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Characterization of K-algebras by self maps II

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Abstract. The notion of a K-algebra was introduced in [2] and it was characterized by its left and right mappings in [3] when group is abelian. In this paper we first explore some new properties of K-algebras, and then we characterize K-algebras by using their left and right mappings when the group is non-abelian.

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1. Introduction

The notion of a K-algebra $(G, \cdot, \odot, e)$ was first introduced by Dar and Akram [2] in 2003 and published in 2005. A K-algebra is an algebra built on a group $(G, \cdot, e)$ by adjoining an induced binary operation $\odot$ on G which is attached to an abstract K-algebra $(G, \cdot, \odot, e)$. This system is, in general non-commutative and non-associative with a right identity e, if $(G, \cdot, e)$ is non-commutative. For a given group G, the K-algebra is proper if G is not an elementary abelian 2-group. Thus, a K-algebra is abelian and non-abelian purely depends on the base group G. Dar and Akram further renamed *a K-algebra on a group G* as a $K(G)$-algebra [3] due to its structural basis G. The $K(G)$-algebras have already been characterized by their left and right mappings in [3] when group is abelian. In this paper we shall explore some new properties and examples of K-algebras. We shall characterize K-algebras by using their left and right mappings when the group is non-abelian. K-algebras have been extensively studied by authors since 2004 (see [1-7]).

2. Properties of K-algebras

Definition 2.1. [2] *Let $(G, \cdot, e)$ be a group in which each non-identity element is not of order 2. Then a K- algebra is a structure $\mathcal{K} = (G, \cdot, \odot, e)$ on a group G in which induced binary operation $\odot : G \times G \to G$ is defined by $\odot(x, y) = x \odot y = x.y^{-1}$ and satisfies the following axioms:*

(K1) $(x \odot y) \odot (x \odot z) = (x \odot ((e \odot z) \odot (e \odot y))) \odot x$,
(K2) $x \odot (x \odot y) = (x \odot (e \odot y)) \odot x$,
(K3) $(x \odot x) = e$,
(K4) $(x \odot e) = x$,
(K5) $(e \odot x) = x^{-1}$
for all $x,\ y,\ z \in G$.

Definition 2.2. [4] *A K-algebra $\mathcal{K}$ is called abelian if and only if $x \odot (e \odot y) = y \odot (e \odot x)$ for all $x,\ y \in G$.*

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If a K-algebra $\mathcal{K}$ is abelian, then the axioms $(K1)$ and $(K2)$ can be written as:
$(\overline{K1})$ $(x \odot y) \odot (x \odot z) = z \odot y$.
$(\overline{K2})$ $x \odot (x \odot y) = y$.

Remark 2.1. (a) *Let $G = \{e, a, b, c\}$ be a Klein four group. Consider a K-algebra on G with the following Cayley table:*

$\odot$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

This is an improper K-algebra on Klein four group since it is elementary abelian 2-group, i.e., $x \odot y = x.y^{-1} = x.y$.
(b) *A K-algebra is proper if G is not an elementary abelian 2-group.*

Example 2.1. *Let $G_1 = \{< a >: a^3 = e\}$ and $G_2 = \{< a >: a^2 = e\}$ be two cyclic groups. Then $G = G_1 \times G_2 = \{(e, e), (a, e), (a^2, e), (e, b), (a, b), (a^2, b)\}$ is a cyclic group of order 6. Consider the K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ on $G = \{e, v, w, x, y, z\}$, where $e = (e, e)$, $v = (a, e)$, $w = (a^2, e)$, $x = (e, b)$, $y = (a, b)$, $z = (a^2, b)$, and $\odot$ is given by the following Cayley's table:*

$\odot$	e	v	w	x	y	z
e	e	w	v	x	z	y
v	v	e	w	y	x	z
w	w	v	e	z	y	x
x	x	z	y	e	w	v
y	y	x	z	v	e	w
z	z	y	x	w	v	e

Example 2.2. *Consider the K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ on the Dihedral group $G = \{e, a, u, v, b, x, y, z\}$ where $u = a^2$, $v = a^3$, $x = ab$, $y = a^2b$, $z = a^3b$, and $\odot$ is given by the following Cayley's table:*

$\odot$	e	a	u	v	b	x	y	z
e	e	v	u	a	b	x	y	z
a	a	e	v	u	x	y	z	b
u	u	a	e	v	y	z	b	x
v	v	u	a	e	z	b	x	y
b	b	x	y	z	e	v	u	a
x	x	y	z	b	a	e	v	u
y	y	z	b	x	u	a	e	v
z	z	b	x	y	v	u	a	e

Example 2.3. *Let $G = V_3(R) = \{(x, y, z) : x, y, z \in R\}$ be the set of all 3-dimensional real vectors which forms an additive $(+)$ abelian group. Define the operation $\odot$ on $V_3(R)$ by $a \odot b = a - b$ for all a, $b \in V_3(R)$.*
Then $(G, +, \odot, e)$ is a K-algebra $\mathcal{K}$.

We give the following theorem without proof.

Theorem 2.1. *Let G_1 and G_2 be two groups and let $\mathcal{K}_1$ and $\mathcal{K}_2$ be K-algebras constructed on G_1 and G_2, respectively. Then $\mathcal{K}_1 \cong \mathcal{K}_2$ if $G_1 \cong G_2$, but its converse is not true.*

Proposition 2.1. *In K-algebras $\mathcal{K}$ the following statements are equivalent:*
(a) *A K-algebra $\mathcal{K}$ is abelian,*
(b) $x \odot (e \odot y) = y \odot (e \odot x)$,
(c) $x \odot (x \odot y) = y$,
(d) $(x \odot y) \odot z = (x \odot z) \odot y$,
(e) $(e \odot x) \odot (e \odot y) = e \odot (x \odot y)$,
(f) $(x \odot y) \odot (x \odot z) = z \odot y$
for all x,y, $z \in G$.

Proof. The proof is easy and hence omitted. □

Proposition 2.2. *If the class of K-algebras $\mathcal{K}$ is an abelian. Then the following identities hold:*
1. $x \odot (e \odot y) = y \odot (e \odot x)$,
2. $(x \odot y) \odot z = (x \odot z) \odot y$,
3. $(x \odot (x \odot y)) \odot y = e$,
4. $e \odot (x \odot y) = (e \odot x) \odot (e \odot y) = y \odot x$

for all x, y, $z \in G$.

Proof. The proof is easy and hence omitted. □

Proposition 2.3. *In an abelian K-algebra $\mathcal{K}$ the following assertions are equivalent:*
5. $x \odot (y \odot z)$
6. $(x \odot y) \odot (e \odot z)$
7. $z \odot (y \odot x)$

Proof. (5) ⇒ (6) since

$$\begin{aligned} x \odot (y \odot z) &= (x \odot (e \odot (z \odot y)) \quad [by\ 4] \\ &= (z \odot y) \odot (e \odot x) \quad [by\ 1] \\ &= (z \odot (e \odot x)) \odot y) \quad [by\ 2] \\ &= (x \odot (e \odot z)) \odot y) \quad [by\ 1] \\ &= (x \odot y) \odot (e \odot z) \quad [by\ 2] \end{aligned}$$

(6) ⇒ (7) since

$$\begin{aligned} (x \odot y) \odot (e \odot x) &= (e \odot (y \odot x)) \odot (e \odot z) \quad [by\ 4] \\ &= z \odot (e \odot (e \odot (y \odot x))) \quad [by\ 1] \\ &= z \odot (y \odot x) \quad [by\ 4] \end{aligned}$$

(7) ⇒ (5) since

$$z \odot (y \odot x) = x \odot (y \odot z) by (7)\ and\ (6).$$

□

We now formulate the following propositions without their proofs when the group is non abelian.

Proposition 2.4. *Let $\mathcal{K}$ be a K-algebra on non-abelian group G. Then the following identities hold in $\mathcal{K}$ for all x, y, $z \in G$:*
(a) $x \odot (y \odot z) = (x \odot (e \odot z)) \odot y$.

(b) $(x \odot y) \odot z = x \odot (z \odot (e \odot y))$.
(c) $e \odot (x \odot y) = y \odot x$.

Proposition 2.5. *Let $\mathcal{K}$ be a K-algebra on non-abelian group G. Then the following identities hold in $\mathcal{K}$ for all x, y, $z \in G$:*
(d) $e \odot (e \odot x) = x$.
(e) $x \odot (x \odot (e \odot x)) = e \odot x$.
(f) $x \odot (z \odot (e \odot x)) = (e \odot x) \odot (z \odot x) = e \odot z$.
(g) $(x \odot y) \odot (z \odot y) = x \odot z$.
(h) $(x \odot y) \odot (e \odot y) = x$.
(i) $x \odot y = e = y \odot x \Longrightarrow x = y$.

3. Characterization of K-algebras using self maps

It is known that all the bijective mappings on a group form a group under the binary operation of their usual composition. The sets of all left and right mappings on group G coincide elementwise if $(G, \cdot, e)$ is an abelian group. In this Section, we extend the concept of left and right mappings to K-algebra $\mathcal{K}$ when G is non-abelian.

Right mappings of K-algebras

Definition 3.1. *Let $\mathcal{K}$ be a K-algebra. For a fixed element $x \in \mathcal{K}$, the mapping $R_x : \mathcal{K} \to \mathcal{K}$ defined by $R_x(y) = y \odot x$ for all $y \in \mathcal{K}$, is called right map on $\mathcal{K}$. The set of all right mappings on K- algebra $\mathcal{K}$ is denoted by R.*

Definition 3.2. *The binary operation of composition ($\circ$) of R on K-algebras built on non- abelian group G behaves in the following way:*

$$(g)R_x \circ R_y = ((g)R_x)R_y = (g \odot x) \odot y = g \odot (y \odot (e \odot x)) = (g)R_{y\odot(e\odot x)}.$$

Example 3.1. *Consider the K-algebra $\mathcal{K} = (S_3, \cdot, \odot, e)$ on the symmetric group $S_3 = \{e, a, b, x, y, z\}$ where $e = (1)$, $a = (123)$, $b = (132)$, $x = (12)$, $y = (13)$, $z = (23)$, and $\odot$ is given by the following Cayley's table:*

$\odot$	e	x	y	z	a	b
e	e	x	y	z	b	a
x	x	e	a	b	z	y
y	y	b	e	a	x	z
z	z	a	b	e	y	x
a	a	z	x	y	e	b
b	b	y	z	x	a	e

The set of all right mappings of a K-algebra is

$$\mathrm{R} = \{\mathrm{R_e}, \mathrm{R_x}, \mathrm{R_y}, \mathrm{R_z}, \mathrm{R_a}, \mathrm{R_b}\}$$

where

$$\begin{aligned}
\mathrm{R_e} &: (e)(a)(b)(x)(y)(z) = I,\\
\mathrm{R_x} &: (e\ x)(y\ b)(z\ a),\\
\mathrm{R_y} &: (e\ y)(x\ a)(z\ b),\\
\mathrm{R_z} &: (e\ z)(x\ b)(y\ a),\\
\mathrm{R_a} &: (e\ b\ a)(x\ z\ y),\\
\mathrm{R_b} &: (e\ a\ b)(x\ y\ z).
\end{aligned}$$

By routine calculations, it is easy to see that

$$(R, \circ) = \{< R_a, R_z >: R_a^3 = I = R_z^2 = (R_a \circ R_z)^2\} \cong S_3.$$

Theorem 3.1. *Let $\mathcal{K}$ be a K-algebra on non-abelian group G. Let R be the set of all right mappings of K-algebra with the binary operation of composition $(\circ)$ of the right mappings defined by*

$$R_x \circ R_y = R_{y \odot (e \odot x)}.$$

Then

a. *the system $(R, \circ)$ forms non- abelian group.*
b. $(R, \circ) \cong G$.

Proof. (a) Since R_x, R_y on K-algebra are composed by $R_x \circ R_y = R_{y \odot (e \odot x)}$. So it is easy to see that:

(i) the composition is non commutative, that is,

$$R_x \circ R_y = R_{y \odot (e \odot x)} \neq R_{x \odot (e \odot y)} = R_y \circ R_x \quad \forall\ x,\ y \in G.$$

(ii) the composition is associative, that is,

$$(R_x \circ R_y) \circ R_z \quad = \quad R_x \circ (R_y \circ R_z)$$

for all x, y, $z \in G$.

(iii) If R_e is the identity element of $(R, \circ)$ and $R_x^{-1} = R_{e \odot x}$ is the inverse of $(R, \circ)$ for all $x \in G$, then

$$R_x \circ R_{e \odot x} \quad = \quad R_e = R_{e \odot x} \circ R_x.$$

Hence $(R, \circ)$ forms non-abelian group.

(b) In order to show that $(R, \circ) \cong G$, we consider the map $\phi : R \to G$, from R into G defined by $\phi(R_x) = e \odot x$ for all $R_x \in R$. We notice that:

(i)clearly ϕ is well-defined.

(ii) ϕ is a homomorphism since for $R_x, R_y \in R$

$$\phi(R_x \circ R_y) = \phi(R_{y \odot (e \odot x)}) = e \odot (y \odot (e \odot x)) = (e \odot x) \cdot (e \odot y) = \phi(R_x) \circ \phi(R_y).$$

(iii) ϕ is one-to-one since

$$\begin{aligned} \phi(R_x) = \phi(R_y) \quad &\Rightarrow \quad e \odot x = e \odot y \\ &\Rightarrow \quad (e \odot x) \odot (e \odot y) = e \\ &\Rightarrow \quad y \odot x = e \\ &\Rightarrow \quad R_{y \odot x} = R_e \\ &\Rightarrow \quad R_y \circ R_{e \odot x} = R_e \\ &\Rightarrow \quad R_y \circ R_x^{-1} = R_e \\ &\Rightarrow \quad R_x = R_y. \end{aligned}$$

Hence

$$(R, \circ) \cong G.$$

□

We give the following Theorem without proof.

Theorem 3.2. *Let $\mathcal{K}$ be a K-algebra on abelian group G. Let R be a set of all right mappings of K-algebra $\mathcal{K}$. Then $(R, \odot)$ is a K-algebra $\mathcal{K}$ on R if and only if the system $(R, \circ)$ on $\mathcal{K}$ is isomorphic to the group G.*

Left mappings of K-algebras

Definition 3.3. *Let $\mathcal{K}$ be a K-algebra. For a fixed element $x \in \mathcal{K}$, the mapping $L_x : \mathcal{K} \to \mathcal{K}$ defined by $L_x(y) = x \odot y$ for all $y \in \mathcal{K}$, is called left map on $\mathcal{K}$. The set of all left mappings on K- algebra $\mathcal{K}$ is denoted by L.*

Example 3.2. *Consider the K-algebra $\mathcal{K} = (S_3, \cdot, \odot, e)$ on the symmetric group $S_3 = \{e, a, b, x, y, z\}$ where $e = (1)$, $a = (123)$, $b = (132)$, $x = (12)$, $y = (13)$, $z = (23)$, and $\odot$ is given by the Cayley's table in Example 3.2. The set of all left mappings of a K-algebra is*

$$\mathrm{L} = \{\mathrm{L_e}, \mathrm{L_x}, \mathrm{L_y}, \mathrm{L_z}, \mathrm{L_a}, \mathrm{L_b}\}$$

where

$$\begin{aligned}
\mathrm{L_e} &: (e)(x)(y)(z)(a\ b),\\
\mathrm{L_x} &: (e\ x)(y\ a\ z\ b),\\
\mathrm{L_y} &: (e\ y)(x\ b\ z\ a),\\
\mathrm{L_z} &: (e\ z)(x\ a\ y\ b),\\
\mathrm{L_a} &: (e\ a)(x\ z\ y)(b),\\
\mathrm{L_b} &: (e\ b)(x\ y\ z)(a).
\end{aligned}$$

It is easy to verify the following:

- $L_x \circ L_e = R_x$, $L_y \circ L_e = R_y$, $L_z \circ L_e = R_z$, $L_a \circ L_e = R_a$, $L_b \circ L_e = R_b$.

By routine calculations, it is easy to see that $(L, \circ)$ does not form a group.

Definition 3.4. *The binary operation of composition $(\circ)$ of L on a K-algebra built on a non-abelian group behaves in the following way:*

$$\begin{aligned}
L_x \circ L_y(z) &= L_x(L_y(z)) = x \odot (y \odot z) = (x \odot (e \odot z)) \odot y)\\
&= R_y \circ L_x \circ L_e(z).
\end{aligned}$$

It is easy to see the following identities:

- $L_x(y) = R_y(x)$
- $R_z \circ L_x = L_x \circ L_z \circ L_e$

for all x, y, $z \in G$. In order to extend further to the mutual interactions of the left and right mappings of a K-algebra, we include the following:

Proposition 3.1. *Let $\mathcal{K}$ be a K-algebra. Then the left mappings of the set $(L, \circ)$ compose on $\mathcal{K}$ holding the following interacting properties to $(R, \circ)$ for all x, y, $z \in G$:*

(1) $L_e^2 = R_e$,
(2) $L_e \circ L_x = R_x$,
(3) $L_{x \odot y} = L_x \circ R_{e \odot y} = L_x \circ L_{e \odot y} \circ L_e$,
(4) $L_e(x \odot y) = y \odot x = L_y(x) = R_x(y)$,
(5) $L_x^2 \circ L_e = R_x \circ L_x$,
(6) $L_x \circ L_z \circ L_e = R_z \circ L_x$.

Proof. Routine. □

Lemma 3.1. [3] *Let $\mathcal{K}$ be a K-algebra on an abelian group G and let $(L, \circ)$ be the set of all left mappings of $\mathcal{K}$. Then $L_e \in L$ is the only non-identity automorphism of $\mathcal{K}$.*

Theorem 3.3. *Let $\mathcal{K}$ be a K-algebra on a non-abelian group G and let $(L, \circ)$ be the set of all left mappings of $\mathcal{K}$. Then $L_e \in L$ is the only non-identity endomorphism of $\mathcal{K}$.*

Proof. L_e is an endomorphism of $\mathcal{K}$ since

$$L_e(x \odot y) = e \odot (x \odot y) = (e \odot x) \odot (e \odot y) = L_e(x) \odot L_e(y).$$

Hence L_e is an endomorphism of K-algebras $\mathcal{K}$. □

From the mutual intersection of the left mappings of a K-algebra $\mathcal{K}$, it is easy to note that, if $L_e \circ L_x = \overline{L}_x$, then $\overline{L}_x(y) = e \odot (x \odot y)$ for all $x \in G$. Let $L/L_e = \overline{L} = \{\overline{L}_x : x \in G\}$, then by routine simplification process one can find in $(\overline{L}, \circ)$ that:

- $L_e \circ L_e = \overline{L}_e=$ Identity$=R_e$
- $\overline{L}_x \circ \overline{L}_y = \overline{L}_{y \odot (e \odot x)}$
- $(\overline{L}_x \circ \overline{L}_y) \circ \overline{L}_z = \overline{L}_x \circ (\overline{L}_y \circ \overline{L}_z)$
- $\overline{L}_x \circ \overline{L}_{e \odot x} = \overline{L}_e = \overline{L}_{e \odot x} \circ \overline{L}_x$

Example 3.3. *In Example 3.8, we see that the set of left mappings L of a K-algebra on non-abelian G does not form group. We generate the group of left mappings formed by $L/L_e = \overline{L} = \{L_e \circ L_x : x \in G\}$. By routine computations, It is easy to see the following:*

$$\begin{aligned}
\overline{\mathrm{L}}_{\mathrm{e}} &= \mathrm{L}_{\mathrm{e}} \circ \mathrm{L}_{\mathrm{e}} = I \\
\overline{\mathrm{L}}_{\mathrm{x}} &= \mathrm{L}_{\mathrm{e}} \circ \mathrm{L}_{\mathrm{x}} = (e\ x)(y\ a)(z\ b) \\
\overline{\mathrm{L}}_{\mathrm{y}} &= \mathrm{L}_{\mathrm{e}} \circ \mathrm{L}_{\mathrm{y}} = (e\ y)(x\ b)(z\ a) \\
\overline{\mathrm{L}}_{\mathrm{z}} &= \mathrm{L}_{\mathrm{e}} \circ \mathrm{L}_{\mathrm{z}} = (e\ z)(x\ a)(y\ b) \\
\overline{\mathrm{L}}_{\mathrm{a}} &= \mathrm{L}_{\mathrm{e}} \circ \mathrm{L}_{\mathrm{a}} = (e\ a\ b)(x\ z\ y) \\
\overline{\mathrm{L}}_{\mathrm{b}} &= \mathrm{L}_{\mathrm{e}} \circ \mathrm{L}_{\mathrm{b}} = (e\ b\ a)(x\ y\ z)
\end{aligned}$$

By routine calculations, it is easy to see that

$$(\overline{L}, \circ) = \{< \overline{L}_a, \overline{L}_z >: \overline{L}_a^3 = I = \overline{L}_z^2 = (\overline{L}_a \overline{L}_z)^2\} \cong S_3.$$

We state the following theorem without proof.

Theorem 3.4. *Let $\mathcal{K}$ be a K-algebra on non-abelian group G. and let*

$$\overline{L} = L/L_e = \{L_e \circ L_x : x \in G\}$$

be the set of all left mappings of a K-algebra with the binary operation of composition $(\circ)$ of left mappings defined by

$$\overline{L}_x \circ \overline{L}_y = \overline{L}_{y \odot (e \odot x)}.$$

Then

a. *the system $(\overline{L}, \circ)$ forms non-abelian group.*
b. $(\overline{L}, \circ) \cong (R, \circ) \cong G$.

Example 3.4. *Let $\overline{L} = \{I = \overline{L}_e, \overline{L}_x, \overline{L}_y, \overline{L}_z, \overline{L}_a, \overline{L}_b\}$ be the set of all left mappings of $\mathcal{K}$ on the symmetric group $S_3 = \{e, a, b, x, y, z\}$ where $e = (1)$, $a = (123)$, $b = (132)$, $x = (12)$, $y = (13)$, $z = (23)$. Consider K-algebra $\mathcal{K}$ on $\overline{L}$, and $\odot$ is given by the following Cayley table:*

$\odot$	$\overline{L}_e$	$\overline{L}_x$	$\overline{L}_y$	$\overline{L}_z$	$\overline{L}_a$	$\overline{L}_b$
$\overline{L}_e$	$\overline{L}_e$	$\overline{L}_x$	$\overline{L}_y$	$\overline{L}_z$	$\overline{L}_b$	$\overline{L}_a$
$\overline{L}_x$	$\overline{L}_x$	$\overline{L}_e$	$\overline{L}_a$	$\overline{L}_b$	$\overline{L}_z$	$\overline{L}_y$
$\overline{L}_y$	$\overline{L}_y$	$\overline{L}_b$	$\overline{L}_e$	$\overline{L}_a$	$\overline{L}_x$	$\overline{L}_z$
$\overline{L}_z$	$\overline{L}_z$	$\overline{L}_a$	$\overline{L}_b$	$\overline{L}_e$	$\overline{L}_y$	$\overline{L}_x$
$\overline{L}_a$	$\overline{L}_a$	$\overline{L}_z$	$\overline{L}_x$	$\overline{L}_y$	$\overline{L}_e$	$\overline{L}_b$
$\overline{L}_b$	$\overline{L}_b$	$\overline{L}_y$	$\overline{L}_z$	$\overline{L}_x$	$\overline{L}_a$	$\overline{L}_e$

Thus we state the following Theorem without proof.

Theorem 3.5. *Let $\mathcal{K}$ be a K-algebra on non-abelian group G. Let $\overline{L}$ be a set of all left mappings of K-algebra $\mathcal{K}$. Then $(\overline{L}, \odot)$ is a K-algebra $\mathcal{K}$ on $\overline{L}$ if and only if the system $(\overline{L}, \circ)$ on $\mathcal{K}$ is isomorphic to the group G.*

In closing this paper, we state the following Theorem which can be easily proved. We hence omit the details.

Theorem 3.6. *Let G be a group and let R and L be the sets of right and left mappings of K-algebras. Then $R \cong L$.*

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