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Analytical approximate solutions of systems of differential-algebraic equations by Laplace homotopy analysis method

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Abstract. This paper presents a numerical technique for solving system of differential-algebraic equations (DAEs) by employing the Laplace homotopy analysis method (LHAM). The biggest advantage over the existing standard analytical techniques is that it overcomes the difficulty arising in calculating complicated terms. Numerical examples are examined to highlight the significant features of this method. Moreover, the solution procedure is easier, more effective and straightforward.

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Introduction

Differential–algebraic equations are normally obtained when modeling chemical engineering systems. Chemical processes are modeled dynamically using differential–algebraic equations. Chemical processes are inherently nonlinear and multivariable and are typically modeled by coupled differential and algebraic equation. A system of DAEs is characterized by its index, which is the number of differentiations required to convert it into a system of ODEs. DAEs with index > 1 are generally hard to solve and are still under active research.

In the past decades, both mathematicians and physicists have devoted considerable effort to the study of explicit and numerical solutions to DAEs. Many powerful methods have been presented [1-16, 19].

The subject of DAEs has researched and solidified only very recently (in the past 35 years). Through many exact solutions for linear DAEs has been found, in general, there exists no method that yields an exact solution for nonlinear DAEs.

The objective of the present paper is to modify the LHAM to provide symbolic approximate solutions for linear and nonlinear differential–algebraic equations. The LHAM is a combination of HAM [17,18] and Laplace transforms.

Therefore, in this work we will introduce a new alternative procedure for solving DAEs. The newly developed technique by no means depends on complicated tools from any field. This can be the most important advantage over the other methods. It is worth mentioning that the proposed algorithm is an elegant combination of Laplace transform method and the homotopy analysis method. Some DAE are examined to illustrate the effectiveness, accuracy and convenience of this method, and in all cases, the presented technique performed excellently.

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1. Laplace Homotopy analysis method

In this section, we employ the Laplace homotopy analysis method to the discussed problem. To show the basic idea, let us consider the DAEs

$$\begin{aligned} u_i'(t) &= f_i(t, u_1, u_2, \dots, u_n, u_1', u_2', \dots, u_n'),\ i = 1, 2, \dots, n-1, \\ 0 &= g(t, u_1, u_2, \dots, u_n), \end{aligned} \tag{1}$$

subject to the initial conditions

$$u_i(0) = a_i,\ i = 1, 2, \dots, n$$

where f_i are known analytical functions.

Applying the Laplace transform to both sides of system (1) and using linearity of Laplace transforms we get

$$\begin{aligned} \pounds[u_i'(t)] &= \pounds[f_i(t, u_1, u_2, \dots, u_n, u_1', u_2', \dots, u_n')],\ i = 1, 2, \dots, n-1, \\ 0 &= g(t, u_1, u_2, \dots, u_n), \end{aligned}$$

we get

$$\begin{aligned} U_i(s) &= \frac{a_i}{s} + \frac{1}{s}\pounds[f_i(t, u_1, u_2, \dots, u_n, u_1', u_2', \dots, u_n')],\ i = 1, 2, \dots, n-1, \\ 0 &= g(t, u_1, u_2, \dots, u_n), \end{aligned} \tag{2}$$

where $U_i(s) = \pounds(u_i(t))$.

The so-called zeroth-order deformation equations of the Laplace Equations(2) are

$$\begin{aligned} (1-q)[\Phi_i(s,q) - U_{i,0}(s)] &= qh_i[\Phi_i(s,q) - \frac{a_i}{s} \\ &\quad - \frac{1}{s}\pounds[f_i(t, \phi_1(t;q), \dots, \phi_n(t;q), \frac{\partial}{\partial t}\phi_1(t;q), \dots, \frac{\partial}{\partial t}\phi_n(t;q))]], \\ (1-q)[\phi_n(t;q) - u_{n,0}(t)] &= -qh_n g(t, \phi_1(t;q), \dots, \phi_n(t;q)),\ i = 1, 2, \dots, n-1, \end{aligned} \tag{3}$$

where $q \in [0,1]$ is an embedding parameter, when $q = 0$ and $q = 1$, we have

$$\begin{aligned} &\Phi_i(s,0) = U_{i,0}(s), \Phi_i(s,1) = U_i(s),\ i = 1, 2, \dots, n-1, \\ &\phi_n(t;0) = u_{n,0}(t), \phi_n(t;0) = u_n(t). \end{aligned}$$

Expanding $\Phi_i(s,q), i = 1, 2, \dots, n-1$ and $\phi_n(t;q)$ in Taylor series with respect to q we get

$$\begin{aligned} \Phi_i(s;q) &= U_{i,0}(s) + \sum_{m=1}^{\infty} U_{i,m}(s)q^m,\ i = 1, 2, \dots, n-1, \\ \phi_n(t;q) &= u_{n,0}(t) + \sum_{m=1}^{\infty} u_{n,m}(t)q^m, \end{aligned} \tag{4}$$

where

$$\begin{aligned} U_{i,m}(s) &= \frac{1}{m!}\frac{\partial^m \Phi_i(s;q)}{\partial q^m}|_{q=0},\ i = 1, 2, \dots, n-1, \\ u_{n,m}(t) &= \frac{1}{m!}\frac{\partial^m \phi_n(t;q)}{\partial q^m}|_{q=0}. \end{aligned}$$

If the initial guesses and the nonzero auxiliary parameters h_i are properly chosen so that the power series (4) converges at $q = 1$, then we have, under these assumptions the solution series

$$U_i(s) = \Phi_i(s;1) = U_{i,0}(s) + \sum_{m=1}^{\infty} U_{i,m}(s),\ i = 1,2,\ldots,n-1,$$
$$u_n(t) = \phi_n(t;1) = u_{i,0}(t) + \sum_{m=1}^{\infty} u_{i,m}(t)$$

For brevity, define the vectors

$$\overrightarrow{U}_{i,m}(s) = \{U_{i,0}(s), U_{i,1}(s), U_{i,2}(s), \ldots, U_{i,m}(s)\},\ i = 1,2,\ldots,n-1,$$
$$\overrightarrow{u}_{n,m}(s) = \{u_{n,0}(s), u_{n,1}(s), u_{m,2}(s), \ldots, u_{n,m}(s)\},$$

Differentiating the zero-order deformation equation (3) m times with respective to q, dividing by $m!$ and finally setting $q = 0$, we have the so-called high-order deformation equation

$$U_{i,m}(s) = \chi_m U_{i,m-1}(s) + h_i \Re_{i,m}(\overrightarrow{U}_{i,m-1}(s)),\ i = 1,2,\ldots,n-1, \tag{5}$$
$$u_{n,m}(t) = \chi_m u_{n,m-1}(t) + h_n \Re_{n,m}(\overrightarrow{u}_{n,m-1}(t))$$

where

$$\Re_{i,m}(\overrightarrow{U}_{i,m-1}(s)) = U_{i,m-1}(s) - \frac{1}{s}\left[\frac{1}{(m-1)!}\frac{\partial^{m-1}}{\partial q^{m-1}}(\pounds[f_i(t,\phi_1(t;q),\ldots,\phi_n(t;q)),\right.$$
$$\left.\frac{\partial}{\partial t}\phi_1(t;q),\ldots,\frac{\partial}{\partial t}\phi_n(t;q))])|_{q=0}\right] - \frac{a_i}{s}(1-\chi_m),\ i = 1,2,\ldots,n-1,$$
$$\Re_{n,m}(\overrightarrow{u}_{n,m-1}(t)) = \frac{-1}{(m-1)!}\frac{\partial^{m-1}}{\partial q^{m-1}}[g(t,\phi_1(t;q),\ldots,\phi_n(t;q))]|_{q=0},$$

and

$$\chi_m = \begin{cases} 0, & m \le 1 \\ 1, & m > 1 \end{cases}.$$

Finally, applying the inverse Laplace transforms of (5), then we have a power series solution

$$u_i(t) = \sum_{m=0}^{\infty} u_{i,m}(t),\ i = 1,2,\ldots,n \tag{6}$$

Note that we have great freedom to choose the value of the auxiliary parameters h_i. Mathematically the value of $u_i(t)$ at any finite order of approximation depends upon the auxiliary parameter h_i, because the zeroth and high order deformation equations contain h_i. Let R_{h_i} denote the set of all values of h_i which ensure the convergence of the HAM series solution (6) of $u_i(t)$. Let h_i be the variable of the horizontal axis and the limit of the series solution (6) of $u_i(t)$ be the variable of vertical axis. Plot the curve $u_i(t) \sim h_i$, where $u_i(t)$ denotes the limit of the series (6). Because the limit of all convergent series solutions (6) is the same for a given a, there exists a horizontal line segment above the region $h \in R_{h_i}$. So, by plotting the curve $u_i(t) \sim h_i$ at a high enough order approximation, one can find an approximation of the set R_{h_i}.

2. Applications

In this part, we introduce some applications on LHAM to solve differential-algebraic equations with fractional derivatives.

Example 1. Consider the following system of linear DAEs :

$$\begin{aligned} u_1'' - (t+1)\, u_2' - tu_1 &= 0, \\ u_2' + (t-1)\, v + u_1 &= (t-1)\exp(t), \\ v - tu_1 &= \exp(t) - t\sin(t)\,, y_1(0) = 0, y_1'(0) = 1, y_2(0) = 2, y_3(0) = 1, \end{aligned} \tag{7}$$

the exact solution is $u_1(t) = \sin(t)\,, u_2(t) = \cos(t) + 1, u_3(t) = \exp(t)\,.$

To derive the solution, take the Laplace transform of both sides of (7) we get

$$\begin{aligned} s^2 U_1(s) - 1 - \pounds\left(tu_2'(t)\right) - U_2(s) + 2 - \pounds\left(tu_1(t)\right) &= 0, \\ sU_2(s) - 2 + \pounds\left(tv(t)\right) - V(s) + U_1(s) &= \pounds((t-1)\exp(t)), \\ v(t) - ty_1 &= \exp(t) - t\sin(t)\,, \end{aligned}$$

or

$$\begin{aligned} U_1(s) &= \frac{1}{s^2}\pounds\left(tu_2'(t)\right) + \frac{1}{s^2}U_2(s) - \frac{1}{s^2} + \frac{1}{s^2}\pounds\left(tu_1(t)\right), \\ U_2(s) &= \frac{2}{s} - \frac{1}{s}\pounds\left(tv(t)\right) + \frac{1}{s}V(s) - \frac{1}{s}U_1(s) + \pounds((t-1)\exp(t)), \\ v(t) &= tu_1 + \exp(t) - t\sin(t)\,, \end{aligned}$$

In this line the $mth-$order deformation equation has the form

$$\begin{aligned} U_{1,m}(s) - \chi_m U_{1,m-1}(s) &= h_1 \Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)), \\ U_{2,m}(s) - \chi_m U_{2,m-1}(s) &= h_2 \Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)), \\ v_m(t) - \chi_m v_{m-1}(t) &= h_3 \Re_{3,m}(\overrightarrow{v}_{m-1}(t)),\ m = 1, 2, 3, \ldots \end{aligned}$$

or

$$\begin{aligned} U_{1,m}(s) &= \chi_m U_{1,m-1}(s) + h_1 \Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)), \\ U_{2,m}(s) &= \chi_m U_{2,m-1}(s) + h_2 \Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)), \\ v_m(t) &= \chi_m v_{m-1}(t) + h_3 \Re_{3,m}(\overrightarrow{v}_{m-1}(t)),\ m = 1, 2, 3, \ldots \end{aligned} \tag{8}$$

where

$$\begin{aligned} \Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)) &= U_1(s) - \frac{1}{s^2}\pounds\left(tu_{2,m-1}'(t)\right) - \frac{1}{s^2}U_{2,m-1}(s) \\ &\quad - \frac{1}{s^2}\pounds\left(tu_{1,m-1}(t)\right) - \frac{1}{s^2}\left(1 - \chi_m\right), \\ \Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)) &= U_{2,m-1}(s) + \frac{1}{s}\pounds\left(tv_{m-1}(t)\right) - \frac{1}{s}V_{m-1}(s) + \frac{1}{s}U_{1,m-1}(s) \\ &\quad - \left(\frac{2}{s} + \pounds((t-1)\exp(t))\right)\left(1 - \chi_m\right), \\ \Re_{3,m}(\overrightarrow{v}_{m-1}(t)) &= v_{m-1}(t) - tu_{1,m-1} - \left(\exp(t) - t\sin(t)\right)\left(1 - \chi_m\right), \end{aligned}$$

According to the initial condition in (8), we can choose the initial guess of $U_i(s)$ and $v(t)$ as follows:

$$U_{1,0}(s) = \frac{1}{s^2},\ U_{2,0}(s) = \frac{2}{s},\ v_0(t) = 1.$$

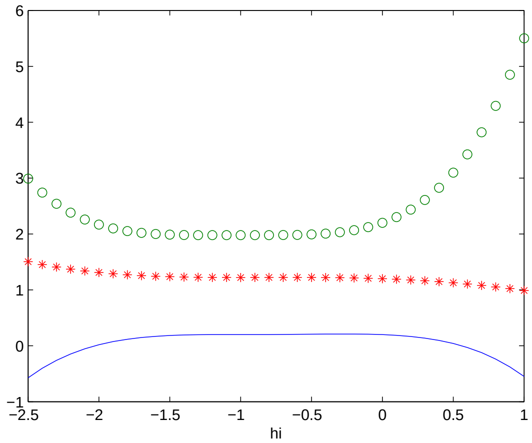

FIGURE 1. The h_i -curve of 5th-order approximation for $u_i(0.2)$ and $v(0.2)$.

Then the solution is

$$u_i(t) = \sum_{m=0}^{\infty} u_{i,m}(t), \;\; i = 1, 2,$$
$$v\,(t) = \sum_{m=0}^{\infty} v_m\,(t)\,.$$

The proper values of h_1, h_2, h_3 found from the h_i-curve shown in Figure 1, it is clear that the series of $u_i(t), v(t)$ convergent when $-1.6 \leq h_i \leq -0.3 \quad i = 1, 2, 3$. Using $h_1 = h_2 = h_3 = -1$ in (8) we find that

$$u_1\,(t) = \sum_{m=0}^{\infty} u_{1,m}\,(t) = t - \frac{t^3}{6} + \frac{t^5}{120} - \frac{t^7}{5040} + \frac{t^9}{362880} - \cdots$$
$$u_2\,(t) = \sum_{m=0}^{\infty} u_{2,m}\,(t) = 1 + 1 - \frac{t^2}{2} + \frac{t^4}{24} - \frac{t^6}{720} + \frac{t^8}{40320} - \cdots$$
$$v\,(t) = \sum_{m=0}^{\infty} v_m\,(t) = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{24} + \frac{t^5}{120} + \cdots$$

The obtained series solutions are the Taylor series expansion of the exact solutions $u_1\,(t) = \sin\,(t)\,, u_2\,(t) = \cos\,(t) + 1, v\,(t) = \exp\,(t)\,.$

Example 2. Consider the system of nonlinear DAEs of three variables:

$$\begin{aligned} &u_1' - u_2 + t^2 v = t^3, \\ &u_2' - (t+1)\,u_1 = -t\sinh\,(t)\,, \\ &u_2 v - t\cosh\,(t) = 0,\;\; u_1\,(0) = 0,\;\; u_2\,(0) = 1,\;\; v\,(0) = 0, \end{aligned} \tag{9}$$

the exact solution is given as $u_1\,(t) = \sinh\,(t)\,,\;\; u_2\,(t) = \cosh\,(t)\,,\;\; v\,(t) = t.$

To derive the solution, take the Laplace transform of both sides of (9) we get

$$
\begin{aligned}
sU_1(s) - U_2(s) + \pounds(t^2 v(t)) &= \pounds(t^3),\\
sU_2(s) - 1 - \pounds((t+1)u_1(t)) &= \pounds(-t\sinh(t)),\\
\pounds(u_2(t)v(t)) - \pounds(t\cosh(t)) &= 0,
\end{aligned}
$$

or

$$
\begin{aligned}
U_1(s) &= \frac{1}{s}U_2(s) - \frac{1}{s}\pounds(t^2 v(t)) + \pounds(t^3),\\
U_2(s) &= \frac{1}{s}\pounds((t+1)u_1(t)) + \frac{1}{s}\pounds(-t\sinh(t)) + \frac{1}{s},\\
\pounds(u_2(t)v(t)) - \pounds(t\cosh(t)) &= 0.
\end{aligned}
$$

The LHAM has the form

$$
\begin{aligned}
U_{1,m}(s) - \chi_m U_{1,m-1}(s) &= h_1 \Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)),\\
U_{2,m}(s) - \chi_m U_{2,m-1}(s) &= h_2 \Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)),\\
v_m(t) - \chi_m v_{m-1}(t) &= h_3 \Re_{3,m}(\overrightarrow{v}_{m-1}(t)),\ m = 1,2,3,\ldots
\end{aligned}
$$

where

$$
\begin{aligned}
\Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)) &= U_{1,m-1}(s) - \frac{1}{s}U_{2,m-1}(s) + \frac{1}{s}\pounds(t^2 v_{m-1}(t)) - \pounds(t^3)(1-\chi_m),\\
\Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)) &= U_{2,m-1}(s) - \frac{1}{s}\pounds((t+1)u_{1,m-1}(t)) - \frac{1}{s}(\pounds(-t\sinh(t)) + 1)(1-\chi_m),\\
\Re_{3,m}(\overrightarrow{v}_{m-1}(t)) &= \pounds(\sum_{i=0}^{m-1} u_{2,i}(t)v_{m-i-1}(t)) - \pounds(t\cosh(t))(1-\chi_m),\ m = 1,2,3,\ldots
\end{aligned}
$$

According to the initial condition in (9), we can choose the initial guess of $U(s)$ and $v(t)$ as follows:

$$
U_{1,0}(s) = 0,\ U_{2,0}(s) = \frac{1}{s},\ v_0(t) = 0.
$$

Hence, the mth-order deformation equations can be given by

$$
\begin{aligned}
U_{i,m}(s) &= \chi_m U_{i,m-1}(s) + h_i \Re_{i,m}(\overrightarrow{U}_{m-1}(s)),\ i = 1,2,\\
v_m(t) &= \chi_m v_{m-1}(t) + h_3 \Re_{3,m}(\overrightarrow{v}_{m-1}(t)),\ m = 1,2,3,\ldots
\end{aligned}
\tag{10}
$$

subject to the initial condition

$$
u_{i,m}(0) = v_m(0) = 0,\ i = 1,2.
$$

If $h_1 = h_2 = h_3 = -1$ in (10), then we obtain the following series solution

$$
\begin{aligned}
u_1(t) &= u_{1,0}(t) + \sum_{m=1}^{\infty} u_{1,m}(t) = t + \frac{t^3}{6} + \frac{t^5}{120} + \frac{t^7}{5040} + \frac{t^9}{362880} + \cdots\\
u_2(t) &= u_{2,0}(t) + \sum_{m=1}^{\infty} u_m(t) = 1 + \frac{t^2}{2} + \frac{t^4}{24} + \frac{t^6}{720} + \frac{t^8}{40320} + \frac{t^{10}}{3628800} + \cdots\\
v(t) &= v_0(t) + \sum_{m=1}^{\infty} v_m(t) = t + 0 + 0 + \cdots
\end{aligned}
$$

The obtained series solutions are the Taylor series expansion of the exact solutions $u_1(t) = \sinh(t),\ u_2(t) = \cosh(t),\ v(t) = t.$

Example 3. Consider the following system of differential-algebraic equations

$$\begin{aligned} u_1'(t) &= u_1(t) - u_2(t)v(t) + \sin(t) + t\cos(t), \\ u_2'(t) &= tv(t) + u_1^2(t) + \sec^2(t) - t^2(\sin^2(t) + \cos(t)) \\ v(t) &= u_1(t) + t(\cos(t) - \sin(t)), u_1(0) = u_1(0) = v(0) = 0, \end{aligned} \tag{11}$$

the exact solution $u_1(t) = t\sin(t), u_2(t) = \tan(t), v(t) = t\cos(t)$.

To derive the solution, take the Laplace transform of both sides of (11) we get

$$\begin{aligned} sU_1(s) &= U_1(s) - \pounds(u_2(t)v(t)) + \pounds(\sin(t) + t\cos(t)), \\ sU_2(s) &= \pounds(tv(t)) + \pounds(u_1^2(t)) + \pounds(\sec^2(t) - t^2(\sin^2(t) + \cos(t))), \\ v(t) &= u_1(t) + t(\cos(t) - \sin(t)), \end{aligned}$$

or

$$\begin{aligned} U_1(s) &= \frac{1}{s}U_1(s) - \frac{1}{s}\pounds(u_2(t)v(t)) + \frac{1}{s}\pounds(\sin(t) + t\cos(t)), \\ U_2(s) &= \frac{1}{s}\pounds(tv(t)) + \frac{1}{s}\pounds(u_1^2(t)) + \frac{1}{s}\pounds(\sec^2(t) - t^2(\sin^2(t) + \cos(t))), \\ v(t) &= u_1(t) + t(\cos(t) - \sin(t)). \end{aligned}$$

The LHAM has the form

$$\begin{aligned} U_{1,m}(s) - \chi_m U_{1,m-1}(s) &= h_1 \Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)), \\ U_{2,m}(s) - \chi_m U_{2,m-1}(s) &= h_2 \Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)), \\ v_m(t) - \chi_m v_{m-1}(t) &= h_3 \Re_{3,m}(\overrightarrow{v}_{m-1}(t)), m = 1, 2, 3, \ldots \end{aligned}$$

where

$$\begin{aligned} \Re_{1,m}(\overrightarrow{U}_{1,m-1}(s)) &= U_{1,m-1}(s) - \frac{1}{s}U_{1,m-1}(s) + \frac{1}{s}\pounds(\sum_{i=0}^{m-1} u_{2,i}(t)v_{m-i-1}(t)) \\ &\quad - \frac{1}{s}\pounds(\sin(t) + t\cos(t))(1 - \chi_m), \\ \Re_{2,m}(\overrightarrow{U}_{2,m-1}(s)) &= U_{2,m-1}(s) - \frac{1}{s}\pounds(tv_{m-1}(t)) - \frac{1}{s}\pounds(\sum_{i=0}^{m-1} u_{1,i}(t)u_{1,m-i-1}(t)) \\ &\quad - \frac{1}{s}\pounds(\sec^2(t) - t^2(\sin^2(t) + \cos(t)))(1 - \chi_m), \\ \Re_{3,m}(\overrightarrow{v}_{m-1}(t)) &= v_{m-1}(t) - u_{1,m-1}(t) - (t(\cos(t) - \sin(t)))(1 - \chi_m), m = 1, 2, 3, \ldots \end{aligned}$$

According to the initial condition in (11), we can choose the initial guess of $U(s)$ and $v(t)$ as follows:

$$U_{1,0}(s) = 0, \ U_{2,0}(s) = 0, \ v_0(t) = 0.$$

Hence, the mth-order deformation equations can be given by

$$\begin{aligned} U_{i,m}(s) &= \chi_m U_{i,m-1}(s) + h_i \Re_{i,m}(\overrightarrow{U}_{i,m-1}(s)), i = 1, 2, \\ v_m(t) &= \chi_m v_{m-1}(t) + h_3 \Re_{3,m}(\overrightarrow{v}_{m-1}(t)), \ m = 1, 2, 3, \ldots \end{aligned} \tag{12}$$

subject to the initial condition

$$u_{i,m}(0) = v_m(0) = 0, \ i = 1, 2.$$

By plotting the h_i-curves at high enough order approximation, one can find the proper values h_1, h_2, h_3. It is clear that the series of $u_i(t), v(t)$ are convergent when

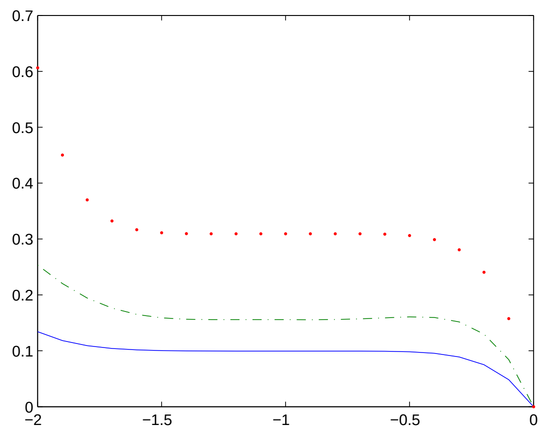

FIGURE 2. The h_i -curve of 8th-order approximation for $u_1(0.1), u_2(0.4)$ and $v(0.3)$ of example 3.

$-1.4 \leq h_i \leq -0.45$, $i = 1, 2, 3$; so, if we set $h_1 = h_2 = h_3 = -1$ in (12), then we obtain the following series solutions

$$u_1(t) = u_{1,0}(t) + \sum_{m=1}^{\infty} u_{1,m}(t) = t^2 - \frac{t^4}{6} + \frac{t^6}{120} - \frac{t^8}{5040} + \frac{t^{10}}{362880} + \cdots$$
$$u_2(t) = u_{2,0}(t) + \sum_{m=1}^{\infty} u_m(t) = t + \frac{t^3}{3} + \frac{2t^5}{15} + \frac{17t^7}{315} + \frac{62t^9}{2835} + \cdots$$
$$v(t) = v_0(t) + \sum_{m=1}^{\infty} v_m(t) = t - \frac{t^3}{2} + \frac{t^5}{24} - \frac{t^7}{720} + \frac{t^9}{40320} + \cdots$$

which are the same as the solutions given by F. Soltanian, S.M. Karbassi, M.M. Hosseini [19] using He's variational iteration method.

3. Conclusion

A combined form of the Laplace transform method with Homotopy analysis method is effectively used to handle linear and nonlinear for system of differential-algebraic equations. The main advantage of the method is its fast convergence to the solution. Moreover, it avoids the volume of calculations that required by other existing analytical methods. In practice, the utilization of the method is straightforward if some symbolic software as Matlab is used to implement the calculations. The new method leads to higher accuracy and simplicity, and in all cases the solutions obtained are easily programmable approximates to the analytic solutions of the original problems with the accuracy required. The proposed scheme can be applied for other nonlinear equations.

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