Annals of the University of Craiova, Mathematics and Computer Science Series
Volume 38(3), 2011, Pages 94–100
ISSN: 1223-6934, Online 2246-9958

Non-cooperative games and stygmergetic programming for a transport optimization problem

Costin-Radu Boldea

Abstract. The stigmergy, witch describes a class of mechanisms that mediate animal to animal interaction through the environment has used in modelling multi-agent systems, as it provides a simple framework for agent interaction and coordination. In this paper stigmergetic mechanisms are combined with the Nash's theory of non-cooperative games in order to model a concurrent multi-player transport problem. A new version of Ant Colony Optimization using multiple species is proposed.

2000 *Mathematics Subject Classification.* Primary 68Q80; Secondary 37B15.
Key words and phrases. Ant Colony Optimization, Non-cooperative games, transport problem.

1. Introduction

The diffculty to solve multiple objective combinatorial optimization problems with traditional techniques has urged researchers to look for alternative, better performing approaches for them. Many of the problems arising in real-life applications are NP-hard. Hence, one usually solves large instances with the use of approximate methods that return near-optimal solutions in a relatively short time. Algorithms of this type are called heuristics. The upgrade of a heuristic is a metaheuristic [1]: a set of algorithmic concepts that can be used to define a heuristic method applicable to a wider set of different problems. A particularly successful metaheuristic based on stigmergy is observed in colonies of real ants.

Stigmergy is a method of communication in decentralized systems in which the individual parts of the system communicate with one another by modifying their local environment. It was first observed in nature as a class of mechanisms that mediate animalanimal interactions (e.g., ant trails, termite nest-building, ant corpse-gathering). Recently, several algorithms have been proposed which are based on the Ant Colony Optimization (ACO) metaheuristic, one of the most used method of stigmergetic programming. Other algorithms are based on termite or bees wasp nest building behavior [6]. Ant colony optimization (ACO) is a metaheuristic inspired by the shortest path searching behavior of various ant species. Since the initial work of Dorigo, Maniezzo, and Colorni on the first ACO algorithm, the Ant System, several researchers have developed different ACO algorithms that performed succesfully when solving many different combinatorial problem [2].

The aim of the current contribution is to analyze the application of these proposals to a more complex problem, the multi-objective shortest path problem (SPP) with concurrent players, using a modified version of the adaptative ACO first proposed by P. Vrancx, K. Verbeeck and A. Now in 2006 [6]. The concrete problem studied int

Received July 03, 2011. Revision received August 26, 2011.

his paper can be formulated as: *Finding the shortest path from a resources center to the own commercial center in the presence of a concurrent firm, without trespassing trough the territory strictly controlled by the other.* The structure of the model used suppose that the locations are the vertices of a graph, the paths are formed by some connected edges, and the territorial occupation is dynamical. The game theory of Nash [4] was used to simulate the direction decision problem for individual agents (ants) in conflict situation.

2. The Stigmergetic Algorithm

Several different approaches have been proposed to apply stigmergy to multiagent systems. In most of the algorithms based on the systems mentioned above a set of common elements can be isolated:

- The agent environment is subdivided in a number of discrete locations, which agents can visit.
- Each location contains a local state that can be accessed and updated by agents visiting that location.
- An interconnection scheme between locations is defined, allowing agents to travel between locations.

The basic idea of Ant Colony Optimization (ACO) [2] is to model the problem to solve as the search for a minimum cost path in a graph, and to use artificial ants to search for good paths. The behavior of artificial ants is inspired from real ants: they lay pheromone on components (edges and/or vertices) of the graph and they choose their path with respect to probabilities that depend on pheromone trails that have been previously laid by the colony. Intuitively, this indirect stigmergetic communication means aims at giving information about the quality of path components in order to attract ants, in the following iterations, towards the corresponding areas of the search space. Artificial ants also have some extra-features that do not find their counterpart in real ants. In particular, they are usually associated with data structures that contain the memory of their previous actions, and they may apply some daemon procedures, such as local search, to improve the quality of computed paths.

The so-called Ant Stigmergy Algorithm (ASA), a generalization of the ACO [5], consists of two main phases: (a) initialization and (b) searh and optimization. The algorithm can be shortly described by:

```
graph = Initialization(parameters)
GraphInitialization(initial pheromone amount)
WHILE not ending condition do
      FOR ALL ants in colony do
          path = FindNextLocation(probability rule)
          Evaluate(path)
      END FOR
     UpdatePheromone(all found paths vertices)
     DaemonAction(select best path)
     ResetCondition(all vertices)
END WHILE
```

In the particular case modeled in this paper, the agents of the concurrent firms are modeled as "artificial ants", bellowing to one of the two species (Blue and Red).

The locations are the vertices of a graph, each vertex V_i, $i - 1 \dots N$, possessing a multivalued state retaining the number of each species ant located in a given (discrete) moment in the vertex (f_1, respectively f_2) , and the level of the "pherormonic" trace, both for the Blue ants ant the Red ones (s_1, respectively s_2).

The *initialization* process generate the two species of ants and initializes with zero the path associated with each one. In the beginning, all m_1 Blue ants and the m_2 Red ants are located in the corespondent "Ant's Hill" (V_1 for the Blue's and V_N for the Red's). An arbitrary location of the resources location V_H was chosen.

The *search of a path and optimization* phases consists in moving each ant from a vertex to another, in order to find the resource location, and to extracts the shortest path founded witch verifies some conditions. Each ant "choose" probabilistically the direction of moving depending on the information about the destination edge (the level of pherormones of his own race and the number of ants always presents in the location). The competition between the agents and the strategy of choosing the path are simulated using a noncooperative games.

3. The non-cooperative game and the "fire" automata

The algorithm described in the previous section is the generic stigmergetic algorithm. In the clasical ACO algorithms, all agents have the same goal and they alter their environment (by leaving pheromones) to share information and coordinate their actions. In this paper we examine the complex problems where agents have the same goals, but do not cooperate with the opposite agents. One such system was proposed by P. Vrancx, K. Verbeeck, A. Now in [6]. The idea behind their model is to move decision making from the agents (ants) to the local environment states. Each local state must contains one learning automata. When an agent visits a location it activates the learning automaton that resides in that location. This automaton then decides the action the agent should take in that location. Transition to the next location triggers an automaton from that location to become active and take some action.

In our case, the automaton can block or allows the access of each specie to the respective location, acting more as a "fire" automaton. The states of a vertex's automaton are characterized by the number of each species ant in the givem vertex (f_1, respectively f_2), and the level of the "pherormonic" trace, both for the Blue ants ant the Red ones (s_1, respectively s_2). When the level of the pherormonic trace of one of the species is significative greater that the level of pherormonic trace of the other:

$$|S_1 - S_2| > h \tag{1}$$

(where h is a fixed "threshold" limit), the automaton block the access of the second specie to the vertex to the future. The first specie "colonizes" the location, assuring a permanent "territorial occupation". The vertex become "safe" for the dominant ants ant interdicted for the others.

This process simulate the elimination of the concurrence from a local market. The final purpose change for the both opponents: each one must find the shortest path to the resources location that pass trough his own safe territory, or through the neutral vertices.

The selection of the next location for each ant is a process that imply an pondered aleatory decision. If a specified ant is located in the vertex V_j, the direction of movement i is determined by

$$max\{r_i * V_i.s_k | V_i \text{ is connected to } V_j\} \tag{2}$$

(where r_i are aleatory value between 0 and 1, $k = 1$ for Blues, $k = 2$ for Reds, and $V_i.s_k$ is pherormonic level in V_i for the k specie). When the ant visits the next location, it modify the ferormonic state of the location by adding a reward trace, function of the precedent state associated to the location. For example, if the ant is Blue, the pherormonic trace in the V_i location changes according to:

$$V_i.s_1 \leftarrow V_i.s_1 + a\frac{V_i.f_1}{V_i.f_2} \tag{3}$$

if the blue ant is searching the food source, and

$$V_i.s_1 \leftarrow V_i.s_1 + b\frac{V_i.f_1}{V_i.f_2} \tag{4}$$

if the blue ant is returning with food. Here a and b are reward parameters that determine the strategy of the ant; in general $a < b$, for example $a = 0.3$ when $b = 1$. The difference between the two parameters favorites the return to the ants hills. The choose of these parameters can be justified by the noncooperative games theory of non-zero-sum (Nash [4]), and is preferable to assume the equilibrium between the two players (the two species). The precedent values was choused for almost equal small populations in order to minimize the time of search. At a logical condition, it can be proved that

$$a/b \simeq \frac{2E}{N(N-1)}$$

where E is the number of edges and N is the number of vertices of the graph.

4. Execution and Results

The algorithm described before was implemented in C++, using arbitrary generated graphs with selected number of vertices. In the figures 1, 2 and 3 are represented an example of the execution result for a 12 node graph, with $m_1 = 20$ Blues ants and $m_2 = 24$ Red ants.

After a given number of steps, the territorial occupation become stable and the search for the shortest "safe" path can begin, as ilustrated in the Fig.2.

The shortest path are easy identified in few more steps. A condition of stop after a number of steps was introduced, because the solution is generally not-unique and sometime the shift between equivalent short path can produce. The given solution is illustrated in the Fig. 3.

The execution time is increasing with the number of ants and the number of vertices, but an inferior number of ants that the number of vertices for each species produces a significative incrementation of the execution time. For $m_K > 2N, k = 1, 2$, the complexity of the algorithm is $\mathcal{O}m_1 \cdot m_2 \cdot N$, giving a polynomial dependence on the number of vertices N.

Note that the solution obtained can be partial: due to the structure of the graph, or a great difference between the numbers of Blue and Red ants, one of the Colony can block permanently the access of the other to the "Food" location. In this case, the second colony have no "safe" path to access the goal location. The Fig. 4 illustrate this case for a 10 vertices graph.

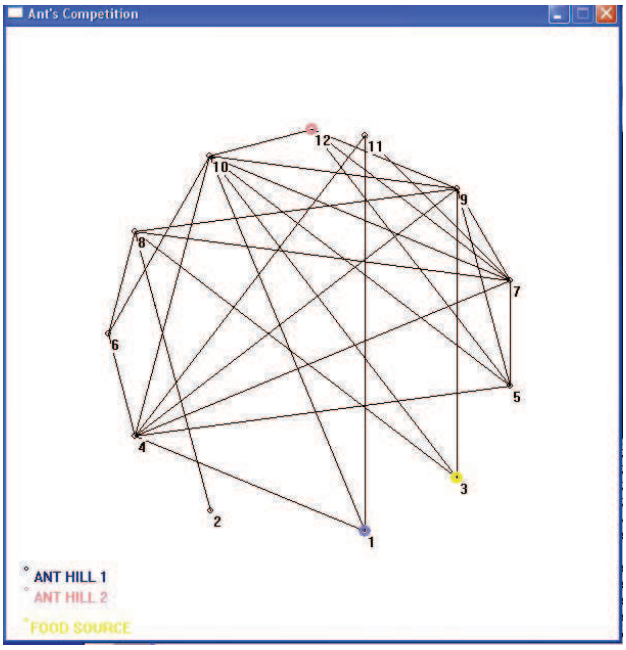

FIGURE 1. The generated graph and the initial positions

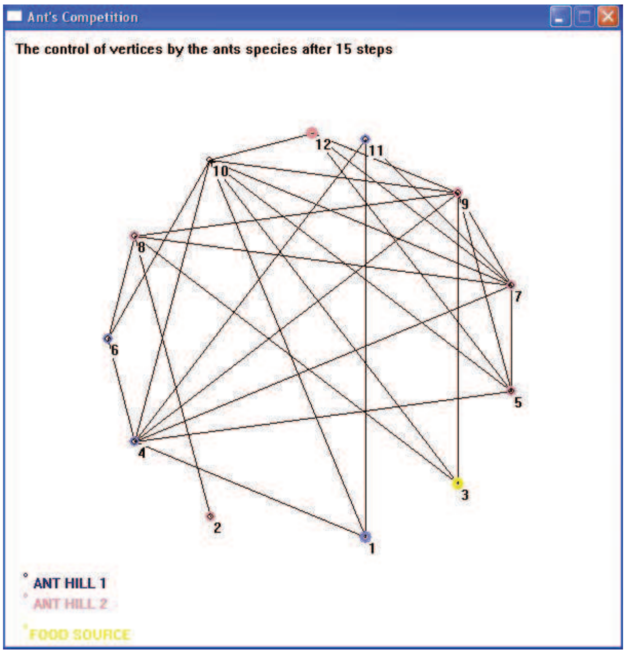

FIGURE 2. The control of the vertices by the two colonies after 15 steps

5. Conclusion

We have proposed in this paper a generic stigmergetic algorithm inspired by the ACO for solving a concurrent shortest path problem with multiple conflicting agents.

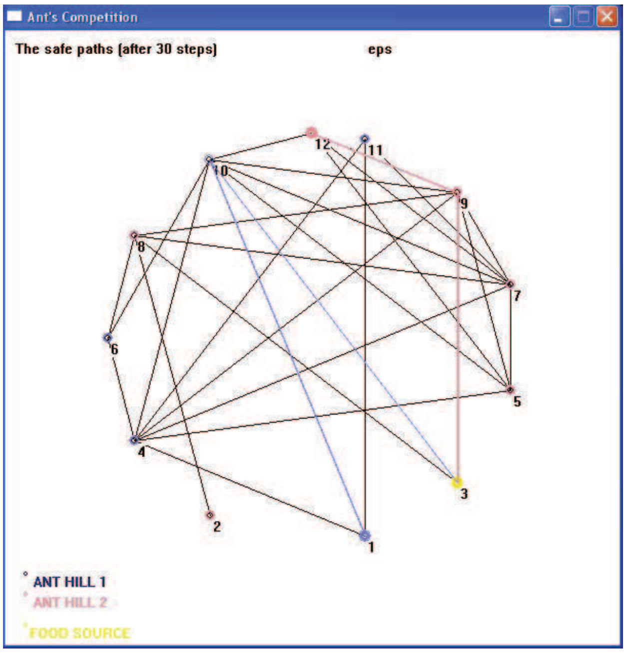

FIGURE 3. The safe paths founded after 30 steps

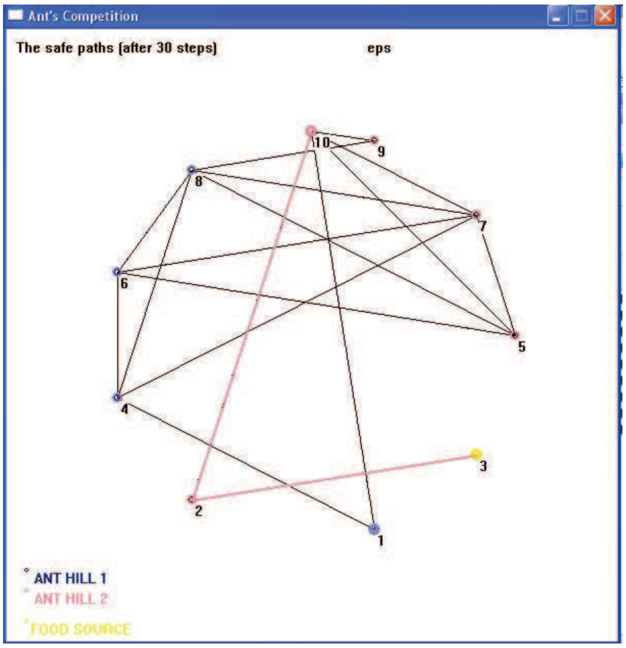

FIGURE 4. No solution for the Blue ants

This algorithm is parameterized by the number of ant colonies and the parameters of a non-cooperative game between the colonies. We have tested the variants of this algorithm for variable dimensions of the graph and the ants colonies, and we obtained different solutions in function of the umber of ants from each colony. The method offer the possibility to implement and simulate variate case of conflicting networks of distribution between two or more economic agents, being easy adaptable to other similar economic problems.

References

[1] C. Blum and A. Roli, Metaheuristics in combinatorial optimization: Overview and conceptual comparison, *ACM Comput. Surv.* **35** (2003), 268-308.

[2] M. Dorigo and T. Stützle, The Ant Colony Optimization Metaheuristic: Algorithms, Applications and Advances, In: F. Glover, G.A. Kochenberger (Eds.), *Handbook of Metaheuristics*, Kluwer, 2003.

[3] K. Najim and A. Poznyak, *Learning Automata - Theory and Applications*, Pergamon Press, Oxford, 1994.

[4] J.F. Nash, *Non-Cooperative Games*, 1951.

[5] J. Šilc and P. Korošec, The Distributed Stigmergic Algorithm for Multi-parameter Optimization, *Parallel Processing and Applied Mathematics, 6th International Conference, PPAM 2005, Poznan*, *Lecture Notes in Computer Science 3911*, Springer-Verlag, 2006, 92–99.

[6] P. Vrancx, K. Verbeeck and A. Now, Analyzing Stigmergetic Algorithms Through Automata Games, *Knowledge Discovery and Emergent Complexity in Bioinformatics*, *Lecture Notes in Bioinformatics 4366*, Springer-Verlag, 2006, 145–156.

(Costin-Radu Boldea) Department of Computer Sciences, University of Craiova, Al.I. Cuza Street, No. 13, Craiova RO-200585, Romania, & Fax: 40-251412673
E-mail address: cboldea@central.ucv.ro