Annals of the University of Craiova, Mathematics and Computer Science Series
Volume 38(3), 2011, Pages 101–108
ISSN: 1223-6934, Online 2246-9958

On a Mann type implicit iteration process for strictly pseudocontraction semigroups

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Abstract. The purpose of this paper is to study the strong and weak convergence of an implicity iteration process for strictly pseudocontractive semigroups in general Banach spaces. The results presented in this paper extend and improve recent results of some people. Zhang [*Acta Mathematica Sinica, English Series,* **26** 337-344 (2010)], Zhang [*Appl. Math. Mech. -Engl. Ed.*, **30**, 145-152 (2009)], Zhou [*Nonlinear Anal.*, **68**, 2977-2983 (2008)], Chen, et al. [*J. Math. Anal.,* **314**, 701-709 (2006)], Osilike [*J. Math. Appl.,* **294**, 73-81 (2004)] and Xu and Oir [*Numer. Funct. Anal. Optim.,* **22**, 767-773 (2001)].

2010 Mathematics Subject Classification. Primary 60J05; Secondary 60J20.
Key words and phrases. Strictly pseudocontraction semigroup, demiclosed principle, common fixed point, Opial's condition, implicit iteration process.

1. Introduction

Let E be a real Banach, E^* is the dual space of E, C is a nonempty closed convex subset of E. $J : E \to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|.\|f\|, \|x\| = \|f\|\}, x \in E.$$

Let $T : C \to C$ be a mapping. We use $F(T)$ to denote the set of fixed points of T; that is, $F(T) := \{x \in C : x = Tx\}$.

Definition 1.1. *One-parameter family $\{T(t) : t \geq 0\}$ of mappings from C into itself is said to be a strictly pseudo-contraction semigroup on C, if the following conditions are satisfied: (i) $T(0)x = x$ for each $x \in C$;*

(ii) $T(t + s)x = T(t)T(s)x$ for any $t, s \in \mathbb{R}_+$ and $x \in C$;

(iii) for each $x \in E$, the mapping $T(.)x$ from $\mathbb{R}_+$ into C is continuous;

(iv) there exists a bounded function $\lambda : [0, \infty) \to \left(0, \frac{1}{2}\right)$ such that for any given $x, y \in C$ there exists $j(x - y) \in J(x - y)$ such that

$$\langle T(t)x - T(t)y, j(x - y)\rangle \|x - y\|^2 - \lambda(t)\|[I - T(t)]x - [I - T(t)]y\|,$$

for each $t > 0$.

Throughout this paper, we denote $F(T) := \underset{t\geq 0}{\cap} F(T(t))$ and $\lambda := \inf_{t\geq 0}\{\lambda(t)\}$, we also assume that $\lambda > 0$.

Received April 04, 2011. Revision received August 27, 2011.
The presented work has been conducted in the context of the GRANT-xy funded by The National University Research Council from Romania, under the contract 01/2008.

Definition 1.2. *(v) A mapping $T : C \to C$ is said to be a pseudo-contraction, if for any $x, y \in C$, there exists $j(x-y) \in J(x-y)$ such that*

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2. \tag{1}$$

(vi) $T : C \to C$ is said to be strongly pseudocontractive, if there exists $k \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq k\|x-y\|^2, \tag{2}$$

for each $x, y \in C$ and for some $j(x-y) \in J(x-y)$.

(vii) $T : C \to C$ is said to be strictly pseudocontractive in the terminology of Browder and Petryshyn [2, 14], if there exists $\lambda > 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \lambda\|(I-T)x - (I-T)y\|^2, \tag{3}$$

for every $x, y \in C$ and for some $j(x-y) \in J(x-y)$.

It is easy to see that every strictly pseudocontractive map is $L-$ Lipschitzian and continuous. Indeed, from (3) we have

$$\lambda\|(x-y) - (Tx - Ty)\|^2 \leq \|(x-y) - (Tx - Ty)\|\|j(x-y)\|$$

on ther other hand

$$\lambda[\|Tx - Ty\| - \|x-y\|] \leq \lambda\|(x-y) - (Tx - Ty)\|$$

therefore, we get

$$\lambda[\|Tx - Ty\| - \|x-y\|] \leq \|x-y\|$$

i.e.,

$$\|Tx - Ty\| \leq L\|x-y\|, \quad L = \frac{1+\lambda}{\lambda}. \tag{4}$$

Since (4), if $\{T(t) : t \geq 0\}$ be a strictly pseudocontractive semigroup on C then for each $t > 0$, we get

$$\|T(t)x - T(t)y\| \leq L(t)\|x-y\|$$

for all $x, y \in C$. In the sequel, we denote $M := \sup_{t\geq 0}\{L(t)\} < \infty$.

Lemma 1.1. (Deimling [3]). *Let E be a real Banach space, C be a nonempty closed convex subset of E and $T : C \to C$ be a continuous strongly pseudocontractive mapping. Then T has a unique fixed point in C.*

Let E be a real Banach space, C be a nonempty closed convex subset of E and $\{T(t) : t \geq 0\}$ be a strictly pseudocontractive semigroup. For every $u \in C, t \in (0, \infty)$ and $s \in (0,1)$, we define a mapping $U_s : C \to C$ by

$$U_s = su + (1-s)T(t)x, \quad x \in C.$$

It is easy to see that U_s is a continuous strongly pseudocontractive mpping. By using Lemma 1.1, there exists a unique fixed point $x_s \in C$ of U_s such that

$$x_s = su + (1-s)T(t)x_s. \tag{5}$$

Let $\{T(t) : t \geq 0\}$ be a strictly pseudocontractive semigroup, let $\{\alpha_n\}$ be a real sequence in $(0,1)$, and $\{t_n\}$ be a real sequence in $(0, +\infty)$. By virtue of 5, we can define an implicit iterative sequence $\{x_n\}$ by

$$\begin{cases} x_0 \in C, \\ x_n = \alpha_n x_{n-1} + (1-\alpha_n)T(t_n)x_n, \quad n \geq 1. \end{cases} \tag{6}$$

It should be pointed out that the following implicit iteration process:

$$\begin{cases} x_0 \in C, \\ x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n, \quad n \geq 1 \end{cases} \tag{7}$$

was firstly introduced by Xu and Ori [11] for a finite family of nonexpansive mappings $\{T_i\}_{i=1}^N$ in a Hilbert space framework, where $T_n = T_{n \bmod N}$. In 2004, Osilike [6] extended the above sequence (7) from the class of nonexpansive mappings to more general class of strictly pseudocontractive mappings. In 2006, Chen, el al. [2] extended the results of Osilike [6] to more general Banach spaces. In 2008, Zhou [14] further extended the results of Chen, el al. [2] from strictly pseudocontractive mapping to Lipschitzian pseudocontractions, and from $q-$ uniformly smooth Banach space to uniformly convex Banach spaces with a Frchet differentiable norm.

Recently, Zhang [12] extended and improve recent results of Zhou [14], Chen, el al. [2], Osilike [6], Xu and Ori [11]. He proved the following results.

Theorem 1.1. (Zhang [12]). *Let E be a reflexive Banach space satisfying the Opial condition. Let C be a nonempty closed convex subset of E and $\{T(t) : t \geq 0\}$ be a strictly pseudocontractive semigroup with a strictly pseudocontractive function $\lambda(t) : [0, \infty) \to \left(0, \frac{1}{2}\right)$ such that $F(t) \neq \emptyset$. Let $\{\alpha_n\}$ be a sequence in $(0, 1)$ and $\{t_n\}$ be a sequence in $(0, \infty)$ satisfying the following conditions:*

(a) $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \frac{\alpha_n}{t_n} = 0.$

(b) $\lim_{n\to\infty} \frac{\lambda(t_n)}{\alpha_n} = K$, *where K is a positive constant.*

Then the sequence $\{x_n\}$ defined by (6) converges weakly to a common fixed point of strictly pseudocontractive semigroup $\{T(t) : t \geq 0\}$.

Theorem 1.2. (Zhang [13]). *Let E be a reflexive Banach space satisfying the Opial condition. Let C be a nonempty closed convex subset of E and $\{T(t) : t \geq 0\}$ be a strictly pseudocontractive semigroup with a strictly pseudocontractive function $\lambda(t) : [0, \infty) \to \left(0, \frac{1}{2}\right)$ such that $F(t) \neq \emptyset$. Let $\{\alpha_n\}$ be a sequence in $(0, 1)$ and $\{t_n\}$ be a sequence in $(0, \infty)$ satisfying the following conditions:*

(a) $\limsup_{n\to\infty} \alpha_n < 1$;

(b) $\sup_{x\in D} \|T(s+t_n)x - T(t_n)x\| \to 0$, *for all $t \in \mathbb{R}_+$, where $D = \{x \in E : \|x\| \leq \gamma\}$ and* $\gamma = \sup_{n\geq 1} \|x_n\|$;

(c) $\lim_{n\to\infty} \frac{\lambda(t_n)}{\alpha_n} = K$, *where K is a positive constant.*

Then the sequence $\{x_n\}$ defined by (6) converges weakly to a common fixed point of strictly pseudocontractive semigroup $\{T(t) : t \geq 0\}$.

In this paper, motivated by the above results, we prove several another weak and strong convergence results for the iterative scheme (6) for a strictly pseudocontractive semigroup in a Banach space.

In the sequel, we will need the following definition and result.

Definition 1.3. *A Banach space E is said to satisfy Opial's condition if whenever $\{x_n\}$ is a sequence in E which converges weakly to x, as $n \to \infty$, then*

$$\limsup_{n\to\infty} ||x_n - x|| < \limsup_{n\to\infty} ||x_n - y||, \quad \forall y \in E, y \neq x. \tag{8}$$

It is well known that Hilbert space and $l^p(1 < p < \infty)$ space satisfy Opial's conditions.

Lemma 1.2. *If $J : E \to 2^{E^*}$ is a normalized duality mapping, then for all $x, y \in E$,*

$$\|x+y\|^2 \le \|x\|^2 + 2\langle y, j(x+y)\rangle, \quad \forall j(x+y) \in J(x+y).$$

Lemma 1.3. (Zhou [14]). *Let E be a real reflexive Banach space with the Opial condition. Let C be a nonempty closed convex subset of E and $T : C \to C$ be a continuous pseudocontractive mapping. Then T is demiclosed at zero, i.e., for any sequence $\{x_n\} \subset E$, if $x_n \rightharpoonup y$ and $\|(I-T)x_n\| \to 0$, then $(I-T)y = 0$.*

2. Main results

2.1. Weakly convergence theorems.

Theorem 2.1. *Let E be a reflexive Banach space satisfying the Opial condition. Let C be a nonempty closed convex subset of E and $\{T(t) : t \ge 0\}$ be a strictly pseudocontractive semigroup with a strictly pseudocontractive function $\lambda(t) : [0,\infty) \to \left(0, \frac{1}{2}\right)$ and suppose that $F := \bigcap_{t\ge 0} Fix(T(t)) \ne \emptyset$. Let $\{\alpha_n\}$ and $\{t_n\}$ be sequences of real numbers satisfying $\{\alpha_n\} \subset (0,1), t_n > 0$, $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \frac{\alpha_n}{t_n} = 0$. Suppose that for any bounded subset $D \subset C$,*

$$\lim_{s\to 0} \sup_{x\in D} \|T(s)x - x\| = 0. \tag{9}$$

Then the sequence $\{x_n\}$ defined by (6) converges weakly to the element of F.

Proof. Claim 1. For each $p \in F$ then the limit $\lim_{n\to\infty} \|x_n - p\|$ exists.

$$\begin{aligned}\|x_n - p\|^2 &= \langle \alpha_n x_{n-1} + (1-\alpha_n)T(t_n)x_n - p, j(x_n - p)\rangle \\ &= (1-\alpha_n)\langle T(t_n)x_n - p, j(x_n - p)\rangle + \alpha_n\langle x_{n-1} - p, j(x_n - p)\rangle \\ &\le (1-\alpha_n)\|x_n - p\|^2 + \alpha_n\|x_{n-1} - p\|.\|x_n - p\|.\end{aligned}$$

So

$$\|x_n - p\|^2 \le \|x_{n-1}\|.\|x_n - p\|. \tag{10}$$

If $\|x_n - p\| = 0$, the result is apparent. Next let $\|x_n - p\| > 0$; it follows from (10) that

$$\|x_n - p\| \le \|x_{n-1} - p\|$$

which implies that the limit $\lim_{n\to\infty} \|x_n - p\|$ exists, and so the sequence $\{x_n\}$ is bounded. This implies that $\{T(t_n)x_n\}$ is bounded.

Claim 2. For each $t > 0$,

$$\lim_{n\to\infty} \|T(t)x_n - x_n\| = 0. \tag{11}$$

In fact, we have

$$\begin{aligned}
&\|x_n - T(t)x_n\| \\
&\le \sum_{k=0}^{[\frac{t}{t_n}]-1} \|T((k+1)t_n)x_n - T(kt_n)x_n\| + \left\|T\left(\left[\frac{t}{t_n}\right]t_n\right)x_n - T(t)x_n\right\| \\
&\le \left[\left[\frac{t}{t_n}\right]||T(t_n)x_n - x_n|| + ||T\left(t - \left[\frac{t}{t_n}\right]t_n\right)x_n - x_n||\right]M \\
&\le \left[t\frac{\alpha_n}{t_n}\|x_{n-1} - T(t_n)x_n\| + \max\{||T(s)x_n - x_n|| : 0 \le s \le t_n\}\right]M,
\end{aligned}$$

for all $n \in \mathbb{N}$. From the condition $\lim\limits_{n\to\infty} \dfrac{\alpha_n}{t_n} = 0$ and (20), we get

$$\lim_{n\to\infty} \|T(t)x_n - x_n\| = 0.$$

Claim 3. $\{x_n\}$ converges weakly to a common fixed point of semigroup $\{T(t) : t \ge 0\}$.

Indeed, since E is reflexive and C is closed and convex and $\{x_n\}$ is bounded, there exists a subsequence $\{x_{n_j}\} \subset \{x_n\}$ such that $x_{n_j} \rightharpoonup x \in C$. We prove that $x \in F$. From (11), for any $t > 0$ we have

$$\|T(t)x_{n_j} - x_{n_j}\| \to 0 \text{ as } j \to \infty.$$

By virtue of Lemma 1.3, $T(t)x = x$. Therefore $x \in F(T)$. We next prove $\{x_n\}$ converges weakly to x. Suppose that there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $x_{n_i} \rightharpoonup q$ and $q \ne x$. By the same method described above we can also prove that $q \in F$. Further, both limits

$$\lim_{n\to\infty} \|x_n - x\|, \quad \lim_{n\to\infty} \|x_n - q\|$$

exists. We have

$$\begin{aligned}
\lim_{n\to\infty} \|x_n - x\| &= \limsup_{j\to\infty} \|x_{n_j} - x\| < \limsup_{j\to\infty} \|x_{n_j} - q\| \\
&= \lim_{n\to\infty} \|x_n - q\| = \limsup_{i\to\infty} \|x_{n_i} - q\| \\
&< \limsup_{i\to\infty} \|x_{n_i} - x\| = \lim_{n\to\infty} \|x_n - x\|.
\end{aligned}$$

This contradiction shows that $q = x$, hence $x_n \rightharpoonup x$. Theorem 2.1 is proved. □

Theorem 2.2. *Let E be a reflexive Banach space satisfying the Opial condition. Let C be a nonempty closed convex subset of E and $\{T(t) : t \ge 0\}$ be a strictly pseudocontractive semigroup with a strictly pseudocontractive function $\lambda(t) : [0,\infty) \to \left(0, \frac{1}{2}\right)$ and suppose that $F := \bigcap_{t\ge0} Fix(T(t)) \ne \emptyset$. Let $\{\alpha_n\}$ and $\{t_n\}$ be sequences of real numbers satisfying $\{\alpha_n\} \subset (0, b] \subset (0, 1), t_n > 0, \liminf_{n\to\infty} t_n = 0, \limsup_{n\to\infty} t_n > 0$, and $\lim_{n\to\infty}(t_{n+1} - t_n) = 0$. Suppose that for any bounded subset $D \subset C$,*

$$\lim_{s\to0} \sup_{x\in D} \|T(s)x - x\| = 0. \tag{12}$$

Then the sequence $\{x_n\}$ defined by (6) converges weakly to the element of F.

Proof. It can be proved as in Theorem 2.1, that for each $p \in F$ then the limit $\lim\limits_{n\to\infty} \|x_n - p\|$ exists. Next, we show that

$$\lim_{n\to\infty} \|x_n - T(t_n)x_n\| = 0. \tag{13}$$

Indeed, From (3) we have for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ such that

$$\begin{aligned}\langle (I-T(t_n))x-(I-T(t_n))y\rangle &\geq \lambda(t_n)\|(I-T(t_n))x-(I-T(t_n))y\|^2\\ &\geq \lambda\|(I-T(t_n))x-(I-T(t_n))y\|^2. \end{aligned} \tag{14}$$

On the other hand, From equation (6) we have

$$x_{n-1} = \frac{1}{\alpha_n}x_n + \left(1-\frac{1}{\alpha_n}\right)T(t_n)x_n. \tag{15}$$

It follows from (15) that

$$x_n - x_{n-1} = \left(1-\frac{1}{\alpha_n}\right)(x_n - T(t_n)x_n)$$

$$\begin{aligned}\langle x_n - x_{n-1}, j(x_n-p)\rangle &= \left(1-\frac{1}{\alpha_n}\right)\langle x_n - T(t_n)x_n, j(x_n-p)\rangle\\ &= -\frac{1-\alpha_n}{\alpha_n}\langle x_n - T(t_n)x_n, j(x_n-p)\rangle. \end{aligned} \tag{16}$$

From Lemma 1.2 and (14) and (16), we have for all $p \in F(T)$ there exists $j(x_n-p) \in J(x_n-p)$ such that

$$\begin{aligned}\|x_n-p\|^2 &= \|x_{n-1}-p+x_n-x_{n-1}\|^2\\ &\leq \|x_{n-1}-p\|^2 + 2\langle x_n-x_{n-1}, j(x_n-p)\rangle\\ &= \|x_{n-1}-p\|^2 - 2\frac{1-\alpha_n}{\alpha_n}\langle x_n - T(t_n)x_n-(p-Tp), j(x_n-p)\rangle\\ &\leq \|x_{n-1}-p\|^2 - 2\lambda\frac{1-\alpha_n}{\alpha_n}\|x_n-T(t_n)x_n\|^2. \end{aligned} \tag{17}$$

From $0<\alpha_n\leq b<1$ and (17) we obtain

$$2\lambda\frac{1-b}{b}\|x_n-T(t_n)x_n\|^2 \leq \|x_{n-1}-p\|^2 - \|x_n-p\|^2. \tag{18}$$

Passing to the upper limit on both sides of inequality (18), we get

$$2\lambda\frac{1-b}{b}\limsup_{n\to\infty}\|x_n-T(t_n)x_n\|^2 = 0.$$

Thus,

$$\lim_{n\to\infty}\|x_n-T(t_n)x_n\| = 0.$$

Now we shall show that $\{x_n\}$ converges weakly to a common fixed point of semigroup $\{T(t): t\geq 0\}$. Indeed, since $\{x_n\}$ is bounded, we assume that a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ converges weakly to $x\in C$. Put $u_j := x_{n_j}, \beta_j := \alpha_{n_j}$ and $s_j := t_{n_j}$ for $j\in\mathbb{N}$. Without loss of generality, as in [7], we let

$$\lim_{j\to\infty} s_j = \lim_{j\to\infty}\frac{\|u_j - T(s_j)u_j\|}{s_j} = 0. \tag{19}$$

Now, we prove that $x = T(t)x$ for a fixed $t > 0$. Indeed,

$$\begin{aligned}\|u_j - T(t)u_j\| &\le \sum_{k=0}^{\left[\frac{t}{s_j}\right]-1} \|T((k+1)s_j)u_j - T(ks_j)u_j\| \\ &+\left\|T\left(\left[\frac{t}{s_j}\right]s_j\right)x - T(t)u_j\right\| \\ &\le \left[\frac{t}{s_j}\right] M\|T(s_j)u_j - u_j\| \\ &+ M\left\|T\left(t-\left[\frac{t}{s_j}\right]s_j\right)u_j - u_j\right\| \\ &\le Mt\frac{\|T(s_j)u_j - u_j\|}{s_j} + M\max_{0\le s\le s_j}\{\|T(s)u_j - u_j\|\}\end{aligned}$$

for all $j \in \mathbb{N}$. From (19) and (12), we get

$$\lim_{j\to\infty} \|u_j - T(t)u_j\| = 0.$$

By virtue of Lemma 1.3, $T(t)x = x$. Therefore $x \in F$. We next prove $\{x_n\}$ converges weakly to x. Suppose that there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $x_{n_i} \rightharpoonup q$ and $q \neq x$. By the same method argument as given in the proof of Theorem 2.1 we can show that $\{x_n\}$ converges weakly to x. Theorem 2.2 is complete. □

2.2. Song convergence theorems.

Theorem 2.3. *Let E be a real Banach space. Let C be a nonempty compact convex subset of E. Let $\{T(t) : t \ge 0\}$ be a strictly pseudocontractive semigroup on C and suppose that $F := \bigcap_{t\ge 0} Fix(T(t)) \neq \emptyset$. Let $\{\alpha_n\}$ and $\{t_n\}$ be sequences of real numbers satisfying $\{\alpha_n\} \subset (0, b] \subset (0, 1), t_n > 0$ and $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \frac{\alpha_n}{t_n} = 0$.Suppose that for any bounded subset $D \subset C$,*

$$\lim_{s\to 0} \sup_{x\in D} \|T(s)x - x\| = 0. \tag{20}$$

Then the sequence $\{x_n\}$ defined by (6) converges strongly to the element of F.

Proof. Since C is a compact convex subset of E and $\{x_n\}$ is bounded by Theorem 2.1 (Claim 1), then there exists a subsequence $\{x_{n_j}\} \subset \{x_n\}$ such that $x_{n_j} \to x \in C$.

Fix $t > 0$, by the continuity of the mapping $T(t)$ and the norm $\|.\|$, together with $\lim_{j\to\infty} \|x_{n_j} - T(t)x_{n_j}\| = 0$ by Theorem 2.2, we have

$$\|x - T(t)x\| = \lim_{j\to\infty} \|x_{n_j} - T(t)x_{n_j}\| = 0.$$

Therefore $x \in Fix(T(t))$, hence $x \in F$.

Because $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F$, thus we obtain that

$$\lim_{n\to\infty} \|x_n - x\| = \lim_{j\to\infty} \|x_{n_j} - x\| = 0.$$

The proof is complete. □

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