Annals of the University of Craiova, Mathematics and Computer Science Series
Volume 37(2), 2010, Pages 75–82
ISSN: 1223-6934

On strong IS-algebras

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Abstract. IS-algebras with additional condition, so called strong IS-algebras, are introduced, and some properties are investigated. We introduced the notion of a strong IS-algebra endomorphisms. In addition, a congruence relation on a strong IS-algebras is defined. As well as some properties of left and right mappings of strong IS-algebras are investigated.

2010 Mathematics Subject Classification. Primary 06F35, 03G25; Secondary 08A35.
Key words and phrases. IS-algebras, strong IS-algebras, endomorophism, left and right mappings.

1. Introduction

Imai and Iséki [5] in 1966 introduced the notion of a BCK-algebra. In the same year, Iséki introduced BCI-algebras [6] as a super class of the class of BCK-algebras. In 1993, Jun et al. [7] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup/BCI-monoid/ BCI-group. In 1998, for the convenience of study, Jun et al. [9] renamed the BCI-semigroup (respectively, BCI-monoid and BCI-group) as the IS-algebra (respectively, IM-algebra and IG-algebra) and studied further properties of these algebras. Not long ago, Park et al. [10] studied the isomorphism theorems in IS-algebras.

Dar introduced the notions of left and right mappings over BCK-algebras in [1] and further discussed in [2]. The notions of left and right mappings over BCI- algebras have been discussed in [3]. In this paper, we discussed IS-algebras with additional condition, so called strong IS-algebras, and investigated several properties. We introduced the notion of strong IS-algebra endomorphisms. Some more properties of left and right mappings of strong IS-algebras are investigated.

2. Preliminaries

The following definitions and notations will be used throughout this paper.

By a BCI-algebra we mean an algebra $(X, *, 0)$ of type $(2, 0)$ satisfying the following conditions: for every $x, y, z \in X$,
(I) $((x * y) * (x * z)) * (z * y) = 0$,
(II) $(x * (x * y)) * y = 0$,
(III) $x * x = 0$,
(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$.
A BCI-algebra X satisfying $0 \leq x$ for all $x \in X$ is called a BCK-algebra. In any BCI-algebra X one can define a partial order "$\leq$" by putting $x \leq y$ if and only if $x * y = 0$.
A BCI-algebra X has the following properties for any $x, y, z \in X$:

Received March 29, 2010. Revision received May 15, 2010.

$(a1)$ $x * 0 = x$,
$(a2)$ $(x * y) * z = (x * z) * y$,
$(a3)$ $x \leq y$ implies that $(x * z) \leq (y * z)$ and $(z * y) \leq (z * x)$,
$(a4)$ $(x * z) * (y * z) \leq x * y$,
$(a5)$ $x * (x * (x * y)) = x * y$,
$(a6)$ $0 * (x * y) = (0 * x) * (0 * y)$,
$(a7)$ $0 * (0 * ((x * z) * (y * z))) = (0 * y) * (0 * x)$.

A non-empty subset I of a BCK/BCI-algebra X is called an ideal of X if it satisfies: (i) $0 \in I$ and (ii) if $x * y \in I$ and $y \in I$ implies $x \in I$ for all $x, y \in X$. Any ideal I has the property: $y \in I$ and $x \leq y$ imply $x \in I$. A non-empty subset D of a BCI-algebra X is called a subalgebra of X if $x * y \in D$ whenever $x, y \in D$. In general, an ideal I of a BCI-algebra X need not be a subalgebra. If an ideal I is also a subalgebra of a BCI-algebra X, we say that I is a closed ideal, equivalently, an ideal I is closed if and only if $0 * x \in I$ whenever $x \in I$.

Definition 2.1. [9]. *An IS-algebra is a non-empty set X with two binary operations "$*$" and "$\cdot$" and constant 0 satisfying the axioms*

$(b1)$ $(X, *, 0)$ is a BCI-algebra,
$(b2)$ $(X, \cdot)$ is a semigroup,
$(b3)$ the operation "$\cdot$" is distributive (on both sides) over the operation "$*$", that is,

$$x \cdot (y * z) = (x \cdot y) * (x \cdot z) \text{ and } (x * y) \cdot z = (x \cdot z) * (y \cdot z) \text{ for all } x, y, z \in X.$$

Note that the **IS**-algebra is a generalization of the ring (see [9]).

Example 2.1. [11]. *Let $X = \{0, a, b, c\}$ be a set with Cayley tables:*

$*$	0	a	b	c
0	0	0	c	b
a	a	0	c	b
b	b	b	0	c
c	c	c	b	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	b	c
c	0	0	c	b

Then X is an **IS**-algebra.

Lemma 2.1. [7]. *Let X be an IS-algebra. Then we have*

(i) $0 \cdot x = x \cdot 0 = 0$,
(ii) $x \leq y$ implies that $x \cdot z \leq y \cdot z$ and $z \cdot x \leq z \cdot y$, for all $x, y, z \in X$.

Definition 2.2. [9]. *A non-empty subset A of an **IS**-algebra X is called a left (resp. right) I-ideal (here we call it a left (resp. right) IS-ideal) of X if*
(i) $x \cdot a \in A$ (resp. $a \cdot x \in A$) whenever $x \in X$ and $a \in A$,

(ii) for any $x, y \in X$, $x * y \in A$ and $y \in A$ imply that $x \in A$.
Both a left and right IS-ideal is called IS-ideal.

Definition 2.3. [8] *An IS-ideal A of an IS-algebra X is said to be closed if $x \in A$ implies $0 * x \in A$.*

Definition 2.4. [10] *Let X, Y be IS-algebras. A mapping $\theta : X \to Y$ is called a homomorphism if for all $a, b \in X$, $\theta(a * b) = \theta(a) * \theta(b)$ and $\theta(a \cdot b) = \theta(a) \cdot \theta(b)$.*

Theorem 2.1. *If X is an IS-algebra, then the following are equivalent:*

(i) $(\forall x \in X)$ $(\mathcal{A}_x = \{y \in X : y \cdot x = 0\}$ is an IS-ideal of $X)$.
(ii) $(\forall a, b \in X)$ (if $a \cdot b = 0$ implies $a \cdot z \cdot b = 0$ for all $z \in X$).

Proof. $(i) \Rightarrow (ii)$ Let $a, b \in X$ be such that $a \cdot b = 0$. Then $a \in \mathcal{A}_b$. Hence, $a \cdot z \in \mathcal{A}_b$ for every $z \in X$. Thus $a \cdot z \cdot b = 0$ for all $z \in X$.
$(ii) \Rightarrow (i)$ Let $x, z \in X$ and $a \in \mathcal{A}_x$. Then $z \cdot a \cdot x = z \cdot 0 = 0$. Hence, $z \cdot a \in \mathcal{A}_x$. Now by (ii), $a \cdot z \cdot x = 0$ for every $z \in X$. Hence, $a \cdot z \in \mathcal{A}_x$. Therefore, $a \cdot z, z \cdot a \in \mathcal{A}_x$. Let $a, z, x \in X$ be such that $a * z \in \mathcal{A}_x$ and $z \in \mathcal{A}_x$. Then

$$\begin{aligned} 0 &= (a * z) \cdot x \\ &= a \cdot x * z \cdot x \\ &= a \cdot x * 0 \qquad \text{(since } z \in \mathcal{A}_x) \\ &= a \cdot x \qquad \text{(by } (a1)). \end{aligned}$$

Hence, $a \in \mathcal{A}_x$. Therefore, $\mathcal{A}_x$ is an IS-ideal of X. □

3. Strong IS-algebras

Definition 3.1. *An **IS**-algebra X is said to be a strong **IS**-algebra if*

$$0 * (x \cdot y) = (0 * x) \cdot (0 * y) \qquad \text{for all } x, y \in X.$$

Definition 3.2. *A strong **IS**-algebra X is said to have an identity if there is an element $e \in X$ with*

$$e \cdot x = x \cdot e = x \qquad \textit{for all } x \in X.$$

Example 3.1. *Let $X = \{0, a, b, c\}$ be a set with Cayley tables:*

$*$	0	a	b	c
0	0	0	b	b
a	a	0	c	b
b	b	b	0	0
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then, by routine calculations, it can be seen that X is a strong IS-algebra with identity c.

Proposition 3.1. *Let X be a strong **IS**-algebra. Then*

(i) $0 * (x \cdot y) = 0 * (0 * (x \cdot y))$ for any $x, y \in X$.
(ii) If $x \cdot y = 0$, then $x \cdot (y * z) = (0 * x) \cdot (0 * z)$ for any $x, y, z \in X$.
(iii) If $x \cdot z = 0$, then $(x * y) \cdot z = (0 * y) \cdot (0 * z)$ for any $x, y, z \in X$.

Theorem 3.1. *Let X be a strong IS-algebra. Then the set $H = \{x \in X \mid 0 * x = 0\}$ is an IS-ideal of X.*

Proof. (i) Let $y \in X$ and $a \in H$. Then

$$\begin{aligned} 0 * (y \cdot a) &= (0 * y) \cdot (0 * a) \\ &= (0 * y) \cdot 0 \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} 0 * (a \cdot y) &= (0 * a) \cdot (0 * y) \\ &= 0 \cdot (0 * y) \\ &= 0 \end{aligned}$$

Hence, $y \cdot a$, $a \cdot y \in H$.
(ii) For $a, y \in X$, $a * y \in H$ and $y \in H$, we have

$$\begin{aligned} 0 &= 0 * (a * y) \\ &= (0 * a) * (0 * y) \\ &= (0 * a) * 0 \\ &= (0 * a). \end{aligned}$$

Hence, $a \in H$. Therefore, H is an *IS*-ideal of X. □

Let ρ be a congruence relation on X, that is, ρ is an equivalence relation on X such that $(x, y) \in \rho$ implies $(x * z, y * z) \in \rho$, $(z * x, z * y) \in \rho, (x \cdot z, y \cdot z) \in \rho$, and $(z \cdot x, z \cdot y) \in \rho$ for all $z \in X$. The set of all equivalence classes of X with respect to ρ will be denoted by X/ρ. On X/ρ we define two operations, $*$, $\cdot$, as follows: $[x]_\rho * [y]_\rho = [x * y]_\rho$ and $[x]_\rho \cdot [y]_\rho = [x \cdot y]_\rho$ for all $[x]_\rho, [y]_\rho \in X/\rho$. It is clear that such operation is well-defined, but $(X/\rho, *, [0]_\rho)$ may not be a BCI-algebra, because X/ρ does not satisfy the fourth condition of a BCI-algebra. (see [4])

Proposition 3.2. [4] *If ρ is a congruence relation on a BCI-algebra G, then the following are equivalent:*

(1) If $x * y \in [0]_\rho$ and $y * x \in [0]_\rho$, then $(x, y) \in \rho$,
(2) ρ is regular, i.e., $[x]_\rho * [y]_\rho = [0]_\rho = [y]_\rho * [x]_\rho$,
(3) $(G/\rho, *, [0]_\rho)$ is a BCI-algebra.

Theorem 3.2. *Let ρ be a regular congruence relation on a strong IS-algebra X. Then X/ρ is a strong IS-algebra.*

Proof. From Proposition 3.2 it follows that $(X/\rho, *)$ is a BCI-algebra. Also, $(X/\rho, \cdot)$ is a semigroup. For every $[x]_\rho$, $[y]_\rho$, $[z]_\rho \in X/\rho$, we have

$$\begin{aligned} [x]_\rho \cdot \Big([y]_\rho * [z]_\rho\Big) &= [x]_\rho \cdot [y * z]_\rho \\ &= [x \cdot (y * z)]_\rho \\ &= [(x \cdot y) * (x \cdot z)]_\rho \\ &= [x \cdot y]_\rho * [x \cdot z]_\rho \\ &= [x]_\rho \cdot [y]_\rho * [x]_\rho \cdot [z]_\rho \end{aligned}$$

and

$$\begin{aligned} \Big([x]_\rho * [y]_\rho\Big) \cdot [z]_\rho &= [x * y]_\rho \cdot [z]_\rho \\ &= [(x * y) \cdot z]_\rho \\ &= [(x \cdot z) * (y \cdot z)]_\rho \\ &= [x \cdot z]_\rho * [y \cdot z]_\rho \\ &= [x]_\rho \cdot [z]_\rho * [y]_\rho \cdot [z]_\rho . \end{aligned}$$

Hence X/ρ is an **IS**-algebra. For every $[x]_\rho$, $[y]_\rho \in X/\rho$, we have

$$\begin{aligned} [0]_\rho * \left([x]_\rho \cdot [y]_\rho\right) &= [0]_\rho * [x \cdot y]_\rho \\ &= [0 * (x \cdot y)]_\rho \\ &= [(0 * x) \cdot (0 * y)]_\rho \\ &= [0 * x]_\rho \cdot [0 * y]_\rho \\ &= \left([0]_\rho * [x]_\rho\right) \cdot \left([0]_\rho * [y]_\rho\right). \end{aligned}$$

Therefore, X/ρ is a strong IS-algebra. □

Theorem 3.3. *If ρ is a congruence relation on an IS-algebra X, then $[0]_\rho$ is a closed IS-ideal.*

Proof. Let $a \in [0]_\rho, x \in X$. Then $(a, 0) \in \rho$ and hence $(a \cdot x, 0 \cdot x) = (a \cdot x, 0) \in \rho$, $(x \cdot a, x \cdot 0) = (x \cdot a, 0) \in \rho$. Thus, $a \cdot x \in [0]_\rho$ and $x \cdot a \in [0]_\rho$.
Let $x, y \in X$ be such that $x * y \in [0]_\rho$ and $y \in [0]_\rho$. Then $(x * y, 0) \in \rho$ and $(y, 0) \in \rho$. Since, $(y, 0) \in \rho$, it follows that $(x * y, x * 0) = (x * y, x) \in \rho$. So, $(x, 0) \in \rho$. Hence, $x \in [0]_\rho$. Therefore, $[0]_\rho$ is an IS-ideal.
If $x \in [0]_\rho$, then $(x, 0) \in \rho$ and hence $(0 * x, 0 * 0) = (0 * x, 0) \in \rho$, that is, $0 * x \in [0]_\rho$.
Therefore, $[0]_\rho$ is a closed IS-ideal. □

Proposition 3.3. *Let ρ be a regular congruence relation on a strong IS-algebra X. Then the mapping $\theta : X \to X/\rho$ defined by $\theta(x) = [x]_\rho$, for all $x \in X$ is a homomorphism.*

4. Strong IS-algebra endomorphisms

Definition 4.1. *A mapping $\eta : X \to X$ on an strong IS-algebra X is called an endomorphism if for all $x, y \in X$, $\eta(x * y) = \eta(x) * \eta(y)$ and $\eta(x \cdot y) = \eta(x) \cdot \eta(y)$.*

The set of $End(X)$ of all endomorphisms of X forms a semigroup under the binary operation of their composition ($\circ$). Let $\eta : X \to X$ be an endomorphism of strong IS-algebra. Then the set $\{x \in X \mid \eta(x) = 0\}$ is called the kernel of η, and denoted by $\ker \eta$.

Proposition 4.1. *If η is an endomorphism of a strong IS-algebra X then*

(i) $\eta(0) = 0$.
(ii) $\eta(0 * x) = 0 * \eta(x))$ for all $x \in X$.
(iii) If $x \cdot y = 0$, then $\eta(x) \cdot \eta(y) = 0$ for all $x, y \in X$.
(iv) If $\eta(x) = 0$, then $\eta(x \cdot y) = 0$ for all $x, y \in X$.
(v) If $\eta(y) = 0$, then $\eta(x \cdot y) = 0$ for all $x, y \in X$.
(vi) If $x \leq y$, then $\eta(x) \leq \eta(y)$ for all $x, y \in X$.
(vii) If $x \leq y$, then $\eta(x \cdot z) \leq \eta(y \cdot z)$ and $\eta(z \cdot x) \leq \eta(z \cdot y)$ for all $x, y, z \in X$.
($viii$) If A is left (resp. right) IS-ideal of X, then so is $\eta(A)$.
(ix) $\ker \eta$ is a closed IS-ideal of X.

Theorem 4.1. *Let η be an endomorphism of a strong IS-algebra X. Then η is one-to-one if and only if $\ker \eta = \{0\}$.*

Proof. Assume that η is one-to-one and let $x \in \ker \eta$. Then $\eta(x) = 0 = \eta(0)$. Thus $x = 0$, i.e., $\ker \eta = \{0\}$. Conversely suppose that $\ker \eta = \{0\}$. Let $x, y \in X$ such that $\eta(x) = \eta(y)$. It follows that $\eta(x*y) = \eta(x)*\eta(y) = 0$ and $\eta(y*x) = \eta(y)*\eta(x) = 0$. So, $x*y$, $y*x \in \ker \eta$. Thus $x*y = y*x = 0$. Hence, $x = y$. Therefore, η is one-to-one. □

Theorem 4.2. *Let η be an endomorphism of a strong IS-algebra X. If η is idempotent, i.e., $\eta(\eta(x)) = \eta(x)$ for all $x \in X$, then η is one-to-one if and only if η is the identity map.*

Proof. $\Longrightarrow$) Suppose η is one-to-one. For any $x \in X$, we have $\eta(x * \eta(x)) = \eta(x) * \eta(\eta(x)) = \eta(x) * \eta(x) = 0 = \eta(0)$ and so $x * \eta(x) = 0$ for any $x \in X$. Similarly, $\eta(x) * x = 0$ for any $x \in X$. Therefore, $\eta(x) = x$ for any $x \in X$ so that, η is the identity map.
$\Longleftarrow$) Obvious. □

Proposition 4.2. *Let η be an endomorphism of a strong IS-algebra X and $\eta^{-1}(0) = \{0\}$. Then $\eta(x) \leq \eta(y)$ imply $x \leq y$.*

Proof. If $\eta(x) \leq \eta(y)$, then we have $\eta(x * y) = \eta(x) * \eta(y) = 0$. Hence, $x * y = 0$, and so we obtain $x \leq y$. □

Theorem 4.3. *Let X be a strong IS-algebra and $\theta \in End(X)$. Then the set $K_\theta = \{(x, y) \in X \times X \mid \theta(x) = \theta(y)\}$ is a congruence relation on X.*

Proof. Clearly, K_θ is an equivalence relation on X. Let $x, y \in X$ be such that $(x, y) \in K_\theta$. Then $\theta(x) = \theta(y)$, which implies that $\theta(x*z) = \theta(x)*\theta(z) = \theta(y)*\theta(z) = \theta(y*z)$, and $\theta(x \cdot z) = \theta(x) \cdot \theta(z) = \theta(y) \cdot \theta(z) = \theta(y \cdot z)$. It follows that $(x * z, y * z)$ and $(x \cdot z, y \cdot z) \in K_\theta$ for all $z \in X$. The proof of $(z * x, z * y)$, $(z \cdot x, z \cdot y) \in K_\theta$ for all $z \in X$ is similar. Therefore, K_θ is a congruence relation on X. □

Definition 4.2. *Let X be a strong IS-algebra. For a fixed element $x \in X$, the mapping $M_x : X \to X$ defined by $M_x(y) = x * y$ for all $y \in X$, is called left map on X. (see* [1]*).*

Definition 4.3. *Let X be a strong IS-algebra. For a fixed element $x \in X$, the mapping $N_x : X \to X$ defined by $N_x(y) = y * x$ for all $y \in X$, is called right map on X. (see* [1]*).*

It is easy to verify the following: ($N_x(y) = M_y(x)$, for all $x, y \in X$), $N_0 = N_0^{-1} = id_X$.

Proposition 4.3. *Let X be a strong IS-algebra. If $M = \{M_x \mid x \in X\}$ and $N = \{N_x \mid x \in X\}$. Then the left mappings of the set $(M, \circ)$ compose on X holding the following interacting properties to $(M, \circ)$ for all $x, y \in X$.*

(*a*) $N_0 \circ M_x = M_x = M_x \circ N_0$.
(*b*) $N_{x \cdot y}(0) = M_0(x) \cdot M_0(y)$.
(*c*) $M_{x*y} = N_y \circ M_x$.
(*d*) $N_x \circ M_x = M_0$.

Proof. Routine. □

Theorem 4.4. *Let X be a strong IS-algebra. Then for all $x, y \in X$, the following hold: for every $x, y \in X$.*

(1) If $x \leq y$, then $M_0(x) = M_0(y)$.
(2) if $M_x \leq M_y$, then $x \leq y$.
(3) $x = y$ if and only if $M_x = M_y$.

Proof. (1) $M_0(x) = 0 * x = (x * y) * x = (x * x) * y = 0 * y = M_0(y)$.
(2) Suppose $M_x \leq M_y$. Then $M_x(z) * M_y(z) = 0$, for all $z \in X$. Thus we have

$$x * y = (x * y) * 0 = (x * y) * (y * y) = M_x(y) * M_y(y) = 0,$$

and so $x \leq y$.
(3) Necessity is obvious. If $M_x = M_y$, then

$$x * y = M_x(y) = M_y(y) = 0 = M_x(x) = M_y(x) = y * x.$$

It follows that $x * y = y * x = 0$. Therefore, $x = y$. □

Proposition 4.4. *Let X be a strong IS-algebra and $\theta \in End(X)$. Then $\theta \circ M_0 = M_0 \circ \theta$.*

Proof. Let $x \in X$. Then

$$\begin{aligned}(\theta \circ M_0)(x) &= \theta(M_0(x)) \\ &= \theta(0 * x) \\ &= \theta(0) * \theta(x) \\ &= 0 * \theta(x) \\ &= M_0(\theta(x)) \\ &= (M_0 \circ \theta)(x).\end{aligned}$$

□

Theorem 4.5. *Let X be a strong IS-algebra. The only endomorphism of X in M is M_0, where $M = \{M_x \mid x \in X\}$.*

Proof. (i) For any $x, y \in X$, we have

$$\begin{aligned}M_0(x * y) &= 0 * x * y \\ &= (0 * x) * (0 * y) \\ &= M_0(x) * M_0(y).\end{aligned}$$

(ii) For any $x, y \in X$, we have

$$\begin{aligned}M_0(x \cdot y) &= 0 * (x \cdot y) \\ &= (0 * x) \cdot (0 * y) \\ &= M_0(x) \cdot M_0(y).\end{aligned}$$

Therefore, M_0 is an endomorphism of X. Now, suppose that M_z is an endomorphism for non-zero z in X. So, $z = z * 0 = M_z(0) = M_z(0 * 0) = M_z(0) * M_z(0) = 0$. Thus, we have the contradiction $z \neq 0$ and $z = 0$. Therefore, the only endomorphism of X in M is M_0. □

Theorem 4.6. *Let X be a strong IS-algebra, and let $\circledast$, $\odot$ be two binary operations on $End(X)$ defined as $(\gamma \circledast \delta)(x) = \gamma(x) * \delta(x)$ and $(\gamma \odot \delta)(x) = \gamma(x) \cdot \delta(x)$ for all $\gamma, \delta \in End(X)$ and $x \in X$. Then $End(X)$ is a strong IS-algebra.*

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