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The best memorization of the texts on several bands

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Abstract. The article contains an inedited proof of the best memorization of the texts on several bands.

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In memory of Nicolae Ţăndăreanu (1947-2013)

A *positioning* P on m bands of n texts, having some lengths L_1, ..., L_n, corresponds to a partition

$$\{1,...,n\} = P_1 \cup P_2 \cup ... \cup P_m,$$

where P_i is the set of the indices of the memorized texts on the i band, $1 \leq i \leq m$. We define

$$\varphi(P) = \sum_{i=1}^{m} \varphi\left(P_i\right),$$

where $\varphi\left(P_i\right)$ is calculated as for one band [1], that is

$$\varphi(P_i) = \sum_{k=1}^{n_i} \left(n_i - k + 1\right) L_{P_k^{(i)}},$$

where n_i is the number of texts memorized on the i band, while $L_{P_1^{(i)}}$, ..., $L_{P_{n_i}^{(i)}}$ are the lengths of the memorized texts on the i band.

We are looking for a positioning P which minimizes the value $\varphi(P)$.

If the number n of the texts is smaller or equal to the number m of bands, it is normal to memorize the texts one by one on each available band, to retrieve them as quickly as possible.

If the number n of the texts is greater than the number m of the bands, we proceed as follow. The first text is memorized on the first band. The second text is memorized on the second band. We proceed in the same way until the m-th text which is memorized on the m-th band, then the $(m+1)$-th text is memorized on the first band, immediately after the text which was memorized on the first band. We continue in the same way until the last text.

Therefore the i-th text will stay on the $r+1$ band, after the existing texts on this band, where $i-1 = mq+r$, $0 \leq r < m$; therefore $1 \leq r+1 \leq m$ and $r+1 = i - \left\lfloor \frac{i-1}{m} \right\rfloor m$, because $q \leq \frac{i-1}{m} < q+1$. Thus $q = \left\lfloor \frac{i-1}{m} \right\rfloor$, ($q$ is the integer part of the quotient between $i-1$ and m). On the same band, after the i-th text, the texts having the order numbers $i+m$, $i+2m$, ..., will be placed. Therefore after the i-th text, other t texts will be placed on the $r+1$ band, where the number t satisfies the inequalities

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$i + tm \le n < i + (t+1)m$; thus $tm \le n - i < (t+1)m$, or $t \le \frac{n-i}{m} < t+1$, therefore $t = \left\lfloor \frac{n-i}{m} \right\rfloor$.

To obtain the best positioning of n texts on m bands, the Greedy method suggests the following strategy: the texts are memorized in growing order of their lengths, the first text being memorized on the first band, each text staying on the next band on which the preceding text stays, from the last band returning to the first band.

Theorem 1. *If $L_1 \le L_2 \le ... \le L_n$, then the strategy suggested by the Greedy method directs to the best memorization of the texts on several bands.*

Proof. Let P be a positioning of n texts on m bands. Let n_i be the number of the texts staying on the i band and let n_j be the number of the texts staying on j band.

If $n_i > n_j$, moving the first text from the i band on the j band, in front of the texts staying on the j band, we obtain a better P' positioning, namely $\varphi(P') \le \varphi(P)$. In fact,

$$\begin{aligned}
\varphi(P_i) &= \sum_{k=1}^{n_i} (n_i - k + 1)\, L_{P_k^{(i)}} = n_i L_{P_1^{(i)}} + \sum_{k=2}^{n_i} (n_i - k + 1)\, L_{P_k^{(i)}} \\
&= n_i L_{P_1^{(i)}} + \sum_{k=1}^{n_i - 1} (n_i - 1 - k + 1)\, L_{P_{k+1}^{(i)}} \\
&= n_i L_{P_1^{(i)}} + \varphi(P_i'),
\end{aligned}$$

and

$$\begin{aligned}
\varphi(P_j) &= \sum_{k=1}^{n_j} (n_j - k + 1)\, L_{P_k^{(j)}} = \sum_{k=2}^{n_j+1} (n_j + 1 - k + 1)\, L_{P_{k-1}^{(j)}} \\
&= -(n_j + 1) L_{P_1^{(j)}} + \sum_{k=1}^{n_j+1} (n_j + 1 - k + 1)\, L_{P_{k-1}^{(j)}} \\
&= -(n_j + 1) L_{P_1^{(j)}} + \varphi(P_j'),
\end{aligned}$$

having $L_{P_0^{(j)}} = L_{P_1^{(i)}}$ and $P_i \cup P_j = P_i' \cup P_j'$, P_i' being obtained from P_i by removing the first text from the i band, and P_j' being obtained by adding this text on the j band, in front of the texts existing on j band.

Therefore $\varphi(P_i) + \varphi(P_j) = [n_i - (n_j + 1)] L_{P_1^{(i)}} + \varphi(P_i') + \varphi(P_j') \ge \varphi(P_i') + \varphi(P_j')$, because $n_i > n_j$, therefore $n_i \ge n_j + 1$, thus $n_i - (n_j + 1) \ge 0$.

If $n \le m$ it results, according to the preceding presentation, that the best positioning corresponds to the situation when, on most n bands, it is memorized only one text.

If $n > m$ it results, according to the preceding presentation, that in the case of the best positioning, no band can not be unoccupied, because if $n_j = 0$ and $n_i \ge 2$, we go to $n_j' = 1$, $n_i' = n_i - 1$ and we obtain a better positioning.

If $m|n$ (that means m divides n), in the case when $n > m$, according to the preceding presentation, we obtain a better positioning if on each of the m bands the same number of texts is memorized.

If m does not divide n, we have $n = mq + r$, $0 < r < m$, and, according to the preceding presentation, it is memorized on r first bands a number greater with 1 of texts than the number of texts memorized on each of the another bands and we obtain a better positioning. If $n_j = n_i + 1$, we have

$$\varphi(P_i) = \sum_{k=1}^{n_i} (n_i - k + 1)\, L_{P_k^{(i)}}$$

and

$$\begin{aligned}\varphi(P_j) &= \sum_{k=1}^{n_j} (n_j - k + 1)\, L_{P_k^{(j)}} = \sum_{k=1}^{n_i+1} (n_i + 1 - k + 1)\, L_{P_k^{(j)}} \\ &= \sum_{k=0}^{n_i} (n_i - k + 1)\, L_{P_{k+1}^{(j)}}.\end{aligned}$$

We will understand by the k column the stream of the texts which correspond to the coefficient $n_i - k + 1$ on each band. It is clear that if we permute the texts existing on the k column, then the value of $\varphi(P) = \sum_{i=1}^{m} \varphi(P_i)$, does not change.

The smallest length text may be brought on the first band by permutation of the existing texts on the k column, if it is not initially on this band. Then it may be brought on the first place on the first band, because the value $\varphi(P)$ decreases or remains the same, according to what is known from the memorization of the texts on only one band. Proceeding in the same way with the text, having the next length value, to bring it on the second band and then on the first place on the second band, the value $\varphi(P)$ decreases or remains the same. Continuing in the same way, we obtain the best memorization of the texts on several bands, according to the Greedy strategy, because the value $\varphi(P)$ will be the smallest. □

Thus we gave a proof for the optimum strategy of memorization of texts on several bands, inedited compared to the ones in [1].

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