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Fuzzy Logic Controller Based on Association Rules

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Abstract. The task of the standard Mamdani fuzzy logic controller is to find a crisp control action from the fuzzy rule-base and from a set of crisp inputs. In this paper we modify the classical Fuzzy Inference Engine in order to activate a set of rules having the same conclusion; thus we obtain a fuzzy set as output (like as in Generalized Modus Ponens reasoning), which can be defuzzified in order to obtain a crisp value. Usually, the inference rules used in a fuzzy logic controller are given by a domain expert; in our system, these rules are automatically induced as fuzzy association rules starting from a training set. The fuzzy confidence value associated with each rule is used to obtain the fuzzy set inferred by our system.

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1. Introduction

The database of a rule-based system may contain imprecisions which are inherent in the description of the rules given by the expert. Because such an inference can not be made by the methods which use classical two valued logic or many valued logic, Zadeh [17] and Mamdani [13] suggested an inference rule called "compositional rule of inference". Afterwards, Zadeh gave a theory of approximate reasoning [18], that is the deduction of imprecise conclusions from a set of imprecise premises. The theory of approximate reasoning is based on fuzzy logic inference processes. An important part of fuzzy reasoning is represented by Fuzzy Logic Control (FLC), derived from the control theory based on mathematical models of the open-loop process. The Fuzzy Logic Control is very useful when the needed models are not known or when they are too complex for analysis with conventional quantitative techniques. In a fuzzy logic controller, the expert knowledge is of the form

$$IF\ (a\ set\ of\ conditions\ are\ satisfied)\ THEN\ (a\ set\ of\ consequences\ are\ inferred)$$

where the antecedents and the consequences of the rules are associated with fuzzy concepts (linguistic terms). The task of a FLC system is to find a crisp control action from the fuzzy rule-base and from the actual crisp inputs. Because the inputs and the outputs of fuzzy rule-based systems are fuzzy sets, we have to fuzzify the crisp inputs and to defuzzify the fuzzy outputs. A standard FLC system consists from four parts, as it results from the Figure 1. The most known FLC systems are: Mamdani, Tsukamoto, Sugeno and Larsen. The research in the field of Fuzzy Logic Control is the subject of many papers and books ([15, 14, 12, 9].

In this paper we modify the classical Fuzzy Inference Engine of the standard Mamdani controller in order to activate only a set of rules having the same conclusion; thus we obtain a fuzzy set as output (like as in Generalized Modus Ponens reasoning),

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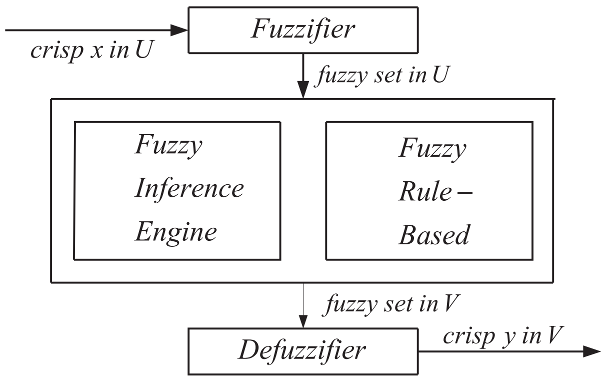


FIGURE 1. Fuzzy logic controller

which can be defuzzified in order to obtain a crisp value. Usually, the inference rules used by FLC are provided by a domain expert. In our case, these rules are induced as fuzzy association rules using a training set.

The our FLC system works as follows:

(1) generate rules used by inference engine
(2) compute the firing level of each rule, corresponding to input data
(3) compute the matching value of the input data with each set of rules having the same conclusion
(4) select the best set of rules used in the inference process
(5) compute the fuzzy set that represents the conclusion inferred and its corresponding crisp value obtained by defuzzification process.

The rest of paper is organized as follows: Section 2 presents the basic concepts with references to our approach. In Section 3 it is described the our FLC system. The Section 4 contains an example about presented system and the last section discusses conclusions and future works.

2. Basic Concepts

In this section we present the basic notion about concepts used in this paper.

2.1. Fuzzy Sets. A *fuzzy set* A in the universe U is defined by membership function $\mu_A : U \to [0,1]$. Because the majority of practical applications work with trapezoidal or triangular distributions and these representations are still a subject of various recent papers ([6], [16] for instance) we will work with membership functions represented by trapezoidal fuzzy numbers. Such a number $N = (\underline{m}, \overline{m}, \alpha, \beta)$ is defined as

$$\mu_N(x) = \begin{cases} 0 \;\; for \;\; x < \underline{m} - \alpha \\ \dfrac{x - \underline{m} + \alpha}{\alpha} \quad for \quad x \in [\underline{m} - \alpha, \underline{m}] \\ 1 \quad for \quad x \in [\underline{m}, \overline{m}] \\ \dfrac{\overline{m} + \beta - x}{\beta} \quad for \quad x \in [\overline{m}, \overline{m} + \beta] \\ 0 \;\; for \;\; x > \overline{m} + \beta \end{cases}$$

Definition 2.1. *A function* $T : [0,1]^2 \to [0,1]$ *is a* ***t-norm*** *iff it is commutative, associative, non-decreasing and* $T(x,1) = x \ \forall x \in [0,1]$.

The t-norms are used to compute the firing levels of the rules. The rules are represented by fuzzy implications. Let X and Y be two variables whose domains are U and V, respectively. A causal link from X to Y is represented as a conditional possibility distribution ([18], [19]) $\pi_{Y/X}$ which restricts the possible values of Y for a given value of X. For the rule

$$if\ X\ is\ A\ then\ Y\ is\ B \tag{1}$$

we have

$$\forall u \in U,\ \forall v \in V,\ \pi_{Y/X}(v,u) = \mu_A(u) \to \mu_B(v) \tag{2}$$

where $\to$ is an implication operator and μ_A and μ_B are the membership functions of the fuzzy sets A and B, respectively.

Definition 2.2. *An* ***implication*** *is a function* $I : [0,1]^2 \to [0,1]$ *satisfying the following conditions for all* $x, y, z \in [0,1]$:

I1: If $x \leq z$ *then* $I(x,y) \geq I(z,y)$

I2: If $y \leq z$ *then* $I(x,y) \leq I(x,z)$

I3: $I(0,y) = 1$ *(falsity implies anything)*

I4: $I(x,1) = 1$ *(anything implies tautology)*

I5: I(1,0)=0 (Booleanity)

Here, we use Lukasiewicz's implication defined as

$$I_L(x,y) = \min(1 - x + y, 1) \tag{3}$$

that is one of most important implication, as is proved in [4].

2.2. Fuzzy Association Rules. Mining of association rules represents one of the most important task in data mining. An association rule describes an interesting relationship among different attributes. The task of discovering boolean association rules was introduced by Agrawal in [2]. Fuzzy association rules can handle both quantitative and categorical data and are expressed in linguistic terms, which are more natural and understandable for human beings.

The basic problem of finding fuzzy association rules was introduced in [11]. Let $\mathcal{DB} = \{t_1, \ldots, t_n\}$ be a database characterized by a set $\mathcal{I} = \{i_1, \ldots, i_m\}$ of categorical or quantitative attributes (items). For each attribute i_k, $(k = 1, \ldots, m)$, we will consider $n(k)$ associated fuzzy sets. Let $F_{i_k} = \{F^1_{i_k}, \ldots, F^{n(k)}_{i_k}\}$ be the set of all these fuzzy sets. For an attribute i_k and a fuzzy set $F^j_{i_k}$, the membership function, denoted $\mu_{F^j_{i_k}}$, is defined as:

$$\mu_{F^j_{i_k}} : dom(i_k) \to [0,1],\ k = 1, \ldots, m,\ j = 1, \ldots, n(k) \tag{4}$$

We call **fuzzy itemset** the tuple $\langle X, F_X \rangle$, where $X \subseteq \mathcal{I}$ is a set of attributes and F_X is a set of fuzzy sets associated with attributes from X.

Definition 2.3. *A* ***fuzzy association rule*** *is an implication of the following form*

$$X \in F_X \Rightarrow Y \in F_Y$$

where $X, Y \in \mathcal{I}$, $X \cap Y = \emptyset$, $X = \{x_1, \ldots, x_p\}$, $Y = \{y_1, \ldots, y_q\}$, *and* $F_X = \{a_1, \ldots, a_p\}$, $F_Y = \{b_1, \ldots, b_q\}$ *are fuzzy sets related to attributes from* X *and* Y *respectively. More exactly,* $a_i \in F_{x_i}$, $(i = 1, \ldots, p)$, *and* $b_i \in F_{y_i}$, $(i = 1, \ldots, q)$.

The left side $X \in F_X$ *of the rule is called* antecedent, *while the right side of the rule* $Y \in F_Y$ *is called* consequent. *We denote this rule with:*

$$\langle X, F_X \rangle \Rightarrow \langle Y, F_Y \rangle$$

An example of a fuzzy association rule is the following:

"IF *Age* is *young* and *Income* is *high* THEN *Cars* is *many*"

Here, $X = \{Age, Income\}$, $Y = \{Cars\}$, $F_X = \{young, high\}$, $F_Y = \{many\}$ and the rule can be represented as:

$$\langle \{Age, Income\}, \{young, high\} \rangle \Rightarrow \langle Cars, many \rangle$$

In order to express the quality of a fuzzy association rule two quality measures, fuzzy support and fuzzy confidence, have been proposed in [11].

Definition 2.4 (Itemset fuzzy support value). *The* ***fuzzy support value*** *of itemset fuzzy itemset* $\langle X, F_X \rangle$ *in* $\mathcal{DB}$ *is:*

$$FS_{\langle X, F_X \rangle} = \frac{\sum_{t_i \in \mathcal{DB}} \prod_{x_j \in X} \alpha_{a_j}(t_i[x_j])}{|\mathcal{DB}|} \tag{5}$$

where

$$\alpha_{a_j}(t_i[x_j]) = \begin{cases} \mu_{a_j}(t_i[x_j]), & \textit{if } \mu_{a_j}(t_i[x_j]) \geq \omega \\ 0, & \textit{otherwise} \end{cases} \tag{6}$$

and ω *is a user specified minimum threshold for the membership function. Thus, the values of membership functions less than this minimum threshold,* ω, *are ignored*

Definition 2.5 (Rule fuzzy support value). *Let* $\langle X, F_X \rangle \Rightarrow \langle Y, F_Y \rangle$ *be a fuzzy association rule. The* ***fuzzy support value of the rule*** *is defined as fuzzy support value of the itemset* $\langle \{X, Y\}, \{F_X, F_Y\} \rangle$ *:*

$$FS_{\langle X, F_X \rangle \Rightarrow \langle Y, F_Y \rangle} = FS_{\langle \{X, Y\}, \{F_X, F_Y\} \rangle}$$

Definition 2.6 (Rule Fuzzy Confidence). *Let* $\langle X, F_X \rangle \Rightarrow \langle Y, F_Y \rangle$ *, a fuzzy association rule. The* ***fuzzy confidence value*** *of the rule is defined as:*

$$FC_{\langle X, F_X \rangle \Rightarrow \langle Y, F_Y \rangle} = \frac{FS_{\langle Z, F_Z \rangle}}{FS_{\langle X, F_X \rangle}}$$

where $Z = \{X, Y\}$ *and* $F_Z = \{X, Y\}$.

A fuzzy association rule is considered as *interesting* if it has enough support and high confidence value.

3. Proposed System

According with the structure presented in Figure 1 an FLC requires the following operations: fuzzification, reasoning and defuzzification. In our system these operations are implemented as follows:

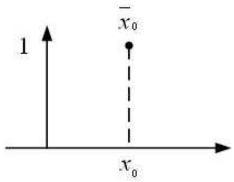


FIGURE 2. Fuzzy singleton as fuzzifier

3.1. Fuzzification and Defuzzification. A fuzzification operator transforms crisp data into fuzzy sets. For instance, $x_0 \in U$ is fuzzified into $\overline{x}_0$ (according to Figure 2).

The fuzzy control action C inferred from the fuzzy control system is transformed into a crisp control action:

$$z_0 = defuzzifier(C),$$

where $defuzzifier$ is a defuzzification operator. One of the most used defuzzification operator, for a discrete fuzzy set $C = \{(c_i, \mu_C(c_i)), i = 1, 2, ..., N\}$ is Middle-of-Maxima: the defuzzified value is defined as mean of all values of the universe of discourse, having maximal membership grades

$$z_0 = \frac{1}{N} \sum_{j=1}^{N} \mu_C(c_j).$$

3.2. Reasoning. In order to perform reasoning a set of rules are necessary. Typically rules for fuzzy logic controllers appear in if-then form and are obtained from the knowledge of experts and operators. As a result, the rules are limited, subjective and inaccurate.

In our system, these rules are automatically induced as fuzzy association rules starting from a training set. We can use any algorithm for mining fuzzy association rules (see [3, 5, 7, 8]) to induce fuzzy association rules. We consider that the training set is described as a set de transactions $\mathcal{DB} = \{t_1, \ldots, t_n\}$ characterized by a set $\mathcal{I} = \{i_1, \ldots, i_m\}$ of attributes. These attributes are represented by the input and output variables of fuzzy logic controller. For each attribute (variable) i_k, $(k = 1, \ldots, m)$, we will consider $n(k)$ linguistic values represented as fuzzy sets.

We generate only rules with input attributes in premise and output attributes in conclusion. The generated rules has the following form:

$$R : if\ X_1\ is\ A_1\ and\ ...\ and\ X_r\ is\ A_r\ then\ Y\ is\ B : \quad (FS, FC) \tag{7}$$

where $X_i, i \in \{1, 2, ..., r\}$ represent the input variables, Y is an output variable, $A_i, i \in \{1, 2, ..., r\}$ and B are linguistic values associated with X_i and Y respectively, and (FS, FC) are the fuzzy support and the fuzzy confidence of rule.

For a given rule R the input data $x = \{x_1, \ldots, x_r\}$ generates the firing level

$$\alpha = T(\mu_{X_1}(x_1), \ldots, \mu_{X_r}(x_r)) \tag{8}$$

where T is a t-norm.

We partition the generated rules in subsets with same conclusion:

$$\mathcal{R}(B) = \begin{cases} R_1 : & if\ X_1^1\ is\ A_1^1\ and\ ...\ and\ X_{r_1}^1\ is\ A_{r_1}^1\ then\ Y\ is\ B : \quad (FS_1, FC_1) \\ \ldots & \\ R_P : & if\ X_1^P\ is\ A_1^P\ and\ ...\ and\ X_{r_P}^P\ is\ A_{r_P}^P\ then\ Y\ is\ B : \quad (FS_P, FC_P) \end{cases} \tag{9}$$

For each rule subset $\mathcal{R}(B) = \{\mathcal{R}_1, \ldots, \mathcal{R}_P\}$ we compute the matching value of input data x as follows:

$$MR(B) = \frac{\sum_{i=1}^{P} \alpha_i * FC_i}{\sum_{i=1}^{P} FC_i} \tag{10}$$

where α_i is the firing level of the rule R_i.

In order to compute the output of FLC for the input x, the system selects the subset of rules $\mathcal{R}(C)$ having the maximum matching measure. This subset will be identified as a single rule having the conclusion C and the firing level $\alpha = MR(C)$. The inferred conclusion C' is given by [9]:

$$\mu_{C'}(v) = I_L(\alpha, \mu_C(v))$$

according with Figure 3

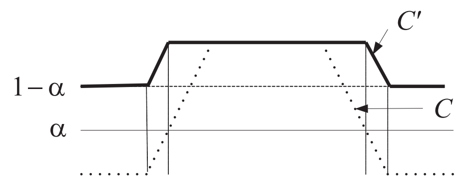


FIGURE 3. Conclusion obtained with Lukasiewicz implication

The crisp value y_0 associated to a conclusion C' inferred by means of the firing level α and the conclusion C represented by the fuzzy number $(\underline{m}_C, \overline{m}_C, \alpha_C, \beta_C)$ is obtained using the Middle-of-Maxima operator [10]:

$$y_0 = \frac{\underline{m}_C + \overline{m}_C + (1-\alpha)(\beta_C - \alpha_C)}{2} \tag{11}$$

If the same maximum matching measure is obtained for two or more subsets of rules, $\mathcal{R}(C_1), \ldots, \mathcal{R}(C_k)$, then we compute the crisp output, y_i, for every subset $\mathcal{R}(C_i)$ and the final crisp output is computed as weighted average:

$$y_0 = \frac{\sum_{i=1}^{k} y_i * FS_{\langle Y, C_i \rangle}}{\sum_{i=1}^{k} FS_{\langle Y, C_i \rangle}} \tag{12}$$

where $FS_{\langle Y, C_i \rangle}$ is fuzzy support value of fuzzy itemset $\langle Y, C_i \rangle$.

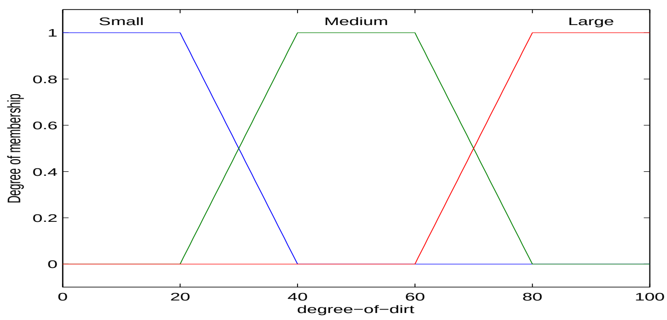
4. A Case Study

In order to show how the proposed system works, we consider an example inspired from [1] concerning washing machines. We consider a FLC with two inputs and one output. The input variables are *degree-of-dirt* (DD) and *type-of-dirt* (TD); the output variable is *washing-time* (WT). We consider the universes of discourse $[0, 100]$ for the input variables and $[0, 60]$ for the output variable.

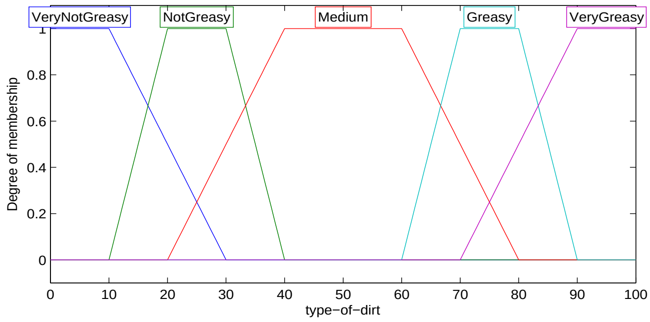
For the input variable DD we can take into consideration the following three linguistic variables (fuzzy sets):

$$F_{DD} = \{Small, Medium, Large\}$$

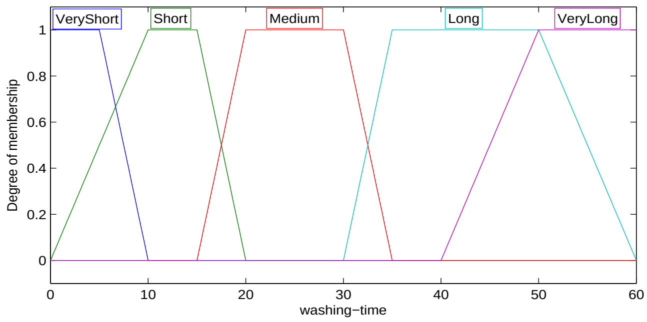
with membership functions defined as the following trapezoidal fuzzy numbers (see Figure 4(a)):



(a) The membership function of the input variable *degree-of-dirt*



(b) The membership function of the input variable *type-of-dirt*



(c) The membership function of the output variable *washing-time*

FIGURE 4. Membership functions for Fuzzy Logic Controller

$Small = (0, 20, 0, 20)$
$Medium = (40, 60, 20, 20)$
$Large = (80, 100, 20, 0)$
Similarly, let

$$F_{TD} = \{VeryNotGreasy, NotGreasy, Medium, Greasy, VeryGreasy\}$$

the set of linguistic variables (fuzzy sets) associated with the input variable TD with membership functions defined as the trapezoidal fuzzy numbers (see Figure 4(b)):

$VeryNotGreasy = (0, 10, 0, 20)$
$NotGreasy = (20, 30, 10, 10)$
$Medium = (40, 60, 20, 20)$
$Greasy = (70, 80, 10, 10)$
$VeryGreasy = (90, 100, 20, 0)$.

For the output variable WT we consider the following set of linguistic variables (fuzzy sets)

$$F_{WT} = \{VeryShort, Short, Medium, Long, VeryLong\}.$$

The membership functions for these linguistic variables are defined as follows (see Figure 4(c)):

$VeryShort = (0, 5, 0, 5)$
$Short = (10, 15, 10, 5)$
$Medium = (20, 30, 5, 5)$
$Long = (35, 50, 5, 10)$
$VeryLong = (50, 60, 10, 0)$.

In order to extract the rules used by the inference engine of FLC, we use a modified implementation of Fuzzy Apriori-T algorithm [3]. This algorithm runs on a training dataset and keep only rules with support and confidence greater than or equal to the minimum support threshold and minimum confidence threshold respectively. A fragment from training dataset is presented in Table 1.

TID	**DD**	**TD**	**WT**
1	10	15	3
2	2	63	31
3	4	73	52
4	89	70	49
5	61	74	73
6	41	22	11

Table 1. Training dataset fragment

The Table 2 contains the fuzzy association rules obtained applying the Fuzzy Apriori-T algorithm on training dataset. In the following we partition these rules in subsets with same conclusion and obtain:
$\mathcal{R}(VeryLong) = \{R_1\}$, $\mathcal{R}(Long) = \{R_2, R_3, R_4\}$, $\mathcal{R}(Medium) = \{R_5, R_6, R_7\}$, $\mathcal{R}(Short) = \{R_8\}$ and $\mathcal{R}(VeryShort) = \{R_9\}$.
Now, we have inference rules for our FLC.

Let consider that we want to compute the output for the following input data $x = (79, 62)$.

First, we compute the firing level for each rule $R_i, i = 1 \ldots 9$ (see Table 3), using the t-norm $t(x, y) = xy$.

After this step, for each partition of rules we compute the matching value of the input data x using the formula (10) (see Table 4) and select the rule set having the maximum matching measure, $\mathcal{R}(Long)$.

Now, the fuzzy output is computed according to the Figure 3 and the crisp output obtained using the formula (11) is 43.1763.

ID	Rule	Confidence
R_1	*If DD is Large and TD is Greasy then WT is VeryLong*	91,27%
R_2	*If DD is Medium and TD is Greasy then WT is Long*	92,06%
R_3	*If DD is Small and TD is Greasy then WT is Long*	91,83%
R_4	*If DD is Large and TD is Medium then WT is Long*	83,54%
R_5	*If DD is Medium and TD is Medium then WT is Medium*	84,52%
R_6	*If DD is Small and TD is Medium then WT is Medium*	92,11%
R_7	*If DD is Large and TD is NotGreasy then WT is Medium*	92,89%
R_8	*If DD is Medium and TD is NotGreasy then WT is Short*	94,94%
R_9	*If DD is Small and TD is NotGreasy then WT is VeryShort*	74,65%

TABLE 2. Fuzzy Association Rules

Rule	Firing Level (α)
$\mathcal{R}_1$	0,1900
R_2	0,0100
R_3	0,0000
R_4	0,8550
R_5	0,0450
R_6	0,0000
R_7	0,0000
R_8	0,0000
R_9	0,0000

TABLE 3. Firing Level for input x

Rule Set	Matching Level
$\mathcal{R}(VeryLong)$	0,1900
$\mathcal{R}(Long)$	0,2705
$\mathcal{R}(Medium)$	0,01417
$\mathcal{R}(Short)$	0,0000
$\mathcal{R}(VeryShort)$	0,0000

TABLE 4. Matching Level for input x

5. Conclusion

This paper presents a fuzzy controller model of Mamdani type. While the standard Mamdani controller activate a set of rules with different conclusions, our model activate a set of rules having the same conclusion; thus we obtain a fuzzy set as output (like as in Generalized Modus Ponens reasoning), which can be defuzzified in order to obtain a crisp value. Moreover, the rules used by Fuzzy Inference Engine are generated using Data Mining techniques. In the future we intend to extend this version in order to work with crisp data, intervals and/or linguistic terms as inputs.

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