\delta_r}$ are specific aerodynamics constants.*

**Remark 3.3.** *$X(\beta)$ is a nonlinear function with argument $\beta$.*

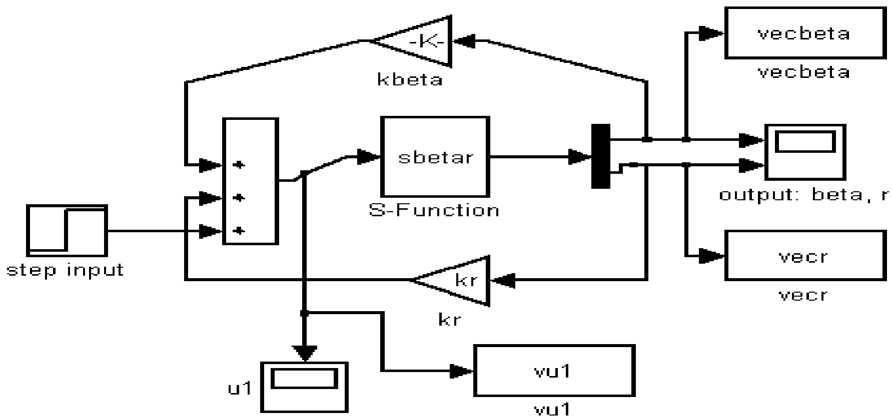


FIGURE 5. The *simulink*$^{TM}$ scheme of the system (3)

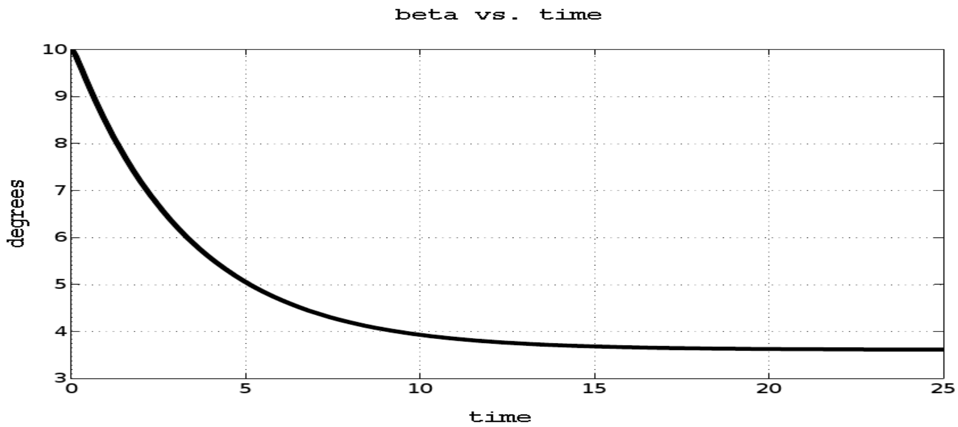


FIGURE 6. $\beta$ vs. time (for the system (3))

## 4. Lateral-directional motion along the OX and OZ axes

Now we consider the following system which describe the coupled motion along the OX and OZ axes:

$$\begin{cases} \dot{\beta} = y_\beta \beta + y_{\delta_r}\delta_r + y_{\delta_a}\delta_a + \frac{g}{V_0}\sin(x_4)\cos(\theta_0) - x_3 + x_2\alpha_0 \\ \dot{p} = a_1 Y_1(\alpha_0)\beta + l_p p + a_1 Y_2(\alpha_0)x_3 + l_{\delta_r}\delta_r + l_{\delta_a}\delta_a \\ \dot{r} = n_\beta \beta + a_3 Y_3(\alpha_0)\beta^2 p + X(\beta)r + n_{\delta_r}\delta_r + n_{\delta_a}\delta_a \\ \dot{\phi} = p + r\cos(\phi)tg(\theta_0) \end{cases} \tag{5}$$

**Remark 4.1.** *We have the following:*
- $Y_1(\alpha_0)$ *is a nonlinear function of* $\alpha_0$
- $Y_2(\alpha_0)$ *is a linear function of* $\alpha_0$
- $Y_3(\alpha_0)$ *is a linear function of* $\alpha_0$

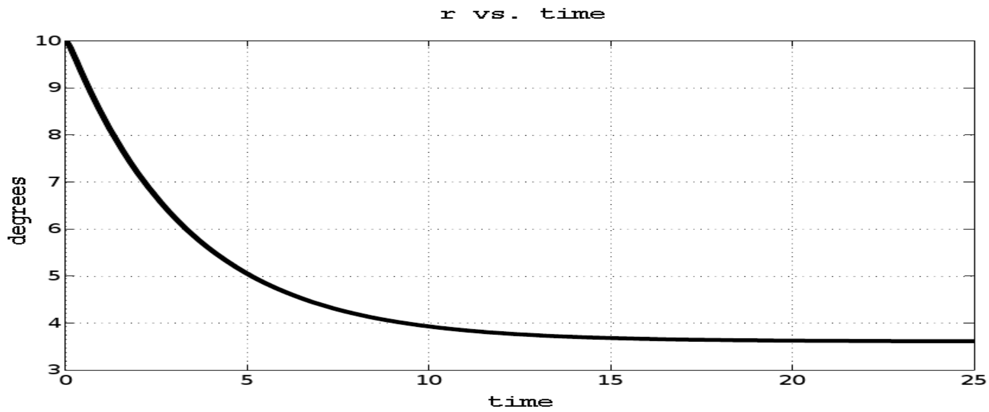


FIGURE 7. r vs. time (for the system (3))

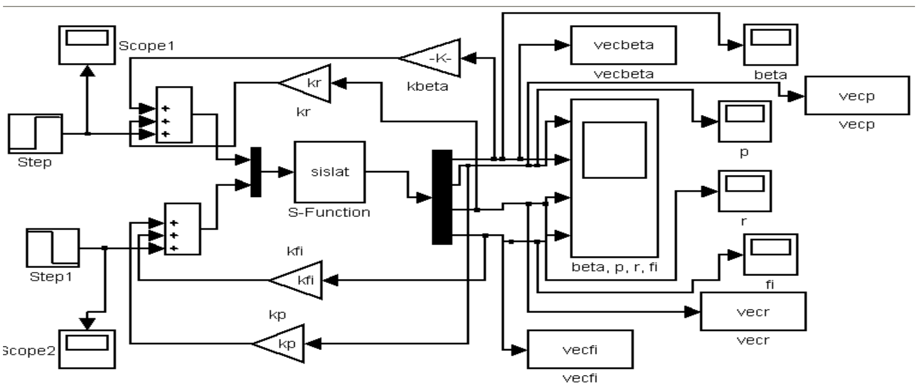


FIGURE 8. The $simulink^{TM}$ scheme of the system (5)

where:

$$\theta_0, \alpha_0 \text{ are values obtained from ([2], 2009)} \tag{6}$$

### 4.1. Considerations.

- Systems (1) and (3) are projections of the system (5) on axes Ox and Oz, respectively.
- Auxiliary sizes of the unit-step type that comes in the additive input are positive, in the case of models from figures (2) and (5), but, in case of the input of the model from Figure (8) the first unit-step is positive, the second being negative.
- The terms of $\delta_a$ and $\delta_r$ from the system (5) are the same with (2) and (4), respectivelly.

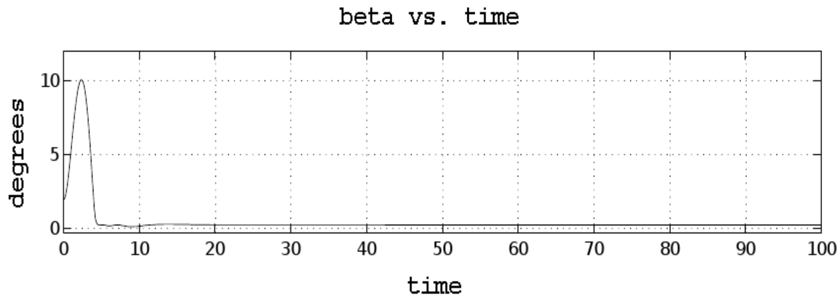


FIGURE 9. $\beta$ vs. time (for the system (5))

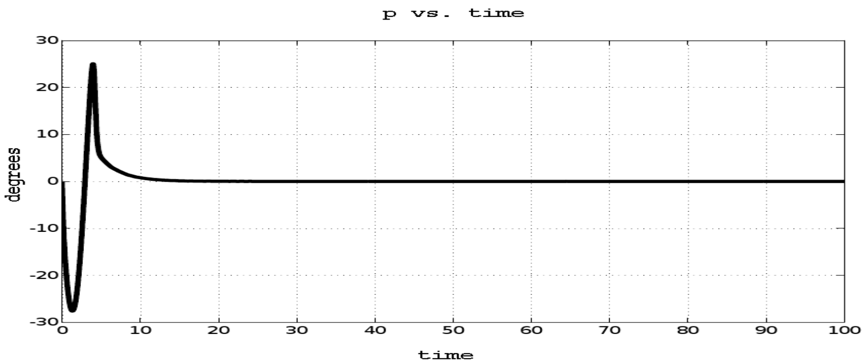


FIGURE 10. p vs. time (for the system (5))

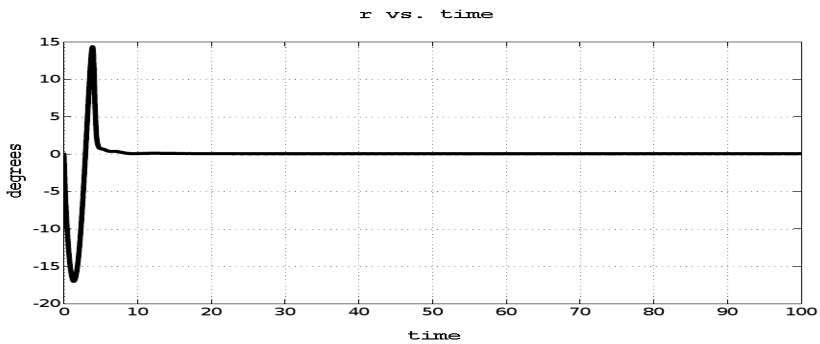


FIGURE 11. r vs. time (for the system (5))

## 5. Conclusions and future work

This work included the following aspects:

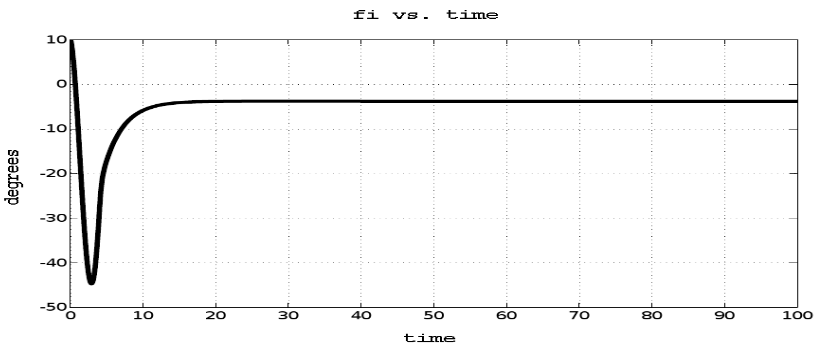


FIGURE 12. $\phi$ vs. time (for the system (5))

- development of mathematical models for lateral-directional movement of the airplane;
- starting from the two decoupled motions (yaw and roll) we build and analyze the coupled motion;
- the study of stability of the movement.

As a future work we propose to extend the simulations in order to do a real-time simulation environment.

## References

[1] The Institute for Theoretical and Experimental Analysis of Aeronautical Structures (Straero), http://www.straero.ro

[2] Iorga,I.: The Pilot Influence on PIO Conditions. A Longitudinal Approach, $9^{th}$ International Conference on Artificial Intelligence and Digital Communications, Craiova, Romania, September 2009 *Submitted*

[3] N.Rahman, J.Whidborne: A Lateral Directional Flight Control System for the MOB Blended Wing Body Planform , available on line at http://www.control2008.org/papers/p67.pdf

[4] Roskam, J.: Airplane Flight Dynamics and Automatic Flight Controls, Design Analysis and Research Corporation, Kansas, S.U.A., 2001

[5] Simulink - MathWorks,http://www.mathworks.com/products/simulink/

[6] Admire Aerodata Model (The Aeronautical Research Institute of Sweden), http://www.foa.se/FOI/Templates/ProjectPage____5062.aspx

(Claudia-Alice State) UNIVERSITY OF CRAIOVA, ALEXANDRU IOAN CUZA 13, 200585 CRAIOVA, ROMANIA
*E-mail address*: cldstate@yahoo.com