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Simulation of flow discharge on Danube River

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ABSTRACT. River flow predictions are needed in many water resources management. This paper presents the results of a numerical method based on Saint-Venant equations to predict flood events on the Danube sector between Iron Gates I and Iron Gates II. A McCormack scheme is used in conjunction with time step control and adaptive roughness coefficient based on water depth and discharge ratio in order to simulate rapid changes of discharge at Iron Gates I and estimate the pick flow at Iron Gates II.

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1. Introduction

Numerical modeling of floods has made major advances in recent decades as a result of increased computational power and the availability of rich and detailed remotely sensed data sources. From the class of all possible inundation models, the optimum set for flood risk management depends on which processes are important for the flood event and the task at hand (e.g. real time forecasting, planning, prediction etc.). In this context a flood model consists of processes that are perceived to be relevant to the application and may be tested by comparison to field data.

The flood wave propagation in natural rivers or, more generally, unsteady flow in open channels has been the subject of numerous studies, ex. Moussa and Bockuillon [9], Fourar and Lahbari [4] and Ngo-Duc et al. [10].

The objective of this paper is to present a hydrodynamic model for simulation the behavior of Danube river during the flooding phases. We consider the river sector between the hydropowers Iron Gates I and Iron Gates II.

Taking into account the variable operation of the system Iron Gates I there is a need for a safety evacuation program in order to prevent flooding in downstream areas. It is also necessary to estimate the time when the wave with the maximum discharge reaches Iron Gates II. Such an operation model will take into account the morphology of Danube river between the two energy systems to hourly determine inflow at Iron Gates II by outflow from Iron Gates I.

In the following we consider the case of an unsteady flow with possible rapid changes in water discharge and depth. From our model point of view, the systems Iron Gates I and II act as sharp discontinuities. We have considered these hydraulic structures as passive static controllers acting at the boundaries of the domain. Our approach is not an algebraic one (Litrico and Fromion [8], p. 159-187). We treat the rapid changes of the boundaries conditions from

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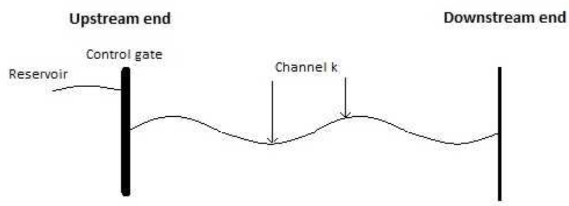

FIGURE 1. 1D Geometry of Danube sector.

upstream end by using the Saint-Venant equations which are known as the governing equations for unsteady flow in open channels. To complete the model geometry (Figure 1) an underflow gate at upstream end (Iron Gates I) is considered in order to simulate the input flow discharge. The flow variables (discharge and water depth) are approximated along the considered river sector in order to determine the maximum values and corresponding time at downstream end (Iron Gates II).

To take into account the rapid changes of inflow variables at upstream end over time (direct consequence of the control at upstream end) we use an adaptive roughness coefficient based on water depth and discharge ratio. Aspects of this approach type are also present in other works, e.g. Ngo-Duc et al. [10], Riggs [12], Lai Sai et al. [7] and Dingman and Sharma [2]. Limitations related to assumption of constant roughness coefficient are reported by Ngo-Duc et al. [10]. Riggs [12] and respectively Lai Sai et al. [7] presented some aspects for variation of the roughness coefficient in relation to water depth and discharge ratio. Ngo-Duc et al. [10] proposed a formula for evaluation of the roughness coefficient and tested a new global river routing model, i.e. TRIP 2.0.

The next sections are organized as follows. In Section 2 we present the computational model and in Section 3 we discuss the tests results.

2. The hydrodynamic model

The behavior of the considered Danube sector is simulated by a suite of 9 open channels flow in order to determine discharge and water depth at specific locations along the stream. Their number depends on the variability of the cross-section of the main channel of the river. An open channel is bounded by two cross-sections at specified locations along the river. Each channel is considered rectangular. The inflow to the junction must equal the outflow from it (continuity condition).

The flood routing problem for a channel is defined as: given channel characteristics (length, width, slope, etc.), flood discharge (Q) and water depth (y) at upstream section, determine the flood discharge and water depth at downstream section. This is same as solving for temporal and spatial variations of Q and y given the channel characteristics, initial conditions (Q and y at all points in the channel at $t = 0$) and boundary conditions (Q and y variation at $x = 0$ for all t).

In order to describe temporal variation of channel variables we make the assumptions :

- the channel bottom slope is small in the concerned river sector,
- the velocity is uniform within a cross section,

- the channel is a rectangular prism,
- there is no lateral inflow or outflow,
- there are rapid changes in water discharge and depth at upstream section over time (rapidly varying flow).

2.1. Methodology. In order to practical assist the flood management along a channel, our model require the prediction of water depth and discharge at particular points. The flow processes of interest are one-dimensional in the down-valley direction. An one-dimensional model may therefore be used to represent the flow.

This kind of dynamics is governed by the well-known Saint-Venant equations (Litrico and Fromion [8], Reidar and Olsen [11]), the *continuity equation*,

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

and *momentum equation*,

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x} - S_0\right) + gAS_f = 0. \tag{2}$$

Here x is the longitudinal abscissa, t is the time, $Q = Q(x,t)$ is the discharge (m^3/s), $A = A(x,t)$ is the cross section area (m^2), S_0 is the friction term due to the bed's slop, S_f is the friction term due to the bed's roughness and g is the gravitational acceleration. In order to calculate S_f, the *Manning formula* can be used (Litrico and Fromion [8], Reidar and Olsen [11]),

$$V = \frac{1}{n} R^{\frac{2}{3}} {S_f}^{\frac{1}{2}}, \tag{3}$$

where $V = V(x,t)$ is the velocity of the flow (m/s), $R = \frac{A}{P}$ is the hydraulic radius (m), n is the *Manning roughness coefficient* and P is the watted perimeter (m).

The equations (1)+(2) can be written in the vector form,

$$U_t + F\left(U\right)_x = R\left(U\right), \tag{4}$$

where $U = (A,Q)^t$ and $F\left(U\right) = \left(A, \frac{Q^2}{A} + gAy\right)^t$, $R\left(U\right) = \left(0, gA\left(S_0 - S_f\right)\right)^t$.

A variant of *McCormack* scheme (Garcia-Navarro et al. [5]) was used to numerically solve the equation (4),

$$\begin{cases} {U_i}^P = {U_i}^n - \frac{\triangle t_n}{\triangle x}\left({F_{i+1}}^n - {F_i}^n\right) + \triangle t_n {R_i}^n \\ {U_i}^C = {U_i}^n - \frac{\triangle t_n}{\triangle x}\left({F_i}^P - {F_{i-1}}^P\right) + \triangle t_n {R_i}^P. \end{cases} \tag{5}$$

In the previous equations i denote the i^{th} cell in x direction and n denote the time step. The first equation is the predictor term and the second equation is the corrector term. The final update is given by,

$${U_i}^{n+1} = \frac{1}{2}\left({U_i}^P + {U_i}^C\right). \tag{6}$$

2.2. Computation aspects. The stability of the numerical process (5)+(6) will be carried out by automatic time step control. Two conditions must be fulfilled, i.e. the *Courant-Friedrichs-Lewy* (CFL) condition for one-dimensional case,

$$C = V\frac{\triangle t_n}{\triangle x} \le C_{max}, \tag{7}$$

where $C_{max} = 1$ is used, and the distance ratio of two consecutive approximation points ${U_i}^n, {U_i}^{n+1}$,

$$e_i{}^n = \frac{|{U_i}^{n+1} - {U_i}^n|}{|{U_i}^{n+1}|} < \varepsilon. \tag{8}$$

If both conditions are met (the second, for all cells in x direction), then adjust the time step smoothly,

$$\triangle t_{n+1} = \triangle t_n \cdot \left(\frac{e_{n-1}}{e_n}\right)^{0.075} \cdot \left(\frac{\varepsilon}{e_n}\right)^{0.150}. \tag{9}$$

In the previous equation, e_n denotes the average error for all cells in x direction, from time step n.

If at least one of the conditions (7), (8) is not achieved, reject the point ${U_i}^{n+1}$ and repeat the step with

$$\triangle t_n \Leftarrow \triangle t_n \cdot \frac{\varepsilon}{e_n}. \tag{10}$$

In most hydraulic studies, the Manning roughness coefficient n from equation (3) is supposed to be globally constant. Since values of n vary between 0.015 and 0.07 in natural streams for flows less than bankfull discharge and reach up to 0.25 for overbank flow, the assumption of a global constant roughness coefficient would contain limitations (Ngo-Duc et al. [10]).

In order to respect the sharp variation of boundary conditions from upstream section of a channel we have used an adaptive roughness coefficient which is based on the water depth and ratio of two consecutive input discharges.

A correlation between the roughness coefficient and the water surface was proposed by Riggs [12]. A similar approach is also mentioned by Lai Sai et al. [7] who present the variation of the roughness coefficient with respect to discharge ratio for three equatorial rivers. The roughness coefficient decreases in main channel as discharge ratio increases. For a gravel-bed river, Kim et al. [6] estimated the roughness coefficient by field measurement and compared with values determined using some empirical methods. One of the conclusions was that estimated roughness coefficient decreases with increasing discharge and remains constant after discharge exceeds a certain value. Similar result is mentioned by Te-Hsing et al. [13].

Dingam and Sharma [2] proposed an equation to determine the river discharge as function of the cross section area, hydraulic radius and channel slope,

$$Q = 1.564 \cdot A^{1.173} \cdot R^{0.400} \cdot {S_0}^{-0.0543 \cdot logS_0}. \tag{11}$$

From equations (3) and (11), [10] derived a formula for evaluation of the roughness coefficient,

$$n = \frac{1}{1.1564} \cdot A^{-0.173} \cdot R^{0.267} \cdot {S_0}^{0.5+0.0543 \cdot logS_0}. \tag{12}$$

From the equations (11) and (12) appears idea of adjusted roughness coefficient to current values of the cross section area and hydraulic radius. To take into account rapid changes of the input flow (i.e. rapid changes of discharge and depth on channel upstream section) and following above results we have used an adaptive roughness coefficient based on the equation (12) but using our own calculation scheme. The next two equations are calibrated after approx. 600 tests using real field data, steady state and flood events of Danube river from downstream Iron Gates I to upstream Iron Gates II. On beginning of computing process the roughness coefficient is initialized by the expression (12) modified as,

$$n = 1.1225 \cdot A^{-0.173} \cdot R^{0.267} \cdot {S_0}^{0.5+0.0543 \cdot logS_0}, \tag{13}$$

where A and R are calculated with starting data. Each time we have new input data on upstream section of a channel, the roughness coefficient n is updated, i.e.

$$n_{current} \Leftarrow n_{prev} - \frac{Q_{i+1} - Q_i}{Q_{i+1}} \cdot \left(1.1225 \cdot A^{-0.173} \cdot R^{0.267} \cdot S_0{}^{0.5+0.0543 \cdot logS_0}\right). \quad (14)$$

Here $\frac{Q_{i+1}-Q_i}{Q_{i+1}}$ is the ratio of two consecutive discharges. In (13) and (14) the cross-section area and hydraulic radius can be expressed as functions of water depth $y(t)$, i.e. $A = A(t) = B \cdot y(t)$, $R = R(t) = \frac{B \cdot y(t)}{2 \cdot y(t)+B}$ where B is the channel width.

3. Model testing

3.1. Boundary and initial conditions. In problems of flood routing, the aim is to predict the level or/and the flow at downstream end of the channel when given the level or/and the flow at the upstream end (Dooge and Napiorkowski [3]). The hydrodynamic model is capable to realize simulations to analyze the behavior of discharge and water depth when the flood wave comes into each channel as function of time.

To complete the equations (1)+(2) we need to introduce boundary and initial conditions for each channel.

In our model, boundary conditions consist of discharge and water depth at upstream section at each hour, $(Q(0,t), y(0,t))$. Boundary conditions for upstream section of the channel 1 are historical data from downstream Iron Gates I. Our computing process starts from hour 0 of the simulated period. The boundary conditions of a channel are the discharge and water depth at 23 hour of the previous day. Generally, the output values of channel k (i.e. solution values at downstream section) at each hour are boundary conditions of channel $k+1$. The output values of the model are solution values from downstream section of channel 9 and represent prediction values from upstream Iron Gates II.

The initial condition of a channel is given in terms of $(Q(x,0), y(x,0))$ for all computational nodes $x \in [0, L]$ where L is the channel length. In practice, initial conditions for a hydraulic model will be incompletely known and some additional assumptions will therefore be necessary (Woodhead and al. [14]). In our case, we did not have historical data for all sections along the considered Danube sector. Thus, the boundary conditions at time $t = 0$ are replicated as initial conditions for all channels.

3.2. Analysis of model results. The total length of the channels which depict the considered Danube sector is 88500 m. The longest channel has 35000 m and the shortest has 5000 m. The mean width of channels is 966 m. For each channel the discretization in x direction is at 1000 m.

In order to prove the validity of the model, the first test presents a hypothetical flood event along three days. The discharge from Iron Gates I is simulated by the equations Akbari and Firoozi [1]

$$\begin{cases} Q(t) = \frac{Q_p}{2} \cdot sin\left(\frac{\pi t}{T_p} - \frac{\pi}{2}\right) + \frac{Q_p}{2} + Q_b, for: 0 \le t \le T_p \\ Q(t) = \frac{Q_p}{2} \cdot cos\left(\pi \frac{t-T_p}{T_b-T_p}\right) + \frac{Q_p}{2} + Q_b, for: T_p < t \le T_b \\ Q(t) = Q_b, for: t > T_b \end{cases} \quad (15)$$

where Q_b is the base discharge, Q_p is the peak discharge, T_p is the time to peak discharge equal to 32 hours and T_b is the time base equal to 72 hours. In Figure 2 the discharge from Iron Gates I is plotted vs the inflow from Iron Gates II simulated by our model. We observe that the maximum propagation time of pick discharge is 3 hours. The corresponding values of the roughness coefficient for the channel 9 of our model are plotted in Figure 2. From

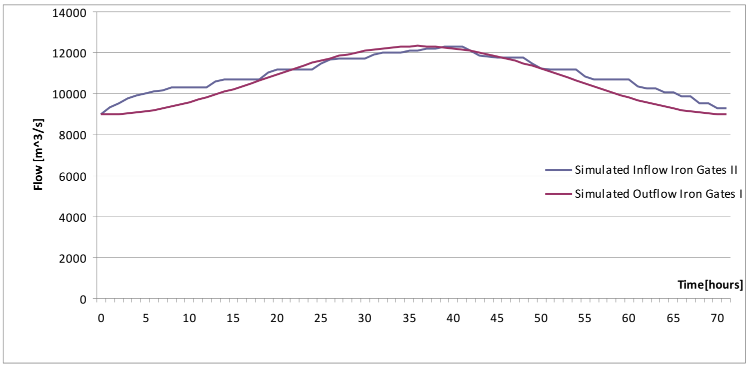

FIGURE 2. Simulated outflow at Iron Gates I vs simulated inflow at Iron Gates II.

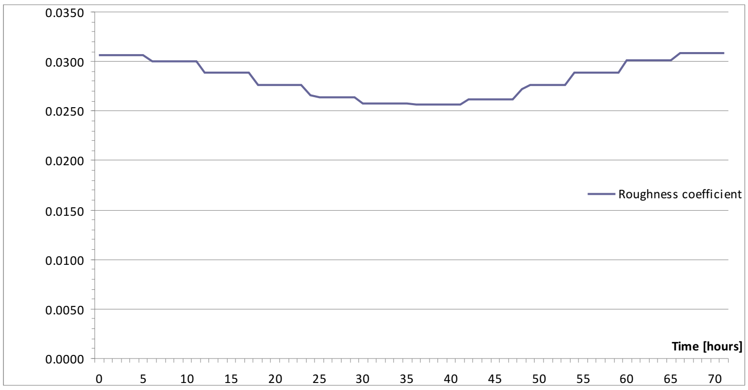

FIGURE 3. The roughness coefficient.

Figures 2 and 3 we observe that the roughness coefficient decreases as discharge increases and inversely.

To assess the performance of the model we make a comparison between the historical data and numerical results. We consider a period of four days with flood events and also we compare the historical outflow from Iron Gates I with the calculated inflow for Iron Gates II.

The initial condition from Iron Gates I was 8982.00 (m^3/s). On the first day (Figure 4) an increase to 10065.00 (m^3/s) is imposed at Iron Gates I. The real effect from Iron Gates II was measured 10097.00 (m^3/s). The calculated inflow for Iron Gates II was 10057.00 (m^3/s) and the propagation time was 3 hours. The percentage of the estimation error is 0.39%. The estimated roughness coefficient for the channel 9 ranges from 0.033594 at 0.030711.

On the second day (Figure 5), the real increase from Iron Gates I was 11409.00 (m^3/s) and from Iron Gates II was 11804.00 (m^3/s). The calculated inflow for Iron Gates II was 11438.00 (m^3/s). The percentage of the estimation error is 3.1%. The roughness coefficient ranges from 0.030711 at 0.027089.

On the third day (Figure 6), the real increase from Iron Gates I was 12323.00 (m^3/s) and from Iron Gates II was 12473.00 (m^3/s). At Iron Gates II, the calculated flow was 12399.00 (m^3/s). The roughness coefficient ranges from 0.027089 at 0.024811.

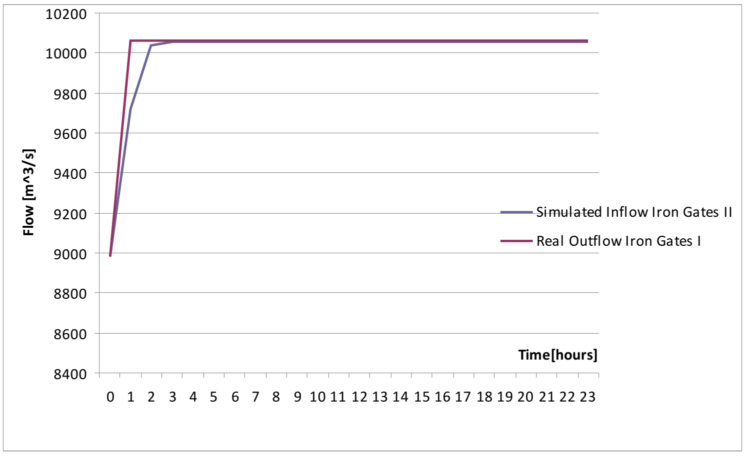

FIGURE 4. Day 1 - Flow variation with time at upstream and downstream ends of the model.

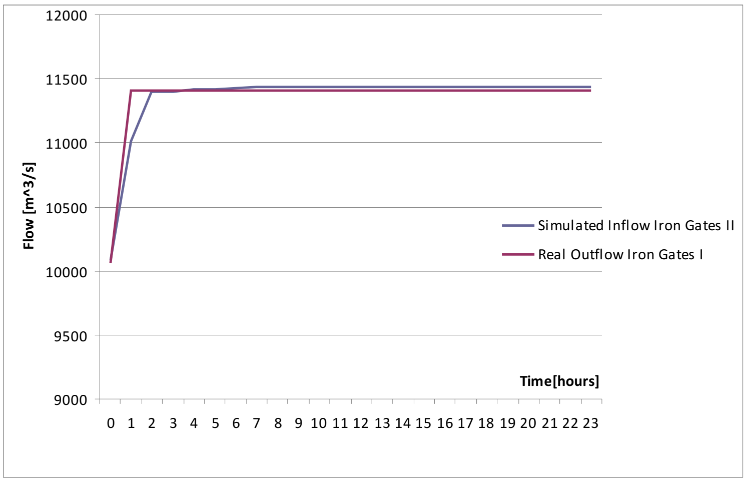

FIGURE 5. Day 2 - Flow variation with time at upstream and downstream ends of the model.

On the fourth day (Figure 7), a decrease from 12323.00 to 11260.00 (m^3/s) is observed at Iron Gates I. The real decrease from Iron Gates II was 11402.00 (m^3/s). For Iron Gates II the calculated decrease was from 12399.00 (m^3/s) to 11150.00 (m^3/s). The percentage of the estimation error is 2.19%. The roughness coefficient ranges from 0.024811 at 0.027705.

4. Conclusion

In this research the Danube sector between Iron Gates I and Iron Gates II is approximated by a suite of 9 open rectangular channels. The solution based on Saint-Venant equations for flood routing problem is presented. Time step control and adaptive roughness coefficient are used. From the results of the numerical method it can be seen that there is a reasonably good matching between the numerical results and the historical data. The maximum percentage of

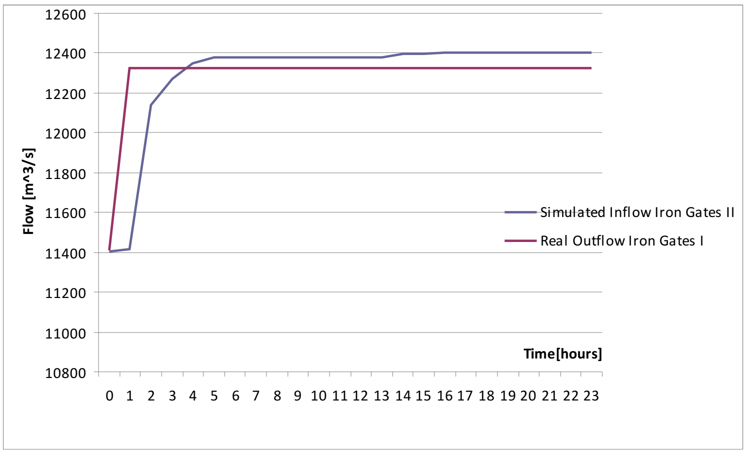

FIGURE 6. Day 3 - Flow variation with time at upstream and downstream ends of the model.

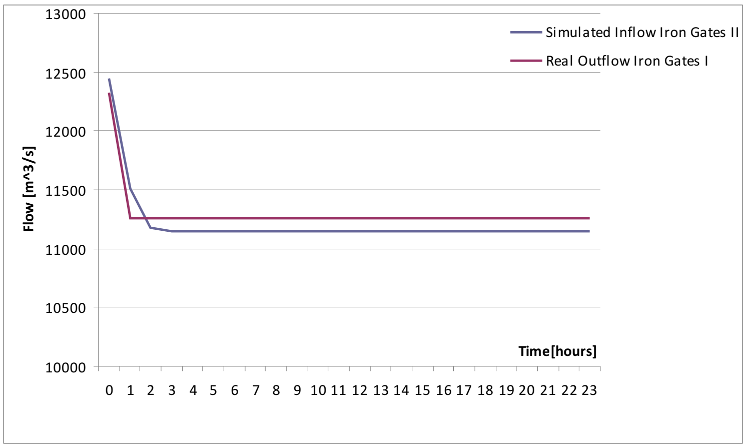

FIGURE 7. Day 4 - Flow variation with time at upstream and downstream ends of the model.

the estimation error is 3.1%. The results show that in the numerical model the arrival time of peak flow from Iron Gates II is maximum three hours later than the discharge from Iron Gates I.

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