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Screen transversal conformal half-lightlike submanifolds

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Abstract. In this paper, we study half-lightlike submanifolds of a semi-Riemannian manifold such that the shape operator of screen distribution is conformal to the shape operator of screen transversal distribution. We mainly obtain some results concerning the induced Ricci curvature tensor and the null sectional curvature of screen transversal conformal half-lightlike submanifolds.

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1. Introduction

Since the intersection of the normal bundle and the tangent bundle of a submanifold of a semi-Riemannian manifold may be not trivial, it is more difficult and interesting to study the geometry of lightlike submanifolds than non-degenerate submanifolds. Two standard methods to deal with the above difficulties were developed by Kupeli [11] and Duggal-Bejancu [4] (see also Duggal-Jin [6] and Duggal-Sahin [8]), respectively.

It is obvious that there are two cases of codimension 2 lightlike submanifolds M since for this type the dimension of their radical distribution is either 1 or 2, a codimension 2 lightlike submanifold is called a half-lightlike submanifold (see [3]) if $\dim(Rad(TM)) = 1$. For more results about half-lightlike submanifolds, we refer the readers to [5] and [9].

There exists a shape operator of the screen distribution in geometry of lightlike submanifolds which is very different from the non-degenerate submanifold case, therefore, the conformality of the operator of screen distribution and the operator of lightlike transversal bundle of a lightlike hypersurface of semi-Riemannian manifolds was introduced by Atindogbe-Duggal [1]. Recently, lots of works (see [2] and [7]) have been done to investigate lightlike submanifolds with screen conformal condition. Moreover, we also refer the reader to [13] by the present authors for some related results on half-lightlike submanifolds satisfying the co-screen conformal condition.

In this paper, we mainly discuss the properties of half-lightlike submanifolds of a semi-Riemannian manifold such that the shape operator of screen distribution and the shape operator of screen transversal distribution are conformal. Under such a geometric condition we prove that the Ricci tensor of a half-lightlike submanifold induced from ambient space is symmetric. Moreover, we obtain some sufficient geometric conditions to characterize screen transversal conformal half-lightlike submanifolds.

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2. Preliminaries

In this section, according to [5] and [10] developed by Duggal-Jin, we collect some notations and fundamental equations of half-lightlike submanifolds of semi-Riemannian manifolds.

A submanifold (M, g) of dimension m immersed in a semi-Riemannian manifold $(\overline{M}, \overline{g})$ of dimension $(m+n)$ is called a lightlike submanifold if the metric g induced from ambient space is degenerate and radical distribution $Rad(TM)$ is of rank r, where $m \geq 2$ and $1 \leq r \leq \min\{m, n\}$. In particular, (M, g) is called a half-lightlike submanifold if $n = 2$ and $r = 1$. It is well known that the radical distribution is given by $Rad(TM) = TM \cap TM^{\perp}$, where $TM^{\perp}$ is called the normal bundle of M in $\overline{M}$. Thus there exist two non-degenerate complementary distributions $S(TM)$ and $S(TM^{\perp})$ of $Rad(TM)$ in TM and $TM^{\perp}$ respectively, which are called the screen and the screen transversal distribution on M respectively. Thus we have

$$TM = Rad(TM) \oplus_{\text{orth}} S(TM), \tag{1}$$

and

$$TM^{\perp} = Rad(TM) \oplus_{\text{orth}} S(TM^{\perp}), \tag{2}$$

where $\oplus_{\text{orth}}$ denotes the orthogonal direct sum.

Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\overline{M}$, it is easy to see that $TM^{\perp}$ is a subbundle of $S(TM)^{\perp}$. As $S(TM^{\perp})$ is a non-degenerate subbundle of $S(TM)^{\perp}$, the orthogonal complementary distribution $S(TM^{\perp})^{\perp}$ to $S(TM^{\perp})$ in $S(TM)^{\perp}$ is also a non-degenerate distribution. Clearly, $Rad(TM)$ is a subbundle of $S(TM^{\perp})^{\perp}$. Choose $L \in \Gamma(S(TM^{\perp}))$ as a unit vector field with $\overline{g}(L, L) = \epsilon = \pm 1$. For any null section $\xi \in Rad(TM)$, there exists a uniquely defined null vector field $N \in \Gamma(S(TM^{\perp})^{\perp})$ satisfying

$$\overline{g}(\xi, N) = 1, \quad \overline{g}(N, N) = \overline{g}(N, X) = \overline{g}(N, L) = 0, \quad \forall\, X \in \Gamma(S(TM)).$$

Denote by $ltr(TM)$ the vector subbundle of $S(TM^{\perp})^{\perp}$ locally spanned by N. Then we show that $S(TM^{\perp})^{\perp} = Rad(TM) \oplus ltr(TM)$. Let $tr(TM) = S(TM^{\perp}) \oplus_{orth} ltr(TM)$. We call N, $ltr(TM)$ and $tr(TM)$ the lightlike transversal vector field, lightlike transversal vector bundle and transversal vector bundle of M with respect to the chosen screen distribution $S(TM)$ respectively. Then $T\overline{M}$ is decomposed as follows:

$$\begin{aligned} T\overline{M} &= TM \oplus tr(TM) = Rad(TM) \oplus tr(TM) \oplus_{\text{orth}} S(TM) \\ &= Rad(TM) \oplus ltr(TM) \oplus_{\text{orth}} S(TM) \oplus_{\text{orth}} S(TM^{\perp}). \end{aligned} \tag{3}$$

Let P be the projection morphism of TM on $S(TM)$ with respect to the decomposition (1). For any $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$, $\xi \in \Gamma(Rad(TM))$ and $L \in \Gamma(S(TM)^{\perp})$, the Gauss and Weingarten formulas of M and $S(TM)$ are given by

$$\overline{\nabla}_X Y = \nabla_X Y + D_1(X, Y)N + D_2(X, Y)L, \tag{4}$$

$$\overline{\nabla}_X N = -A_N X + \rho_1(X)N + \rho_2(X)L, \tag{5}$$

$$\overline{\nabla}_X L = -A_L X + \varepsilon_1(X)N + \varepsilon_2(X)L, \tag{6}$$

$$\nabla_X PY = \nabla^*_X PY + E(X, PY)\xi, \tag{7}$$

$$\nabla_X \xi = -A^*_\xi X + u_1(X)\xi, \tag{8}$$

respectively, where ∇ and ∇^* are induced connection on TM and $S(TM)$ respectively, D_1 and D_2 are called locally second fundamental forms of M, E is called the locally second fundamental form on $S(TM)$. A_N, A^*_ξ and A_L are linear operators on TM and ρ_1, ρ_2, ε_1, ε_2 and u_1 are 1-forms on TM. Note the connection ∇ is torsion free but is not metric tensor, the connection ∇^* is metric. D_1 and D_2 are both symmetric tensors on $\Gamma(TM)$.

It is easy to check $\varepsilon_2 = 0$ and the following:

$$D_1(X,\xi) = 0, \quad D_1(X,PY) = g(A^*_\xi X, PY), \quad \overline{g}(A^*_\xi X, N) = 0, \tag{9}$$

$$\epsilon D_2(X,Y) = g(A_L X, Y) - \varepsilon_1(X)\eta(Y), \quad \overline{g}(A_L X, N) = \epsilon\rho_2(X), \tag{10}$$

$$E(X,PY) = g(A_N X, PY), \quad \overline{g}(A_N X, N) = 0, \quad u_1(X) = -\rho_1(X) \tag{11}$$

for any $X \in \Gamma(TM)$ and $N \in \Gamma(ltr(TM))$, where $\eta(X) = \overline{g}(X,N)$.

From the above equations we see that A^*_ξ and A_N are $\Gamma(TM)$-valued shape operators related to D_1 and D_2 respectively, and A^*_ξ is self-adjoint on TM and satisfies

$$A^*_\xi \xi = 0. \tag{12}$$

Denote by $\overline{R}$ and R the curvature tensor of semi-Riemannian connection $\overline{\nabla}$ of $\overline{M}$ and the induced connection ∇ on M respectively, we obtain the following Gauss-Codazzi equations for M and $S(TM)$.

$$\begin{aligned}\overline{g}(\overline{R}(X,Y)Z,PW) = g(R(X,Y)Z,PW) &+ D_1(X,Z)E(Y,PW) \\ &- D_1(Y,Z)E(X,PW) + \epsilon D_2(X,Z)D_2(Y,PW) \\ &- \epsilon D_2(Y,Z)D_2(X,PW),\end{aligned} \tag{13}$$

$$\overline{g}(\overline{R}(X,Y)Z,N) = \overline{g}(R(X,Y)Z,N) + \epsilon\rho_2(Y)D_2(X,Z) - \epsilon\rho_2(X)D_2(Y,Z), \tag{14}$$

$$\overline{g}(\overline{R}(X,Y)Z,\xi) = g(R(X,Y)Z,\xi) + \varepsilon_1(X)D_2(Y,PZ) - \varepsilon_1(Y)D_2(X,PZ), \tag{15}$$

for any $X,Y,Z,W \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $\xi \in \Gamma(Rad(TM))$.

The Ricci curvature tensor of $\overline{M}$ denoted by $\overline{Ric}$ is defined by

$$\overline{Ric}(X,Y) = \text{trace}\{Z \to \overline{R}(X,Z)Y\}, \tag{16}$$

where $\overline{R}(X,Y,Z,W) = \overline{g}(\overline{R}(X,Y)Z,W)$. Locally, $\overline{Ric}(X,Y)$ is given by

$$\overline{Ric}(X,Y) = \sum_i \epsilon_i \overline{g}(\overline{R}(X,E_i)Y,E_i), \tag{17}$$

where $\{E_1, E_2, \cdots, E_{m+2}\}$ is an orthogonal frame field of $T\overline{M}$ and $\overline{g}(E_i,E_i) = \epsilon_i$ is the sign of E_i for $1 \leq i \leq m+2$. In particular, if $\overline{Ric}(X,Y) = k\overline{g}(X,Y)$ for any $X,Y \in \Gamma(TM)$, then $\overline{M}$ is called an Einstein manifold, where k is a constant on $\overline{M}$.

3. Screen transversal conformal half-lightlike submanifolds

In this section, we consider a class of half-lightlike submanifolds with screen transversal conformal geometric condition defined as follows.

Definition 3.1. Let M be a half-lightlike submanifold of a semi-Riemannian manifold, M is called *screen transversal locally* (resp. *globally*) *conformal* if on any coordinate neighborhood U (resp. $U = M$) there exists a non-zero smooth function θ such that for any null vector field $\xi \in \Gamma(Rad(TM))$ the relation

$$A_L X = \theta A^*_\xi X, \quad \forall\, X \in \Gamma(TM) \tag{18}$$

holds, where L is a unit vector field of screen transversal bundle of M.

In the sequel, by a screen transversal conformal we shall mean globally screen transversal conformal unless otherwise specified. From (12) we know the shape operator A^*_ξ is $S(TM)$-valued, thus for a screen transversal conformal half-lightlike submanifold we obtain $\rho_2 = 0$ following from (10). Also, we have the following conditions to characterize screen transversal conformal half-lightlike submanifolds.

Theorem 3.1. *Let M be a half-lightlike submanifold of a semi-Riemannian manifold, then M is screen transversal conformal if and only if*

$$D_2(X,PY) = \epsilon\theta D_1(X,PY), \quad \rho_2(X) = 0, \quad \forall\, X,Y \in \Gamma(TM),$$

where θ is a non-zero smooth function on M.

Proof. If M is a screen transversal conformal half-lightlike submanifold, then it follows from (9) and (10) that

$$\epsilon D_2(X,PY) = g(A_L X,PY) = \theta g(A^*_\xi X,PY) = \theta D_1(X,PY).$$

Conversely, we know from (10) that if $\rho_2(X) = \epsilon\overline{g}(A_L X,N) = 0$, then the shape operator A_L is $S(TM)$-valued. As $S(TM)$ is a non-degenerate distribution, then $D_2(X,PY) = \epsilon\theta D_1(X,PY)$ implies that $A_L = \epsilon\theta A^*_\xi$. This completes the proof. □

Definition 3.2 (see [8])**.** A lightlike submanifold M in semi-Riemannian manifold $(\overline{M},\overline{g})$ is said to be *irrotational* if $\overline{\nabla}_X\xi \in \Gamma(TM)$ for any $X \in \Gamma(TM)$, where $\xi \in \Gamma(Rad(TM))$.

For a half-lightlike submanifold M, it follows from (9) that the above definition is equivalent to $D_2(X,\xi) = -\epsilon\varepsilon_1(X) = 0$ for any $X \in \Gamma(TM)$. If M is a screen transversal conformal half-lightlike submanifold, then we have $D_2(\xi,PY) = 0$ for any $X \in \Gamma(TM)$ and $\xi \in \Gamma(Rad(TM))$. From (9) and Theorem 3.1, we have the following proposition on screen transversal conformal half-lightlike submanifolds.

Proposition 3.2. *Let M be a screen transversal conformal half-lightlike submanifold, then M is said to be irrotational provided $\varepsilon_1(\xi) = 0$.*

Theorem 3.3. *Let M be a screen transversal conformal half-lightlike submanifold of a semi-Riemannian space form $(\overline{M}(c),\overline{g})$, then*

$$\rho_1(\xi) + \epsilon\theta\varepsilon_1(\xi) = 0,$$

where $\xi \in \Gamma(Rad(TM))$.

Proof. It follows from (4)–(6) and (9)–(11) that

$$\begin{aligned}\overline{g}(\overline{R}(X,Y)PZ,\xi) = (\nabla_X D_1)(Y,PZ) - (\nabla_Y D_1)(X,PZ) + \rho_1(X)D_1(Y,PZ)\\ - \rho_1(Y)D_1(X,PZ) + \varepsilon_1(X)D_2(Y,PZ) - \varepsilon_1(Y)D_2(X,PZ),\end{aligned} \tag{19}$$

and

$$\begin{aligned}\overline{g}(\overline{R}(X,Y)PZ,u) = \epsilon\big((\nabla_X D_2)(Y,PZ) - (\nabla_Y D_2)(X,PZ)\big)\\ + \rho_2(X)D_1(Y,PZ) - \rho_2(Y)D_1(X,PZ)\end{aligned} \tag{20}$$

respectively. By differentiating $D_2(Y,PZ) = \epsilon\theta D_1(Y,PZ)$, we have

$$(\nabla_X D_2)(Y,PZ) = \epsilon X(\theta)D_1(Y,PZ) + \epsilon\theta(\nabla_X D_1)(Y,PZ). \tag{21}$$

From Theorem 3.1 it turns out that if M is screen transversal conformal then $\rho_2 = 0$, thus, equation (20) becomes

$$(\nabla_X D_2)(Y,PZ) - (\nabla_Y D_2)(X,PZ) = 0. \tag{22}$$

Substituting (22) into (21) and using $D_2(Y, PZ) = \epsilon\theta D_1(Y, PZ)$ and (19), we have

$$D_1(X, PZ)\big(\rho_1(Y) + \epsilon\theta\varepsilon_1(Y)\big) - D_1(Y, PZ)\big(\rho_1(X) + \epsilon\theta\varepsilon_1(X)\big) = 0. \tag{23}$$

Replacing Y by ξ in the above equation we have

$$D_1(X, PZ)\big(\rho_1(\xi) + \epsilon\theta\varepsilon_1(\xi)\big) = 0 \tag{24}$$

for any $X, Z \in \Gamma(TM)$. This completes the proof. □

Corollary 3.4. *Let M be an irrotational screen transversal conformal half-lightlike submanifold of a semi-Riemannian space form $(\overline{M}(c), \overline{g})$, then*

$$\rho_1(\xi) = 0,$$

where $\xi \in \Gamma(Rad(TM))$.

Let M be an m-dimensional screen transversal conformal half-lightlike submanifold of semi-Riemannian space form $\overline{M}(c)$ and $S(TM) = span\{e_1, e_2, \cdots, e_m\}$, where $\{e_i\}$ is an orthogonal frame fields of $\Gamma(S(TM))$ and $g(e_i, e_i) = \epsilon_i$. Then from the Gauss-Codazzi equations we have

$$\begin{aligned}\overline{g}(\overline{R}(X, e_i)Y, e_i) = g(R(X, e_i)Y, e_i) &+ D_1(X, Y)\big(E(e_i, e_i) + \epsilon\theta^2 D_1(e_i, e_i)\big) \\ &- D_1(e_i, Y)\big(E(X, e_i) + \epsilon\theta^2 D_1(X, e_i)\big),\end{aligned} \tag{25}$$

and

$$\overline{g}(\overline{R}(X, \xi)Y, N) = \overline{g}(R(X, \xi)Y, N). \tag{26}$$

Making use of the above equations and the definition of Ricci curvature tensor, we have

$$\begin{aligned}Ric(X, Y) =& \sum_i \epsilon_i g(R(X, e_i)Y, e_i) + \overline{g}(R(X, \xi)Y, N) \\ =& \sum_i \epsilon_i \overline{g}(\overline{R}(X, e_i)Y, e_i) + \overline{g}(\overline{R}(X, \xi)Y, N) - D_1(X, Y)\sum_i E(e_i, e_i) \\ & - D_1(X, Y)\sum_i \epsilon\theta^2 D_1(e_i, e_i) + \sum_i D_1(e_i, Y)\big(E(X, e_i) + \epsilon\theta^2 D_1(X, e_i)\big) \\ =& (1 - m)cg(X, Y) - D_1(X, Y)\sum_i \big(E(e_i, e_i) + \epsilon\theta^2 D_1(e_i, e_i)\big) \\ & + \epsilon\theta^2 \sum_i D_1(e_i, X)D_1(e_i, Y) + \sum_i D_1(Y, e_i)E(X, e_i).\end{aligned} \tag{27}$$

If there exist an orthogonal frame fields $\{e_i\}$ of $S(TM)$ such that the associated matrices of shape operators A_N and A^*_ξ, that is, $\big(E(e_i, e_j)\big)_{(m-1)\times(m-1)}$ and $\big(D_1(e_i, e_j)\big)_{(m-1)\times(m-1)}$, can be diagonalized, then we have

$$D_1(PY, e_i)E(PX, e_i) = D_1(PX, e_i)E(PY, e_i), \quad \forall\, X, Y \in \Gamma(TM).$$

It follows from (11) that $E(\xi, PX) = 0$ is equivalent to $A_N\xi = 0$, therefore, we obtain the following theorem.

Theorem 3.5. *Let M be an m-dimensional screen transversal conformal half-lightlike submanifold of semi-Riemannian space form $\overline{M}(c)$ with $A_N\xi = 0$, if the shape operator of screen bundle and the shape operator of lightlike transversal bundle are symmetric, then the induced Ricci curvature tensor is symmetric.*

In particular, we have the following corollary to characterize Einstein half-lightlike surface.

Corollary 3.6. *Let M be a screen transversal conformal half-lightlike surface of 4-dimensional semi-Riemannian space form $\overline{M}(c)$ with $A_N\xi = 0$, then the induced Ricci curvature tensor is symmetric. Moreover, the surface M is an Einstein surface.*

Proof. We obtain from (27) that

$$\begin{aligned} Ric(X,Y) =& - cg(X,Y) - D_1(X,Y)\big(E(e,e) + \epsilon\theta^2 D_1(e,e)\big) \\ &+ D_1(e,Y)\big(E(X,e) + \epsilon\theta^2 D_1(X,e)\big) \\ =& - cg(X,Y), \end{aligned} \tag{28}$$

where e is a unit vector field of $S(TM)$. This proves the corollary. □

Recall the following notion of null sectional curvature shown in [4]. Let $x \in M$ and ξ be a null vector of T_xM. A plane H of T_xM is called a null plane directed by ξ if it contains ξ, $g_x(\xi, W) = 0$ for any $W \in H$ and there exists $W_o \in H$ such that $g_x(W_o, W_o) \neq 0$. Thus the null section curvature of H with respective ξ and the induced connection ∇ of M, is defined as a real number

$$K_\xi(H) = \frac{g_x(R(W,\xi)\xi, W)}{g_x(W,W)},$$

where $W \neq 0$ is any vector in H linearly independent with respect to ξ. From [12] we see that an $n(n \geq 3)$-dimensional Lorentzian manifold is of constant curvature if and only if its null sectional curvatures are everywhere zero.

Theorem 3.7. *Let M be a screen transversal conformal half-lightlike submanifold of semi-Riemannian space form $\overline{M}(c)$, then the null sectional curvature of M is given by*

$$K_\xi(H) = -\epsilon\theta\varepsilon_1(\xi)D_1(W,W).$$

Proof. From (13) we have

$$\begin{aligned} K_\xi(H) = &\overline{g}(\overline{R}(W,\xi)\xi, W) + D_1(\xi,\xi)E(W,PW) - D_1(W,\xi)E(\xi,PW) \\ &+ \epsilon D_2(\xi,\xi)D_2(W,PW) - \epsilon D_2(W,\xi)D_2(\xi,PW). \end{aligned} \tag{29}$$

By (9) and Theorem 3.1, we obtain $K_\xi(H) = -\epsilon\theta\varepsilon_1(\xi)D_1(W,W)$. This completes the proof. □

If M is a screen transversal conformal half-lightlike submanifold, it follows from Theorem 3.1 that $D_2(\xi, PX) = 0$. Making use of Definition 3.2, we have the following corollary.

Corollary 3.8. *Let M be a screen transversal conformal half-lightlike submanifold of semi-Riemannian space form $\overline{M}(c)$, then the null sectional curvature of M vanishes if and only if $D_1(W,W) = 0$ or M is irrotational.*

Definition 3.3. A half-lightlike submanifold M of semi-Riemannian manifold is said to be *totally* D_1 (resp. D_2)*-umbilical* if on each coordinate neighborhood U there exists a smooth function H_1 (resp. H_2) such that $D_1(X,Y) = H_1g(X,Y)$ (resp. $D_2(X,Y) = H_2g(X,Y)$) for any $X, Y \in \Gamma(TM)$. In particular, if $H_1 = 0$ (resp. $H_2 = 0$), then M is said to be D_1 (resp. D_2)*-geodesic.*

Note that M is totally umbilical (resp. geodesic) (see [5]) if and only if M are both D_1 and D_2 umbilical (reps. geodesic). Together with Theorem 3.1 and the above definition, we have the following theorem.

Theorem 3.9. *Let M be a nowhere geodesic half-lightlike submanifold of semi-Riemannian manifold $\overline{M}$, then the the following assertions are equivalent:*

(1) *M is totally umbilical half-lightlike submanifold of $\overline{M}$ with $\rho_2(X) = 0, \forall X \in \Gamma(TM)$;*

(2) *M is screen transversal conformal and totally D_1-umbilical half-lightlike submanifold of $\overline{M}$ with $\varepsilon_1(X) = 0$, $\forall X \in \Gamma(TM)$;*

(3) *M is screen transversal conformal and totally D_2-umbilical half-lightlike submanifold of $\overline{M}$ with $\varepsilon_1(X) = 0$, $\forall X \in \Gamma(TM)$.*

Proof. (1)⇒(2): if M is a totally umbilical half-lightlike submanifold of $\overline{M}$, by Definition 3.3 we have $D_2(X,Y) = \theta D_1(X,Y)$, where $\theta = \frac{H_2}{H_1}$. Then it follows from Theorem 3.1 that M is screen transversal conformal. Also, it follows from (9) that $\varepsilon_1(X) = 0$.

Conversely, if M is screen transversal conformal then we obtain $D_2(X,PY) = \epsilon\theta D_1(X,PY)$. Noticing that M is totally D_1-umbilical if and only if $D_1(X,Y) = H_1 g(X,Y)$, thus, by Theorem 3.1, we prove the equivalence of (1) and (2). The proof of equivalence between (1) and (3) is the same with that of (1) and (2). □

Theorem 3.10. *Suppose that M is a screen transversal conformal half-lightlike submanifold of semi-Riemannian manifold, then M is totally umbilical if and only if $A^*_\xi X = H_1 PX$ and $\varepsilon_1(X) = 0$ for any $X \in \Gamma(TM)$.*

Proof. If M is a totally umbilical half-lightlike submanifold, it follows from (9) that $D_1(X,PY) = H_1 g(X,PY) = g(A^*_\xi X, PY)$. Since the screen distribution is non-degenerate, we have $A^*_\xi X = H_1 PX$. Moreover, it follows from (10) that $D_2(\xi, X) = H_2 g(\xi, PX) = -\epsilon\varepsilon_1(X) = 0$.

Conversely, if $A^*_\xi X = H_1 PX$, it follows from (9) that $D_1(X,PY) = g(A^*_\xi X, PY) = H_1 g(X,PY)$. In view of $D_2(X,PY) = \epsilon\theta D_1(X,PY) = \epsilon\theta H_1 g(X,PY)$ and $\epsilon_1(X) = 0$, we see that M is umbilical. This completes the proof. □

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References

[1] C. Atindogbe and K. L. Duggal, Conformal screen on lightlike hypersurfaces, *Int. J. Pure Appl. Math.* **11** (2004), no. 4, 421–442.

[2] K. L. Duggal, On canonical screen for lightlike submanifolds of codimensional two, *Cent. Eur. J. Math.* **5** (2007), no. 4, 710–719.

[3] K. L. Duggal and A. Bejancu, Lightlike submanifolds of codimensional two, *Math. J. Toyama Univ.* **15** (1992), 59–82.

[4] K. L. Duggal and A. Bejancu, *Lightlike submainfolds of semi-Riemannian mainfolds and applications*, Kluwer Academic Publishers, Dordrecht, 1996.

[5] K. L. Duggal and D. H. Jin, Half lightlike submanifolds of codimensional 2, *Math. J. Toyama Univ.* **22** (1999), 121–161.

[6] K. L. Duggal and D. H. Jin, *Null curves and hypersurfaces of semi-Riemannian manifolds*, World Scientific, Singapore, 2007.

[7] K. L. Duggal and B. Sahin, Screen conformal half-lightlike submanifolds, *Int. J. Math. Math. Sci.* **68** (2004), 3737–3753.

[8] K. L. Duggal and B. Sahin, *Differential geometry of lightlike submanifolds*, Birkhauser Veralag AG, Basel, Boston, Berlin, 2010.

[9] D. H. Jin, Einstein half lightlike submanifold with a Killing co-screen distribution, *Honam Math. J.* **30** (2008), no. 3, 487–504.
[10] D. H. Jin, Geometry of half lightlike submanifolds of a semi-Riemannian space form with a semi-symmetric metric connection, *J. Chungcheong Math. Soc.* **24** (2011),769–780.
[11] D. N. Kupeli, *Singular semi-Riemnnian Geometry*, Mathematics and Its Applications, Kluwer Academic Publishers, Dordrecht, 1996.
[12] B. O'Neill, *Semi-Riemannian Geometry with Application to Relativity*, Academic Press, New York, 1983.
[13] Y. N. Wang and X. M. Liu, Co-screen conformal half lightlike submanifolds, *Tamkang J. Math.* **44** (2013), no. 4, 431–444.
[14] W. J. Wang and X. M. Liu, On parallelism of ascreen half lightlike submanifolds of indefinite cosymplectic manifolds, *Int. Electron. J. Geom.* **6** (2013), no. 2, 70–78.

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