Annals of the University of Craiova, Mathematics and Computer Science Series
Volume 38(3), 2011, Pages 22–30
ISSN: 1223-6934, Online 2246-9958

Intuitionistic Fuzzy Sets for Optional Courses Selection

Ion Iancu and Daniela Dănciulescu

Abstract. Using some similarity measures of intuitionistic fuzzy sets based on L_p metric and on J-divergence, we propose an approach for selecting the optional courses in the master program depending on the optional courses chosen in the license program.

2010 Mathematics Subject Classification. Primary 94D05; Secondary 03B52, 93C42, 68T27, 68T37.
Key words and phrases. fuzzy set, intuitionistic fuzzy set, degree of similarity, divergence, metric.

1. Introduction

Atanassov ([1, 2]) introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set ([24]). Gau and Buehrer [7] introduced the concept of vague set, but Bustine and Burillo [5] showed that vague sets are intuitionistic fuzzy sets. Some multiplicative operational laws [22] and some additive operational laws [23] of intuitionistic fuzzy values were defined by Xu and Yager and by Xu, respectively; in the same time, based on these operational laws, some intuitionistic fuzzy aggregation operators were proposed [23].

Intuitionistic fuzzy systems can be useful in situations when the description of a problem by a linguistic variable given in terms of a membership function only, seems too rough. For example, in decision making problems, sales analysis, new product marketing, financial services, etc., there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. For a greater precision, intuitionistic fuzzy sets are indicated to be used.

In this paper we will present intuitionistic fuzzy sets as a tool for reasoning in the presence of imperfect facts and imprecise knowledge. Such methods with applications in medical diagnosis have been proposed in [6] and [19]. The method from [19] involves intuitionistic fuzzy distances introduced in [17], [18] while the method from [6] is based on the max-min-max composition rule. In [19] the advantages of this approach are pointed out in comparison with the method presented in [6]. Concerning the degree of similarity between IFSs proposed in [9], the authors demonstrated - by numerical experiments - that these measures are resonable in measuring the degree of similarity between IFSs and they give better results than the previous techniques.

Another possibility to compute the similarity between two intuitionistic fuzzy sets was proposed in [10] based on J-divergence. Divergence measures based on the idea of information-theoretic entropy were first introduced in communication theory by Shannon [16] and later by Wiener [21] in Cybernetics. The most popular divergence measure associated with the Shannon entropy function is the Kullback–Leibler divergence (K-L divergence) [11], perhaps because of their simplicity. To measure the

Received June 25, 2011. Revision received August 17, 2011.

uncertainty about the remaining lifetime, Nada and Paul [15] generalized Shannons entropy by using the concept of residual entropy and developed two kinds of generalized residual entropies. Based on Havrda and Charavats entropy function [8], a family of divergences between IFSs, called J_α-divergence, was proposed in [10]. The proposed J_α-divergence can induce some special distance and similarity measures between IFSs. The results of numerical examples from [10] indicated that the proposed measures are good in pattern recognition problems.

Our method is based on the degree of similarity between intuitionistic fuzzy sets, in which the similarity measures are introduced by L_p metric and by J_α.

2. Preliminaries

According to [1] an IFS is given by

Definition 2.1. *An IFS $\widetilde{A}$ in X is defined as*

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x))\}$$

where

$$\mu_{\widetilde{A}} : X \to [0,1], \nu_{\widetilde{A}} : X \to [0,1]$$

with the condition

$$0 \leq \mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \leq 1, \quad \forall x \in X.$$

The numbers $\mu_{\widetilde{A}}(x)$ and $\nu_{\widetilde{A}}(x)$ denote the degree of membership and non-membership of x to $\widetilde{A}$, respectively.

Obviously, a fuzzy set A corresponds to the following IFS $\widetilde{A} = \{(x, \mu_A(x), 1 - \mu_A(x)/x \in X)\}$. For each IFS $\widetilde{A}$ in X,

$$\pi_{\widetilde{A}}(x) = 1 - \mu_{\widetilde{A}}(x) - \nu_{\widetilde{A}}(x)$$

is called the intuitionistic index of x in $\widetilde{A}$; it is a hesitancy degree of x to $\widetilde{A}$ [1, 2, 3, 4] and satisfies the inequality $0 \leq \pi_{\widetilde{A}}(x) \leq 1 \quad \forall x \in X$. Therefore, if we want to describe an intuitionistic fuzzy set we must use any two functions from the triplet: (membership function, non-membership function, intuitionistic index). We denote $IFS(X)$ as the set of all IFSs in X.

Definition 2.2. *The complementary set of the IFS $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x))\}$ is the IFS $\widetilde{A}^c = \{(x, \nu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(x))\}$*

Definition 2.3. *An intuitionistic fuzzy relation $\Re$ between (not necessarily all distinct) universes $U_1, U_2, ..., U_n \quad (n \geq 1)$ is an IFS in the cartesian product $U_1 \times ... \times U_n$. Occasionally, we will use the abbreviation IFR.*

In the study of the similarity between IFSs, Li and Cheng [12] and Mitchell [14] introduced the following definition

Definition 2.4. *A mapping $S : IFS(X) \times IFS(X) \to [0,1]$ is said to be a degree of similarity if it satisfies the following properties, for all $\widetilde{A}, \widetilde{B}, \widetilde{C} \in IFS(X)$:*

(P1) $0 \leq S(\widetilde{A}, \widetilde{B} \leq 1$

(P2) $S(\widetilde{A}, \widetilde{B}) = 1$ *if and only if* $\widetilde{A} = \widetilde{B}$

(P3) $S(\widetilde{A}, \widetilde{B}) = S(\widetilde{B}, \widetilde{A})$

(P4) $S(\widetilde{A}, \widetilde{C}) \leq S(\widetilde{A}, \widetilde{B})$ *and* $S(\widetilde{A}, \widetilde{C}) \leq S(\widetilde{B}, \widetilde{C})$ *if* $\widetilde{A} \subseteq \widetilde{B} \subseteq \widetilde{C}$.

$S(\widetilde{A}, \widetilde{B})$ *is the degree of similarity between* $\widetilde{A}$ *and* $\widetilde{B}$.

Assume that there are two IFSs $\widetilde{A}$ and $\widetilde{B}$ in $X = \{x_1, ..., x_n\}$; the degree of similarity between the two IFSs, $\widetilde{A}$ and $\widetilde{B}$, can be calculated in various forms:

• Li and Cheng [12]:

$$S_d^p(\widetilde{A}, \widetilde{B}) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} |m_{\widetilde{A}}(i) - m_{\widetilde{B}}(i)|^p}$$

where

$$m_{\widetilde{A}}(i) = (1 + \mu_{\widetilde{A}}(x_i) - \nu_{\widetilde{A}}(x_i))/2,$$
$$m_{\widetilde{B}}(i) = (1 + \mu_{\widetilde{B}}(x_i) - \nu_{\widetilde{B}}(x_i))/2$$

and $1 \leq p < \infty$.

• Liang and Shi [13]:

$$S_e^p(\widetilde{A}, \widetilde{B}) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} (\phi_1(i) + \phi_2(i))^p}$$

where

$$\phi_1(i) = |\mu_{\widetilde{A}}(x_i) - \mu_{\widetilde{B}}(x_i)|/2$$

and

$$\phi_2(i) = |\nu_{\widetilde{B}}(x_i) - \nu_{\widetilde{A}}(x_i)|/2$$

• Mitchel [14]:

$$S_{mod}(\widetilde{A}, \widetilde{B}) = \frac{1}{2}(\rho_\mu(\widetilde{A}, \widetilde{B}) + \rho_\nu(\widetilde{A}, \widetilde{B}))$$

with

$$\rho_\mu(\widetilde{A}, \widetilde{B}) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} |\mu_{\widetilde{A}}(x_i) - \mu_{\widetilde{B}}(x_i)|^p}$$

and

$$\rho_\nu(\widetilde{A}, \widetilde{B}) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} |\nu_{\widetilde{A}}(x_i) - \nu_{\widetilde{B}}(x_i)|^p}$$

• Hung and Yang [9]:

$$SH_l(\widetilde{A}, \widetilde{B}) = 1 - d_H(\widetilde{A}, \widetilde{B}),$$

$$SH_e(\widetilde{A}, \widetilde{B}) = \frac{exp(-d_H(\widetilde{A}, \widetilde{B})) - exp(-1)}{1 - exp(-1)},$$

and

$$SH_c(\widetilde{A}, \widetilde{B}) = \frac{1 - d_H(\widetilde{A}, \widetilde{B})}{1 + d_H(\widetilde{A}, \widetilde{B})}$$

where

$$d_H(\widetilde{A}, \widetilde{B}) = \frac{1}{n} \sum_{i=1}^{n} max\{|\mu_{\widetilde{A}}(x_i) - \mu_{\widetilde{B}}(x_i)|, |\nu_{\widetilde{A}}(x_i) - \nu_{\widetilde{B}}(x_i)|\}$$

• Szmidt and Kacprzyk [18]

$$S_{sk}(\widetilde{A}, \widetilde{B}) = l_{IFS}(\widetilde{A}, \widetilde{B}^c) - l_{IFS}(\widetilde{A}, \widetilde{B})$$

where

$$l_{IFS}(\widetilde{A}, \widetilde{B}) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_{\widetilde{A}}(x_i) - \mu_{\widetilde{B}}(x_i)| + |\nu_{\widetilde{A}}(x_i) - \nu_{\widetilde{B}}(x_i)| + |\pi_{\widetilde{A}}(x_i) - \pi_{\widetilde{B}}(x_i)|)$$

As is proved in [9] the measure S_{sk} violates property P2 from Definition 2.4; for this we recommend to use the distance l_{IFS} instead of S_{sk}. Similarity measures based on L_p metric were proposed by Hung and Yang in [9]. The distance $L_p(\widetilde{A}, \widetilde{B})$ between $\widetilde{A}, \widetilde{B} \in IFS(X)$ is defined as follows

$$L_p(\widetilde{A}, \widetilde{B}) = \frac{1}{n} \sum_{i=1}^{n} d_p(I_{\widetilde{A}}(x_i), I_{\widetilde{B}}(x_i))$$

where

$$d_p(I_{\widetilde{A}}(x_i), I_{\widetilde{B}}(x_i)) = (|\mu_{\widetilde{A}}(x_i) - \mu_{\widetilde{B}}(x_i)|^p + |\nu_{\widetilde{A}}(x_i) - \nu_{\widetilde{B}}(x_i)|^p)^{1/p}, \quad p \geq 1.$$

For a monotone decreasing function f, Hung and Yang [9] defined the similarity measure between IFSs $\widetilde{A}$ and $\widetilde{B}$ as follows:

$$SL(\widetilde{A}, \widetilde{B}) = \frac{f(L_p(\widetilde{A}, \widetilde{B})) - f(2^{1/p})}{f(0) - f(2^{1/p})}.$$

For $f(x) \in \{1 - x, exp(-x), \dfrac{1}{1+x}\}$ the corresponding measures between $\widetilde{A}$ and $\widetilde{B}$ are given by [9]

$$SL_l^p(\widetilde{A}, \widetilde{B}) = \frac{2^{1/p} - L_p(\widetilde{A}, \widetilde{B})}{2^{1/p}},$$

$$SL_e^p(\widetilde{A}, \widetilde{B}) = \frac{exp(-L_p(\widetilde{A}, \widetilde{B})) - exp(-2^{1/p})}{1 - exp(-2^{1/p})}$$

and

$$SL_c^p(\widetilde{A}, \widetilde{B}) = \frac{2^{1/p} - L_p(\widetilde{A}, \widetilde{B})}{2^{1/p}(1 + L_p(\widetilde{A}, \widetilde{B}))}.$$

These three measures are verified in [9] on a set of 5 examples from Liang and Shi [13] and Wang and Xin [20] and the conclusion is:

a) in 4 cases the classification is the same as in [13] and [20]

b) in one case the measure S_s^p from [13] cannot classify an example while the measures of Hung and Yang [9] do that.

This is why we will use the measures from [9].

For two IFSs $\widetilde{A}$ and $\widetilde{B}$ in $X = \{x_1, x_2, ..., x_n\}$, the divergence between $\widetilde{A}$ and $\widetilde{B}$, say $J_\alpha(\widetilde{A}, \widetilde{B})$, can be defined as [10]:

$$J_\alpha(\widetilde{A}, \widetilde{B}) = \frac{1}{n} \sum_{i=1}^{n} J_\alpha(A_i, B_i)$$

where $A_i = \{(x_i, \mu_A(x_i), \nu_A(x_i))\}$, $B_i = \{(x_i, \mu_B(x_i), \nu_B(x_i))\}$

$$J_\alpha(A_i, B_i) = \begin{cases} \frac{1}{\alpha-1}((\frac{\mu_{A_i}+\mu_{B_i}}{2})^\alpha - \frac{1}{2}(\mu_{A_i}^\alpha + \mu_{B_i}^\alpha) + \\ (\frac{\nu_{A_i}+\nu_{B_i}}{2})^\alpha - \frac{1}{2}(\nu_{A_i}^\alpha + \nu_{B_i}^\alpha) + \\ (\frac{\pi_{A_i}+\pi_{B_i}}{2})^\alpha - \frac{1}{2}(\pi_{A_i}^\alpha + \pi_{B_i}^\alpha)), \alpha > 0, \alpha \neq 1 \\ \frac{-1}{2}\{(\mu_{A_i}+\mu_{B_i})ln(\frac{\mu_{A_i}+\mu_{B_i}}{2}) - \mu_{A_i}ln(\mu_{A_i}) - \mu_{B_i}ln(\mu_{B_i}) \\ +(\nu_{A_i}+\nu_{B_i})ln(\frac{\nu_{A_i}+\nu_{B_i}}{2}) - \nu_{A_i}ln(\nu_{A_i}) - \nu_{B_i}ln(\nu_{B_i}) \\ +(\pi_{A_i}+\pi_{B_i})ln(\frac{\pi_{A_i}+\pi_{B_i}}{2}) - \pi_{A_i}ln(\pi_{A_i}) - \pi_{B_i}ln(\pi_{B_i})\}, \alpha = 1 \end{cases}$$

and μ_{A_i}, μ_{B_i}, ν_{A_i}, ν_{B_i}, π_{A_i}, π_{B_i} represent the values of functions μ_A, μ_B, ν_A, ν_B, π_A and π_B, respectively, computed in x_i. The similarity measure between IFSs $\widetilde{A}$ and $\widetilde{B}$ is given by [10]

$$SJ(\widetilde{A}, \widetilde{B}) = \frac{f(J_\alpha(\widetilde{A}, \widetilde{B})) - f(U(\alpha))}{f(0) - f(U(\alpha))}$$

where $U(\alpha) = \frac{1}{\alpha-1}(1-\frac{1}{2^{\alpha-1}})$ and f is a monotone decreasing function. For $\alpha \in [1, 2]$ it holds $SJ(\widetilde{A}, \widetilde{B}) \in [0, 1]$. For $f(x) \in \{1-x, exp(-x), \frac{1}{1+x}\}$ one get [10] the following similarity measures

$$SJ_\alpha^l(\widetilde{A}, \widetilde{B}) = \frac{U(\alpha) - J_\alpha(\widetilde{A}, \widetilde{B})}{U(\alpha)},$$

$$SJ_\alpha^l(\widetilde{A}, \widetilde{B}) = \frac{exp(-J_\alpha(\widetilde{A}, \widetilde{B})) - exp(-U(\alpha))}{1 - exp(-U(\alpha))}$$

and

$$SJ_\alpha^c(\widetilde{A}, \widetilde{B}) = \frac{U(\alpha) - J_\alpha(\widetilde{A}, \widetilde{B})}{U(\alpha)(1 + J_\alpha(\widetilde{A}, \widetilde{B}))}.$$

3. Our approach

In order to use the similarity measures for selecting, by a student, the optional courses in a master program depending on the optional courses chosen in the license program, we consider the following information:

- a set of students $\{S_1, S_2, ..., S_n\}$ who graduated license program;
- m pairs of optional courses studied in the undergraduate program $(L_i, L_{i+1}), i \in \{1, 3, ..., 2m-1\}$;
- l pairs of optional courses studied in the graduate program: $(M_i, M_{i+1}), i \in \{1, 3, ..., 2l-1\}$;
- scores of students, between 1 and 10, in optional courses: $MM(i, j)$ represents the score of student S_i at the course L_j;
- percentage of knowledge gained from mandatory courses in the undergraduate program required for optional courses from graduate program: $P(i)$ is the percentage associated to course L_i;

• average score of mandatory courses which are useful for optional courses in undergraduate program: $MN(i,j)$ is the average associated with the student S_i and the course L_i;

• an intuitionistic fuzzy relation Q given from the set of students to the set of courses in the undergraduate program: $\mu_Q(i,j) = MM(i,j)/10$, $\nu_Q(i,j) = MN(i,j) \times P(j)/100$;

• an intuitionistic fuzzy relation R given from the set of courses in the undergraduate program to the set of courses in the graduate program: $\mu_R(i,j)$ represents the percentage of knowledge from the optional course L_i used by course M_j and $\nu_R(i,j)$ represents the percentage of knowledge acquired at the mandatory courses that are necessary during M_j course.

• the IFR Q defines for every student S_i an IFS

$$IQ_i = \{(j, \mu_Q(i,j), \nu_Q(i,j))), j = 1, 2, ..., 2m\}$$

• the IFR R defines for every optional course M_k an IFS

$$IR_k = \{(j, \mu_R(j,k), \nu_R(j,k))), j = 1, 2, ..., 2m\}$$

Our task is to establish for each student $S_i, i = 1, 2, ..., n$ the optional courses from the graduate program. To fulfill this task, we propose to calculate for each student some similarity measures between his optional courses in the undergraduate program to the set of courses in the undergraduate program characteristic for each course in the graduate program; namely, we compute the similarity between each IFS IO_i to the set of IFSs $\{IR_k\}, k = 1, 2, ..., 2l$ and the greatest value for each pair $(M_{2j-1}, M_{2j}), j = 1, ..., l$ point out the appropriate course to selected for the graduate program. We will work separately with measures based on L_p metric and with those based on J_α divergence and we compare the results to validate our model.

4. A case study

We consider the following particular values:

• $n = 3$; the students are S_1, S_2, S_3

• $m = 3$; the optional courses in undergraduate program are (L_1, L_2), (L_3, L_4), (L_5, L_6)

• $l = 2$; the optional courses in graduate program are $(M_1, M_2), (M_3, M_4)$

• scores matrix for the optional courses $L_1, ..., L_6$ is

$$\begin{pmatrix} 9 & 0 & 10 & 8 & 0 & 9 \\ 0 & 7 & 9 & 0 & 8 & 7 \\ 5 & 0 & 0 & 7 & 0 & 7 \end{pmatrix}$$

• the percentages associated with courses $L_1, ..., L_6$ are, respectively

$$(10, \quad 15, \quad 5, \quad 5, \quad 10, \quad 5)$$

• the matrix MN is

$$\begin{pmatrix} 8.5 & 9 & 9.5 & 8 & 9.5 & 9 \\ 6 & 6.5 & 9.5 & 8 & 7.5 & 6 \\ 5.5 & 6 & 5 & 7.5 & 7 & 6.5 \end{pmatrix}$$

• the values μ_Q and ν_Q corresponding to the IFR Q are

$$\begin{pmatrix} 0.9 & 0 & 1 & 0.8 & 0 & 0.9 \\ 0 & 0.7 & 0.9 & 0 & 0.8 & 0.7 \\ 0.5 & 0 & 0 & 0.7 & 0 & 0.7 \end{pmatrix}$$

and

$$\begin{pmatrix} 0.085 & 0.135 & 0 & 0.04 & 0.095 & 0.045 \\ 0.06 & 0.0974 & 0.0475 & 0.04 & 0.075 & 0.03 \\ 0.055 & 0.09 & 0.025 & 0.0375 & 0.07 & 0.0325 \end{pmatrix}$$

respectively
• the values μ_R and ν_R corresponding to the IFR R are

$$\begin{pmatrix} 0 & 0.05 & 0.2 & 0.1 \\ 0 & 0.25 & 0.15 & 0.1 \\ 0 & 0.7 & 0.01 & 0.01 \\ 0.08 & 0.01 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0.07 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\begin{pmatrix} 0.12 & 0.1 & 0.09 & 0.11 \\ 0.12 & 0.1 & 0.09 & 0.11 \\ 0.12 & 0.1 & 0.09 & 0.11 \\ 0.12 & 0.1 & 0.09 & 0.11 \\ 0.12 & 0.1 & 0.09 & 0.11 \\ 0.12 & 0.1 & 0.09 & 0.11 \end{pmatrix}$$

respectively.
We compute

$$L_p(S_i, M_k) = \frac{1}{6}\sum_{j=1}^{6}(|\mu_Q(i,j) - \mu_R(j,k)|^p + |\nu_Q(i,j) - \nu_R(j,k)|^p)^{1/p}.$$

For $p = 2$, the similarity measures between the set of students $\{S_1, S_2, S_3\}$ and the set of courses $\{M_1, M_2, M_3, M_4\}$ are the following
• for the measure SL_l^2

$$\begin{pmatrix} & M_1 & M_2 & M_3 & M_4 \\ S_1 & 0.57 & 0.56 & 0.59 & 0.59 \\ S_2 & 0.64 & 0.66 & 0.63 & 0.62 \\ S_3 & 0.75 & 0.74 & 0.77 & 0.77 \end{pmatrix}$$

• for the measure SL_e^2

$$\begin{pmatrix} & M_1 & M_2 & M_3 & M_4 \\ S_1 & 0.39 & 0.39 & 0.42 & 0.41 \\ S_2 & 0.48 & 0.49 & 0.46 & 0.45 \\ S_3 & 0.61 & 0.59 & 0.63 & 0.63 \end{pmatrix}$$

• for the measure SL_c^2

$$\begin{pmatrix} & M_1 & M_2 & M_3 & M_4 \\ S_1 & 0.35 & 0.34 & 0.37 & 0.37 \\ S_2 & 0.43 & 0.44 & 0.41 & 0.40 \\ S_3 & 0.56 & 0.54 & 0.58 & 0.58 \end{pmatrix}$$

The conclusion given by the measure SL_l^2 is
• for the student S_1 the courses M_1 and (M_3 or M_4) are recommended
• for the student S_2 the courses M_2 and M_3 are recommended
• for the student S_3 the courses M_1 and (M_3 or M_4) are recommended.

The conclusion given by the measure SL_e^2 is
• for the student S_1 the courses (M_1 or M_2) and M_3 are recommended
• for the student S_2 the courses M_2 and M_3 are recommended
• for the student S_3 the courses M_1 and (M_3 or M_4) are recommended.

The conclusion given by the measure SL_c^2 is
• for the student S_1 the courses M_1 and (M_3 or M_4) are recommended
• for the student S_2 the courses M_2 and M_3 are recommended
• for the student S_3 the courses M_1 and (M_3 or M_4) are recommended.

We compute the measures SL for other values of p, namely $p = 3$ and $p = 5$; synthesizing the results given by SL measures we obtain the following conclusion
• for the student S_1 the courses M_1 and M_3 are recommended
• for the student S_2 the courses M_2 and M_3 are recommended
• for the student S_3 the courses M_1 and M_4 are recommended.

Working with the SJ measures and the parameter $\alpha \in \{1.25, 1.5, 1.75\}$ we obtain the same conclusion as in the case of SL measures, which validates the correctness of our proposed model. However, if the student can not make a choice between two courses according to the previous method then he/she can select a course using strictly personal criteria; for instance, applicability of the course in a future preferred profession.

5. Conclusion

The paper solves a decision problem using, separately, a set of similarity measures based on L_p metric and another set based on J divergence in order to compute the compatibility between two intuitionistic fuzzy sets. In the case study examined the two types of measures give the same result which illustrates that the proposed model is reasonable.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20** (1986), no. 1, 87–66.
[2] K. Atanassov, More on intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **33** (1989), no. 1, 37–46.
[3] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **61** (1994), 137-142.
[4] K. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica-Verlag, Heidelberg, New York, 1999.
[5] H. Bustine and P. Burillo, Vague Sets are Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems* **79** (1996), 403–405.

[6] S.K. De, R. Biswas and A.R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, *Fuzzy Sets and Systems* **117** (2001), no. 2, 209–213.
[7] W.L. Gau and D.J. Buehrer, Vague sets, *IEEE Transactions on Systems Man and Cybernetics* **23** (1993), no. 2, 610–614.
[8] J. Havrda and F. Charavat, Quantification method of classification processes: concept of structural α-entropy, *Kybernetika* **3** (1967), 30-35.
[9] W.-L. Hung and M.-S. Yang, Similarity measures of intuitionistic fuzzy sets based on L_p metric, *International Journal of Approximate Reasoning* **46** (2007), 120–136.
[10] W.-L. Hung and M.-S. Yang, On the J-divergence of intuitionistic fuzzy sets with its application to pattern recognition, *Information Sciences* **178** (2008), 1641–1650.
[11] S. Kullback and R.A. Leibler, On information and sufficiency, *Ann. Math. Statist.* **22** (1951), 79-86.
[12] D. Li and C. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognition, *Pattern Recognition Letters* **23** (2002), 221-225.
[13] Z. Liang and P. Shi, Similarity measures on intuitionistic fuzzy sets, *Pattern Recognition Letters* **24** (2003), 2687-2693.
[14] H.B. Mitchell, On the Dengfeng-Chuntian similarity measure and its application to pattern recognition, *Pattern Recognition Letters* **24** (2003), 3101-3104.
[15] A.K. Nada and P. Paul, Some results on generalized residual entropy, *Inform. Sci.* **176** (2006), 27-47.
[16] C.E. Shannon, The mathematical theory of communication, *The Bell System Technical Journal* **27** (1948), 379-423, 623-656.
[17] E. Szmidt and J. Kacprzyk, On measuring distances between intuitionistic fuzzy sets, *Notes on IFS* **3** (1997), no. 4, 1-13.
[18] E. Szmidt and J. Kacprzyk, Distances between intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **114** (2000), no 3, 505-518.
[19] E. Szmidt and J. Kacprzyk, Intuitionistic Fuzzy Sets in Some Medical Applications, *Proceedings of Fifth Int. Conf. on IFSs*, Sofia, 22-23 Sept, 2001, Notes on IFS, Vol. 7 (2001), no. 4, 58–64.
[20] W. Wang and X. Xin, Distance measure between intuitionistic fuzzy sets, *Pattern Recognition Letters* **26** (2005), 2063-2069.
[21] N. Wiener, *Cybernetics*, Wiley, New York, 1949.
[22] Z.S. Xu and R.R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, *International Journal of General Systems* **35** (2006), no. 4, 417–433.
[23] Z.S. Xu, Intuitionistic fuzzy aggregation operators, *IEEE Transactions on Fuzzy Systems* **15** (2007), no. 6, 1–10.
[24] L.A. Zadeh, Fuzzy sets, *Information and Control* **8** (1965), no. 4, 338–356.

(Ion Iancu) Department of Computer Science, University of Craiova, 13 A.I. Cuza Street, Craiova, 200585, Romania
E-mail address: i_iancu@yahoo.com

(Daniela Dănciulescu) Department of Economic Informatics, University of Craiova, 13 A.I. Cuza Street, Craiova, 200585, Romania
E-mail address: danadanciulescu@yahoo.com