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Blind source separation using measure on copulas

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Abstract. The paper introduces a novel BSS algorithm for instantaneous mixtures of both independent and dependent sources. This approach is based on the minimization of Kullback-Leibler divergence between copula densities. This latter takes advantage of copulas to model the dependency structure of the source components. The new algorithm can efficiently achieve good separation standard BSS methods fail. Simulation results are presented showing the convergence and the efficiency of the proposed algorithms.

Key words and phrases. Blind source separation, instantaneous mixtures, Copulas, Kullbak-Leibler divergence.

1. Introduction

The blind source separation problem is a fundamental issue in applications of many different fields such as signal and image processing, medical data analysis, communications, etc. The BSS aims to recover unknown source signals from a set of observations which are unknown linear mixture of the sources. It was introduced and formulated by Bernard Ans, Jeanny Herault and Christian Jutten [1] since the 80's, describing a biological problem. In order to separate the data set, different assumptions on the sources have to be made. The most common assumptions are statistical independence of the sources and the condition that at most one of the components is gaussian, which leads to the field of Independent Component Analysis (ICA), see for instance [2]. Many methods of BSS have been proposed [3, 4, 5, 6], using second or higher order statistics [7], maximizing likelihood [8], maximizing nongaussianity [9], minimizing the mutual information [10], φ-divergences [11], etc. An interesting overview of the problem can be found in [12]. Recently, it has been shown in [13] that, based on copula without the assumption of the independence of the sources, we can still determine the sources (up to scale and permutation indeterminacies) of both independent and dependent sources components. In this paper, we use copulas to model the dependency structure of the source components, and we will focus on the criterion of modified Kullback-Leibler divergence, viewed as measure of difference between copulas, and we will use it to propose a new BSS approach that applies both in the standard case of independent source components, and in the non standard one of dependent source components. The method proceeds in two steps: the first one consists of spatial whitening and the second one consists to apply a series of Givens rotations, minimizing the estimate of the modified Kullback-Leibler divergence.

The outline of this paper goes like that: we briefly review the principle of BSS and its extensions in Section 2. The main conclusions of copula theory are briefly introduced with some of their fundamental properties and examples in Section 3. In Section 4, we describe the new model proposed for BSS, and in Section 5 we present

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some experimental results, in addition we compare our approach with some existing ones in the literature. Finally, we conclude the paper and give some further research directions.

2. Principle of BSS

BSS can be modeled as follows. Denoting $\boldsymbol{A}$ the mixing operator, the relationship between the observations and sources is

$$\boldsymbol{x}(t) := \boldsymbol{A}[\boldsymbol{s}(t)] + \boldsymbol{b}(t),\ t \in \mathbb{R}, \tag{1}$$

where $\boldsymbol{x}$ is a set of observations, $\boldsymbol{s}$ is a set of unknown sources, and $\boldsymbol{b}$ is an additive noise. In this paper, we consider the linear BSS model with instantaneous mixtures, the operator $\boldsymbol{A}$ corresponds then to a scalar matrix, and the additive noise is either considered as an additional set of sources, or it is reduced by applying some form of preprocessing [14]. We assume that the number of sources is equal to the number of observations. The model writes

$$\boldsymbol{x}(t) := \boldsymbol{A}\ \boldsymbol{s}(t),\ \forall t \in \mathbb{R}, \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^p$ represents the observed vector, $\boldsymbol{s} \in \mathbb{R}^p$ is the unknown vector of sources to be estimated, and $\boldsymbol{A}$ is the unknown mixing matrix. The goal of BSS, is therefore to estimate the unknown sources $\boldsymbol{s}(t)$ from the set of observed mixtures $\boldsymbol{x}(t)$. The estimation is performed with no prior information about either the sources or the mixing process $\boldsymbol{A} \in \mathbb{R}^{p\times p}$. Specific restrictions are made on the mixing model and the source signals in order to limit the generality. The separating system is defined by

$$\boldsymbol{y}(t) := \boldsymbol{B}\ \boldsymbol{x}(t),\ \forall t \in \mathbb{R}. \tag{3}$$

The vector $\boldsymbol{y}(t) \in \mathbb{R}^p$ is the output signal vector (estimated source vector) and $\boldsymbol{B} \in \mathbb{R}^{p\times p}$ is called the separating operator. In other words, the problem is to obtain an estimator $\widehat{\boldsymbol{B}}$ closing to the ideal solution $\boldsymbol{A}^{-1}$ using only the observation $\boldsymbol{x}(t)$, which leads to accurate estimation of the source $\boldsymbol{s}(t)$

$$\widehat{\boldsymbol{y}}(t) := \widehat{\boldsymbol{B}}\ \boldsymbol{x}(t) \simeq \widehat{\boldsymbol{s}}(t). \tag{4}$$

3. Recalls on copula

Let's recall some elementary facts about copulas. Let $\boldsymbol{Z} := (Z_1, \ldots, Z_p)^\top \in \mathbb{R}^p$, $p \geq 1$, a random vector, with cumulative distribution function (c.d.f.)

$$F_{\boldsymbol{Z}}(\cdot) :\ \boldsymbol{z} \in \mathbb{R}^p \mapsto F_{\boldsymbol{Z}}(\boldsymbol{z}) := F_{\boldsymbol{Z}}(z_1, \ldots, z_p) := \mathbb{P}(Z_1 \leq z_1, \ldots, Z_p \leq z_p), \tag{5}$$

and continuous marginal functions

$$F_{Z_i}(\cdot) :\ z_i \in \mathbb{R} \mapsto F_{Z_i}(z_i) := \mathbb{P}(Z_i \leq y_i),\ \forall i = 1, \ldots, p. \tag{6}$$

The following characterization theorem of Sklar [15] shows that there exists a unique p-variate function called copula that ties the joint and the margins together.

Theorem 3.1. *Given $\boldsymbol{Z} := (Z_1, \ldots, Z_p)^\top$ a random vector, with joint distribution function $F_{\boldsymbol{Z}}$ and continuous distribution margins $F_{Z_1}, \ldots, F_{Z_p}$. Then there exists a unique copula $\mathbb{C}$ such that for all $\boldsymbol{z} := (z_1, \ldots, z_p)^\top \in \mathbb{R}^p$,*

$$F_{\boldsymbol{Z}}(\boldsymbol{z}) := \mathbb{C}_{\boldsymbol{Z}}(F_{Z_1}(z_1), \ldots, F_{Z_p}(z_p)). \tag{7}$$

A copula function $\mathbb{C}_{\boldsymbol{Z}}$ of $\boldsymbol{z}$ is itself a multivariate probability distribution function $\mathbb{C} : [0,1]^p \longrightarrow [0,1]$, with uniform margins on $[0,1]$. Recall that the copula density $\boldsymbol{c}(\cdot)$, if it exists, is the componentwise derivative of $\mathbb{C}$

$$\boldsymbol{c}(\boldsymbol{u}) := \frac{\partial^p \mathbb{C}(\boldsymbol{u})}{\partial u_1 \dots \partial u_p},\ \forall \boldsymbol{u} \in [0,1]^p. \tag{8}$$

If the components $Z_1, \dots, Z_p$ are statistically independent, then the corresponding copula writes

$$\mathbb{C}_\Pi(\boldsymbol{u}) := \prod_{i=1}^{p} u_i,\ \forall \boldsymbol{u} \in [0,1]^p. \tag{9}$$

It is called the copula of independence, and the independent copula density is the function taking the value one on $[0,1]^p$ and zero otherwise, namely,

$$\boldsymbol{c}_\Pi(\boldsymbol{u}) := 1_{[0,1]^p}(\boldsymbol{u}),\ \forall \boldsymbol{u} \in [0,1]^p. \tag{10}$$

Let $f_{\boldsymbol{Z}}(\cdot)$, if it exists, be the probability density on $\mathbb{R}^p$ of the random vector $\boldsymbol{Z} = (Z_1, \dots, Z_p)^\top$, and, respectively, $f_{Z_1}(\cdot), \dots, f_{Z_p}(\cdot)$, the marginal probability densities of the random variables $Z_1, \dots, Z_p$. Then, a straightforward computation shows that, for all $\boldsymbol{z} := (z_1, \dots, z_p)^\top \in \mathbb{R}^p$, we have

$$f_{\boldsymbol{Z}}(\boldsymbol{z}) = \prod_{i=1}^{p} f_{Z_i}(z_i) \boldsymbol{c}_{\boldsymbol{Z}}(F_{Z_1}(z_1), \dots, F_{Z_p}(z_p)). \tag{11}$$

As previously highlighted, copulas play an important role in the construction of multivariate d.f.'s. Therefore, several investigations have been carried out concerning the construction of different families of copulas and their properties. In the monographs by [16],[17], the reader may find detailed ingredients of the modeling theory as well as surveys of the commonly used semiparametric copulas.

4. The proposed approach

The discrete version of the original problem (2) writes

$$\boldsymbol{x}(n) := A\boldsymbol{s}(n),\ n = 1, \dots, N. \tag{12}$$

The source signals $\boldsymbol{s}(n)$, $n = 1, \dots, N$, will be considered as N copies of the random source vector $\boldsymbol{S}$, and then $\boldsymbol{x}(n)$, $\boldsymbol{y}(n) := \boldsymbol{B}\boldsymbol{x}(n)$, $n = 1, \dots, N$ are, respectively, N copies of the random source vector $\boldsymbol{X}$ and $\boldsymbol{Y} := \boldsymbol{B}\boldsymbol{X}$.

4.1. A separation procedure for independent sources.

Assume that the source components are independent. The mutual information of $\boldsymbol{Y}$ is defined by

$$MI(\boldsymbol{Y}) := \int_{\mathbb{R}^p} -\log \frac{\prod_{i=1}^{p} f_{Y_i}(y_i)}{f_{\boldsymbol{Y}}(\boldsymbol{y})} f_{\boldsymbol{Y}}(\boldsymbol{y})\, \mathrm{d}y_1, \dots, \mathrm{d}y_p. \tag{13}$$

It is called also the modified Kullbak-Leibler divergence (KL_m), between the product of the marginal densities and the joint density of the vector. Note also that $MI(\boldsymbol{Y}) := KL_m\left(\prod_{i=1}^{n} f_{Y_i}, f_{\boldsymbol{Y}}\right)$ is nonnegative and achieves its minimum value zero iff $f_{\boldsymbol{Y}}(.) = \prod_{i=1}^{p} f_{Y_i}(.)$ i.e., iff the components of the vector $\boldsymbol{Y}$ are statistically independent.

Using the relation (11), and applying the change variable formula for multiple integrals, we can show that $MI(\boldsymbol{Y})$ can be written via copula densities as

$$MI(\boldsymbol{Y}) := \int_{[0,1]^p} -\log\left(\frac{1}{\boldsymbol{c_Y}(\boldsymbol{u})}\right)\boldsymbol{c_Y}(\boldsymbol{u})\,\mathrm{d}\boldsymbol{u} =: KL_m\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right), \tag{14}$$

where $\boldsymbol{c_Y}$ is the density copula of $\boldsymbol{Y}$, and $\boldsymbol{c}_\Pi(\boldsymbol{u}) := 1_{[0,1]^p}(\boldsymbol{u})$ is the product copula density.

Moreover, the above criterion (14) can be written as follows:

$$KL_m\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right) := \mathbb{E}\left[\log\left(\frac{c_{\boldsymbol{Y}}(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))}{\boldsymbol{c}_\Pi(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))}\right)\right], \tag{15}$$

where $\mathbb{E}(.)$ denotes the mathematical expectation.

The modified Kullbak-Leibler divergence $KL_m\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right)$ is nonnegative and attains its minimum value zero at $\boldsymbol{B} = \boldsymbol{DPA}^{-1}$, where $\boldsymbol{D}$ and $\boldsymbol{P}$ are, respectively a diagonal and permutation matrix. Therefore, to achieve separation, the idea is to minimize some statistical estimate $\widehat{KL_m}\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right)$, of $KL_m\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right)$, constructed from the data $\boldsymbol{y}(1), \ldots, \boldsymbol{y}(n)$. The separation matrix is then estimated by

$$\widehat{\boldsymbol{B}} = \arg\min_{\boldsymbol{B}} \widehat{KL_m}\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right), \tag{16}$$

leading to the estimated source signals $\widehat{\boldsymbol{y}}(n) = \widehat{\boldsymbol{B}}\,\boldsymbol{x}(n), \quad n = 1, \ldots, N$. In view of (15), we propose to estimate the criterion $KL_m\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right)$ through

$$\widehat{KL_m}\left(\boldsymbol{c}_\Pi, \boldsymbol{c_Y}\right) := \frac{1}{N}\sum_{i=1}^{N} \log\left(\widehat{c}_{\boldsymbol{Y}}(\widehat{F}_{Y_1}(y_1(n)), \ldots, \widehat{F}_{Y_p}(y_p(n)))\right), \tag{17}$$

where

$$\widehat{c}_{\boldsymbol{Y}}(\boldsymbol{u}) := \frac{1}{NH_1\cdots H_p}\sum_{m=1}^{N}\prod_{j=1}^{p} k\left(\frac{\widehat{F}_{Y_j}(y_j(m)) - u_j}{H_j}\right), \forall \boldsymbol{u} \in [0,1]^p, \tag{18}$$

is the kernel estimate of the copula density $c_{\boldsymbol{Y}}(.)$, and $\widehat{F}_{Y_j}(x)$, $j = 1, \ldots, p$, is the smoothed estimate of the marginal distribution function $F_{Y_j}(x)$ of the random variable Y_j, at any real value $x \in \mathbb{R}$, defined by

$$\widehat{F}_{Y_j}(x) := \frac{1}{N}\sum_{m=1}^{N} K\left(\frac{y_j(m) - x}{h_j}\right), \forall j = 1, \ldots, p \tag{19}$$

where $K(.)$ is the primitive of a kernel $k(.)$, a symmetric centered probability density. In our forthcoming simulation study, we will take for the kernel $k(.)$ a standard Gaussian density. A more appropriate choice of the kernel $k(.)$, for estimating the copula density ,can be done according to [19], which copes with the boundary effect. The bandwidth parameters $H_1, \ldots, H_p$ and $h_1, \ldots, h_p$ in (18,19) will be chosen according to Silverman's rule of thumb, see [18], i.e., for all $j = 1, \ldots, p$, we take

$$\begin{cases} H_j = \left(\dfrac{4}{p+2}\right)^{\frac{1}{p+4}} N^{\frac{-1}{p+4}}\widehat{\Sigma}_j, \\ h_j = \left(\dfrac{4}{3}\right)^{\frac{1}{5}} N^{\frac{-1}{5}}\widehat{\sigma}_j, \end{cases} \tag{20}$$

where $\widehat{\Sigma}_j$ and $\widehat{\sigma}_j$ are, respectively, the empirical standard deviation of the data $\widehat{F}_{Y_j}(y_j(1)), \ldots, \widehat{F}_{Y_j}(y_j(N))$ and $y_j(1), \ldots, y_j(N)$.

In order to compute the estimate of the de-mixing matrix $\widehat{\boldsymbol{B}}$, our method proceeds in two steps: the first one consists of spatial whitening and the second one consists to apply a series of Givens rotations, minimizing the estimate of the KL_m-divergence. The whitened mixture vector $\boldsymbol{z}$ can be written as

$$\boldsymbol{z}(n) = \boldsymbol{W}\boldsymbol{x}(n),\ n = 1, \ldots, N, \tag{21}$$

where $\boldsymbol{W}$ is the whitening $p \times p$-matrix. Let $\boldsymbol{U}$ be a unitary $p \times p$- matrix, namely, the matrix $\boldsymbol{U}$ satisfying $\boldsymbol{U}\boldsymbol{U}^\top = \boldsymbol{I}_p$. It can be written as $\boldsymbol{U}(\theta) := \prod_{1 \le i < \le p} G(i, k, \theta_m)$, where $G(i, k, \theta_m)$ is the $p \times p$-matrix with entries

$$G(i,k,\theta_m)_{j,l} := \begin{cases} cos(\theta_m) & \text{if} \quad j = i, l = i \quad \text{or} \quad j = k, l = k; \\ sin(\theta_m) & \text{if} \quad j = i, l = k; \\ -sin(\theta_m) & \text{if} \quad j = k, l = i; \\ 1 & \text{if} \quad j = l; \\ 0 & \text{else,} \end{cases} \tag{22}$$

for all $1 \le j, l \le p$, and $\theta_m \in]-\pi/2, \pi/2[$, $m = 1, \ldots, p(p-1)/2$, are the rotation angles (the components of the vector θ). The estimated source signals take then the form $\boldsymbol{y}(n) = \boldsymbol{U}(\theta)\boldsymbol{z}(n),\ n = 1, \ldots, N$, and the separating matrix is $\boldsymbol{B} = \boldsymbol{U}(\theta)\boldsymbol{W}$. The estimate $\widehat{KL_m}(\boldsymbol{c}_\Pi, \boldsymbol{c}_{\boldsymbol{Y}})$, of $KL_m(\boldsymbol{c}_\Pi, \boldsymbol{c}_{\boldsymbol{Y}})$, can be seen as a function of the parameter vector θ. Let $\widehat{\theta} := \arg\min_{\boldsymbol{\theta}} \widehat{KL_m}(\boldsymbol{c}_\Pi, \boldsymbol{c}_{\boldsymbol{Y}})$ which can be computed by a descent gradient (in θ) algorithm. The de-mixing matrix is then estimated

$$\widehat{\boldsymbol{B}} = \boldsymbol{U}(\widehat{\theta})\boldsymbol{W}, \tag{23}$$

leading to the estimated source signals

$$\widehat{\boldsymbol{y}}(n) = \widehat{\boldsymbol{B}}\,\boldsymbol{x}(n) = \boldsymbol{U}(\widehat{\theta})\boldsymbol{W}\boldsymbol{x}(n), \quad n = 1, \ldots, N. \tag{24}$$

We summarize the above methodology in the following algorithm.

Algorithm 1 The separation algorithm for independent source components.

Data: the observed signals $\boldsymbol{x}(n),\ n = 1, \ldots, N$.
Result: the estimated sources $\widehat{\boldsymbol{y}}(n),\ n = 1, \ldots, N$.
Whitening and Initialization: $\boldsymbol{z}(n) := \boldsymbol{W}\boldsymbol{x}(n), \widehat{\boldsymbol{y}}_0(n) = \boldsymbol{U}(\widehat{\theta_0})\boldsymbol{z}(n)$. Given $\varepsilon > 0$ and $\mu > 0$.
Do: • **Update $\boldsymbol{\theta}$** and $\boldsymbol{y}$

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \mu \frac{d\widehat{KL_m}(\boldsymbol{c}_\Pi, \boldsymbol{c}_{\boldsymbol{y}})}{d\boldsymbol{\theta}}.$$

$$\boldsymbol{y}_{k+1}(n) = \boldsymbol{U}(\theta_{k+1})\boldsymbol{z}(n),\ n = 1, \ldots, N.$$

• **Until** $||\theta_{k+1} - \theta_k|| < \varepsilon$
$\widehat{\boldsymbol{y}}(n) = \boldsymbol{y}_{k+1}(n),\ n = 1, \ldots, N.$

4.2. A separation procedure for dependent sources.

In the case where the source components are dependent, we assume that we dispose of some prior information about the density copula of the random source vector $\boldsymbol{S}$. Note that this is possible for many practical problems, it can be done, from realizations of $\boldsymbol{S}$, by a model selection procedure in semiparametric copula density models $\{\boldsymbol{c}_\alpha(.); \alpha \in \Theta \subset \mathbb{R}^d\}$, typically indexed by a multivariate parameter α, see [20]. The parameter α can be estimated using maximum semiparametric likelihood, see [21]. We denote by $\widehat{\alpha}$, the obtained value of α and $c_{\widehat{\alpha}}(.)$ the copula density modeling the dependency structure of the source components. Obviously, since the source components are assumed to be dependent, $\boldsymbol{c}_{\widehat{\alpha}}(.)$ is different from the density copula of independence $\boldsymbol{c}_{\Pi}(.)$. Hence, we naturally replace in (15), $\boldsymbol{c}_{\Pi}$ by $\boldsymbol{c}_{\widehat{\alpha}}$, then we define the separating criterion $KL_m(\boldsymbol{c}_{\widehat{\alpha}}, \boldsymbol{c}_{\boldsymbol{Y}})$ by

$$KL_m(\boldsymbol{c}_{\widehat{\alpha}}, \boldsymbol{c}_{\boldsymbol{Y}}) := \mathbb{E}\left[\log\left(\frac{c_{\boldsymbol{Y}}(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))}{\boldsymbol{c}_{\widehat{\alpha}}(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))}\right)\right], \tag{25}$$

Moreover, we can show that $KL_m(\boldsymbol{c}_{\widehat{\alpha}}, \boldsymbol{c}_{\boldsymbol{Y}})$, is nonnegative and attains its minimum value zero at $\boldsymbol{B} = \boldsymbol{DPA}^{-1}$. The separation for dependent source components, is reached in

$$\widehat{\boldsymbol{B}} = \arg\min_{\boldsymbol{B}} \widehat{KL_m}(\boldsymbol{c}_{\widehat{\alpha}}, \boldsymbol{c}_{\boldsymbol{Y}}), \tag{26}$$

where

$$\widehat{KL_m}(\boldsymbol{c}_{\widehat{\alpha}}, \boldsymbol{c}_{\boldsymbol{Y}}) := \frac{1}{N}\sum_{i=1}^{N} \log\left(\frac{\widehat{c}_{\boldsymbol{Y}}(\widehat{F}_{Y_1}(y_1(n)), \ldots, \widehat{F}_{Y_p}(y_p(n)))}{\widehat{c}_{\widehat{\alpha}}(\widehat{F}_{Y_1}(y_1(n)), \ldots, \widehat{F}_{Y_p}(y_p(n)))}\right). \tag{27}$$

The estimates of copula density and the marginal distribution functions are defined as before. The solution $\widehat{\boldsymbol{B}}$ can be computed by a descent gradient (in θ) algorithm. The estimated source signals are by $\widehat{\boldsymbol{y}}(n) = \widehat{\boldsymbol{B}}\boldsymbol{x}(n),\ n = 1, \ldots, N$; see Algorithm 2.

Algorithm 2 The separation algorithm for dependent source components.

Data: the observed signals $\boldsymbol{x}(n),\ n = 1, \ldots, N$.
Result: the estimated sources $\widehat{\boldsymbol{y}}(n),\ n = 1, \ldots, N$.
Whitening and Initialization: $\boldsymbol{z}(n) := \boldsymbol{W}\boldsymbol{x}(n), \widehat{\boldsymbol{y}}_0(n) = \boldsymbol{U}(\widehat{\theta_0})\boldsymbol{z}(n)$. Given $\varepsilon > 0$ and $\mu > 0$.
Do: • **Update $\boldsymbol{\theta}$** and $\boldsymbol{y}$

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \mu\frac{d\widehat{KL_m}(\boldsymbol{c}_{\widehat{\alpha}}, \boldsymbol{c}_{\boldsymbol{y}})}{d\boldsymbol{\theta}}.$$

$\boldsymbol{y}_{k+1}(n) = \boldsymbol{U}(\theta_{k+1})\boldsymbol{z}(n),\ n = 1, \ldots, N.$

• **Until** $||\theta_{k+1} - \theta_k|| < \varepsilon$

$\widehat{\boldsymbol{y}}(n) = \boldsymbol{y}_{k+1}(n),\ n = 1, \ldots, N.$

5. Simulation results

In this section, we present representative simulation results for the proposed method. We will limit ourselves to the case of 2 mixtures 2 sources. We start by illustrating the performance of BSS-copula with a simple experiment on independent sources. Then we turn to use BSS-copula to separate dependent sources. The results will be compared with the classical independent MI criterion, see, [10], for the same data. The 2 sources are mixed with the matrix $\boldsymbol{A} := [1\ 0.8; 0.8\ 1]$. A Gaussian noise was also added to the mixtures. The gradient descent parameter is taken $\mu = 0.1$. And the number of samples is $N = 2000$, and all simulations are repeated 20 times. The accuracy of source estimation is evaluated through the signal-noise-ratio (SNR), defined by

$$SNR_i := 10\log_{10}\left(\frac{\sum_{k=1}^{N}\widehat{y}_i(k)^2}{\sum_{k=1}^{N}(\widehat{y}_i(k)^2\ |_{s_i(k)=0})}\right),\ i = 1, 2. \tag{28}$$

5.1. Independent source components:

In this experiment, we consider two mixed signals of two kinds of sample sources: uniform i.i.d with independent components Figure 1; i.i.d sources with independent components drawn from the 4-ASK (Amplitude Shift Keying) alphabet Figure 2. Ve observe from Figure 1 and Figure 2, that the proposed method (Algorithm 1) gives good results for the standard case of independent component sources.

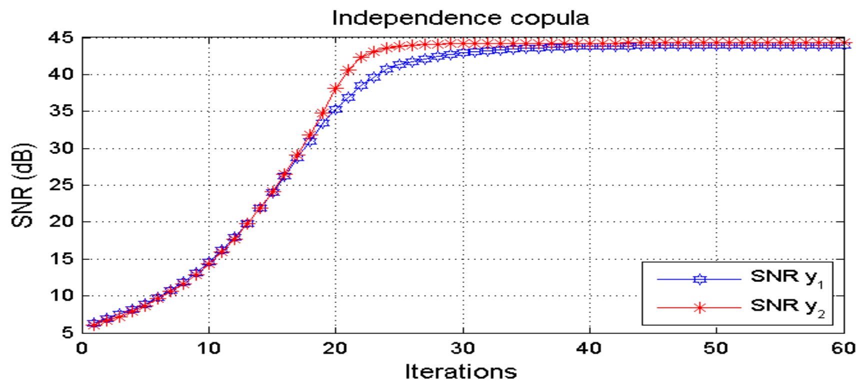

FIGURE 1. Average output SNRs versus iteration number : Uniform independent sources.

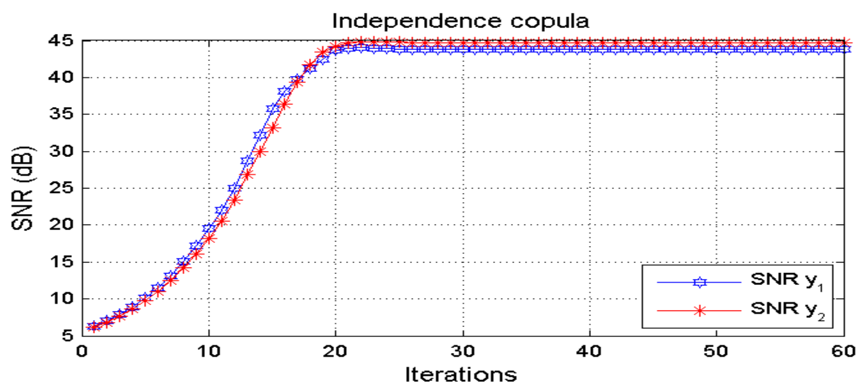

FIGURE 2. Average output SNRs versus iteration number : ASK independent sources.

Figure 3 shows the criterion value vs iterations. We can see that our criterion converges to 0 when the separation is achieved.

5.2. Dependent source components:

In this subsection we show the capability of the proposed method (Algorithm 2 for dependent sources) to successfully separate two dependent mixed signals, we dealt with instantaneous mixtures of four kinds of sample sources:

1 i.i.d.(with uniform marginals) vector sources with dependent components generated from Ali-Mikhail-Haq (AMH) copula with $\widehat{\theta} = 0.8$.

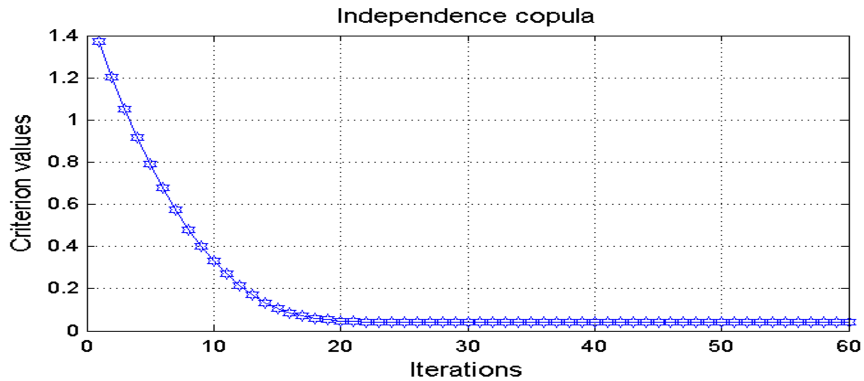

FIGURE 3. The criterion value vs iterations : uniform independent sources.

2 i.i.d.(binary phase-shift keying(BPSK)-marginals) vector sources with dependent components generated from Fairlie-Gumbel-Morgenstern (FGM) copula with $\widehat{\theta} = 0.85$.
3 i.i.d.(with uniform marginals) vector sources with dependent components generated from Clayton copula with $\widehat{\theta} = 2.5$.
4 i.i.d.(with binary phase-shift keying(BPSK)-marginals) vector sources with dependent components generated from Frank copula with $\widehat{\theta} = 3$.

In Figures 4- 7, we have shown the SNRs for each kind of sample sources. It can be seen from the simulations that the proposed method is able to separate, with good performance, the mixtures of dependent source components.

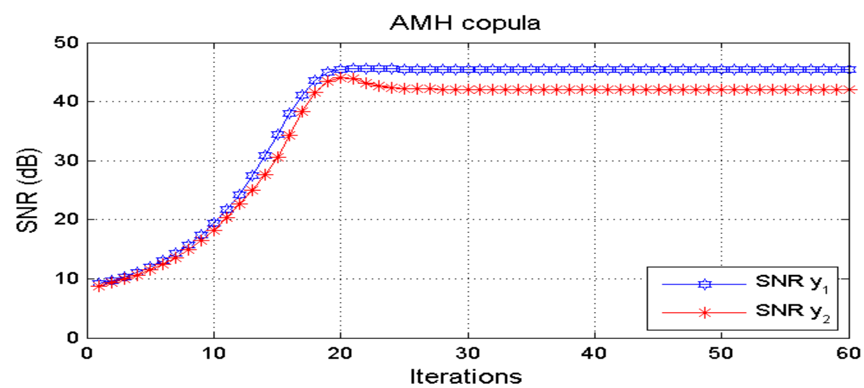

FIGURE 4. Average output SNRs versus iteration number : Uniform dependent sources from AMH-copula.

Moreover, Figures 8-9 show the criterion value versus iterations for AMH and Frank copulas. We can see that our criterion converges to 0 when the separation is achieved.

5.3. Comparison. In this section, both independent and dependent signal sources are tested to confirm the performance of our proposed method, and compared with the MI method proposed by [10] for instantaneous linear mixture, under the same conditions. At the top of Figure 10- 13, we have shown the means of the SNRs of

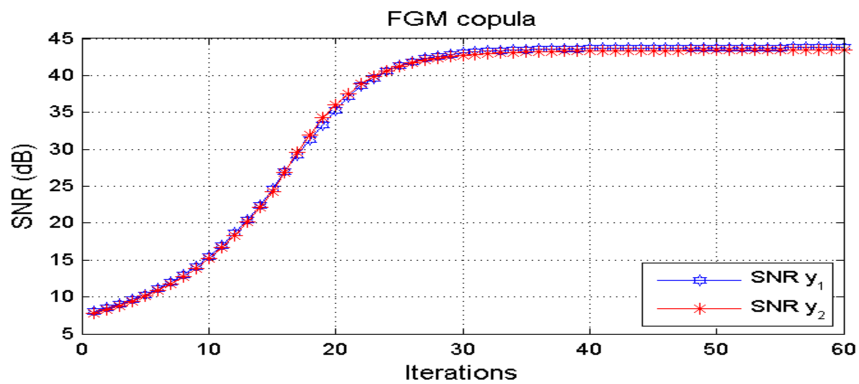

FIGURE 5. Average output SNRs versus iteration number : Bpsk dependent sources from FGM-copula.

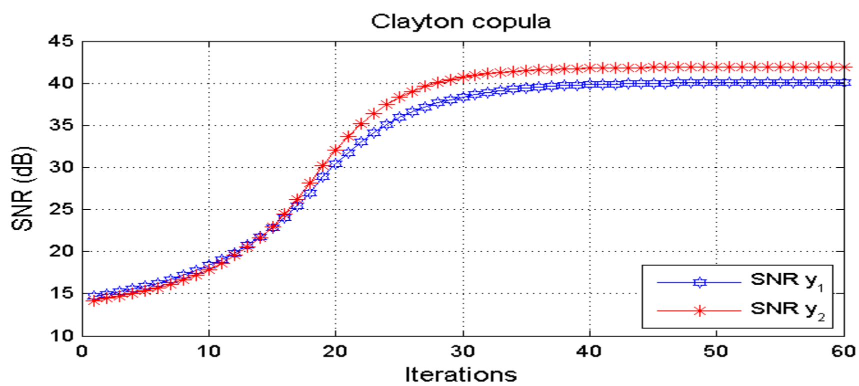

FIGURE 6. Average output SNRs versus iteration number : Uniform dependent sources from Clayton-copula.

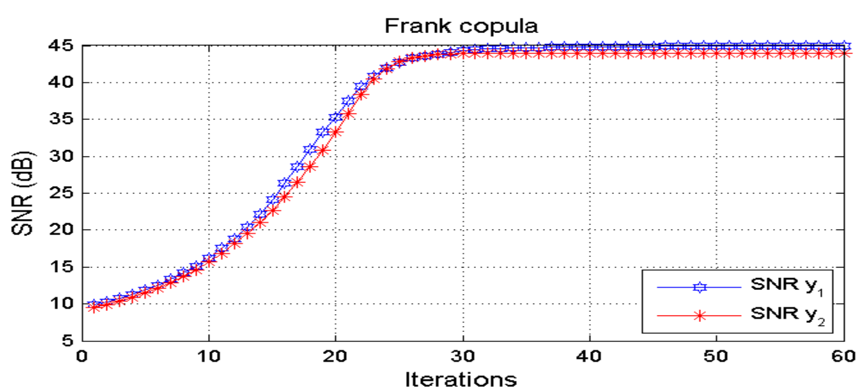

FIGURE 7. Average output SNRs versus iteration number : Bpsk dependent sources from Frank-copula.

two sources for each kind of sample sources. It can be seen from the simulations of Figure 10 (the standard case of independent component sources), that the method

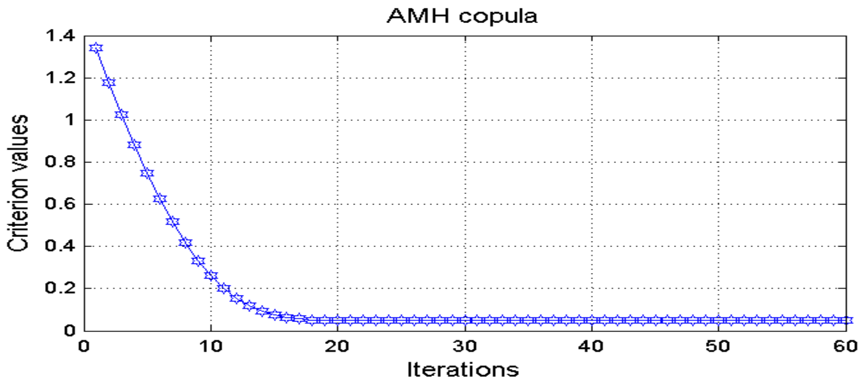

FIGURE 8. The criterion value vs iterations : Uniform dependent sources from AMH-copula.

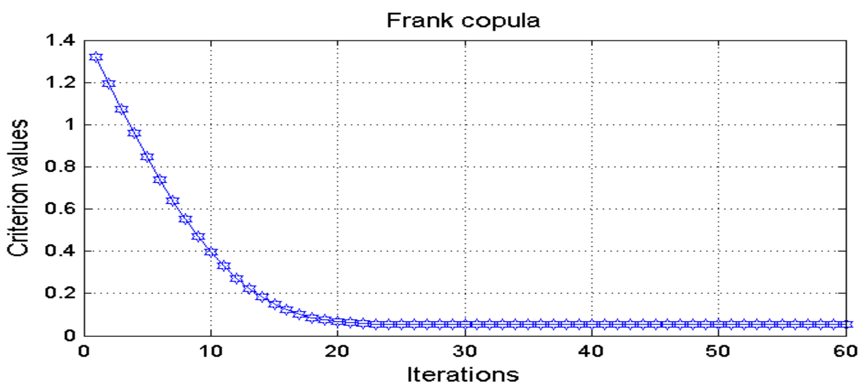

FIGURE 9. The criterion value vs iterations : BPSK dependent sources from Frank-copula.

proposed achieves the separation with same similar accuracy as [14]. Likewise in the case of dependent component sources, one can seen from the simulations of Figure 11 to Figure 13 that our method exhibits better performance than the MI one. At the bottom of Figure 11- 13, we show the criterion value vs iterations. As we can see, the both criteria of the two methods converges to zero when the separation is achieved. But the proposed method gives two well separate sources, unlike the MI one provides two independent sources very far from the sources. And that, is clearly seen at the top of Figure 11- 13, representing, the means of the SNRs of the two sources for each kind of sample sources.

6. Conclusions

We have presented a new BSS algorithm. The approach is able to separate instantaneous linear mixtures of both independent and dependent source components. It proceeds in two steps: the first one consists of spatial whitening and the second one consists to apply a series of Givens rotations, minimizing the estimate of the modified Kullback-Leibler divergence. In Section 5, the accuracy and the consistency of the obtained algorithms are illustrated by simulation, for 2×2 mixture-source. It should

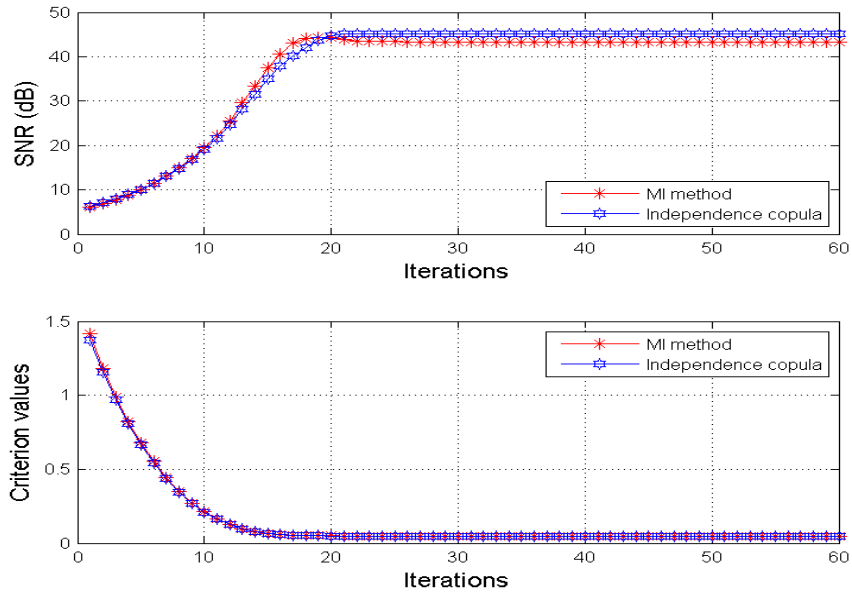

FIGURE 10. Average output SNRs versus iteration number: uniform independent sources.

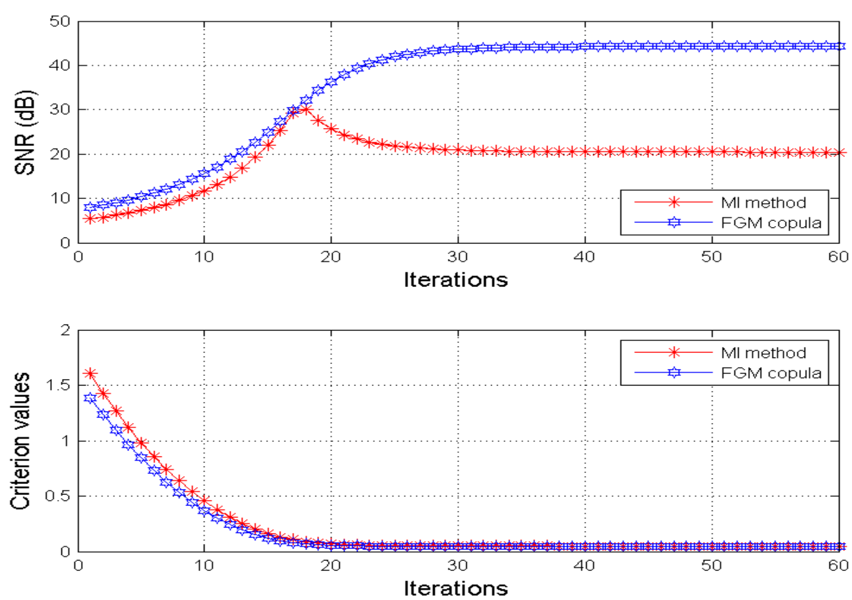

FIGURE 11. Average output SNRs versus iteration number: BPSK dependent sources from FGM-copula.

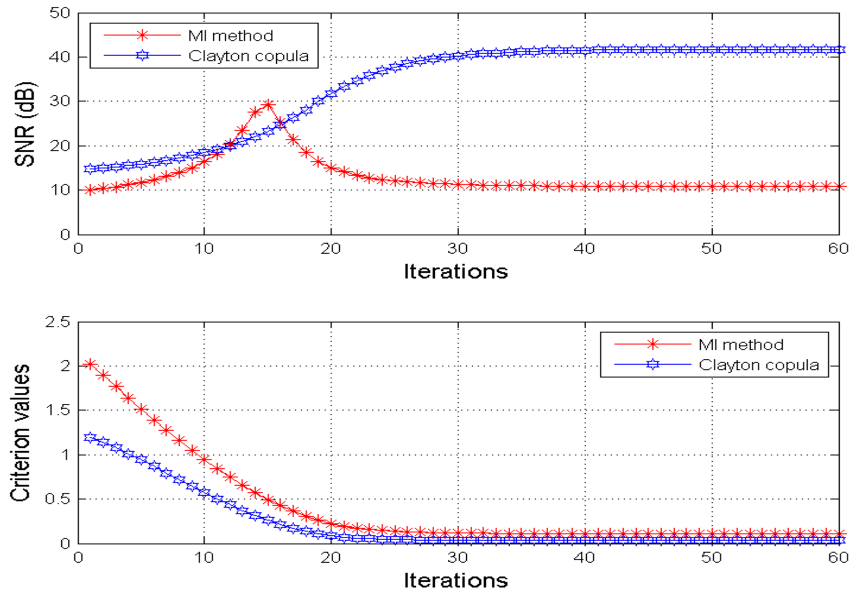

FIGURE 12. Average output SNRs versus iteration number: uniform dependent sources from Clayton-copula.

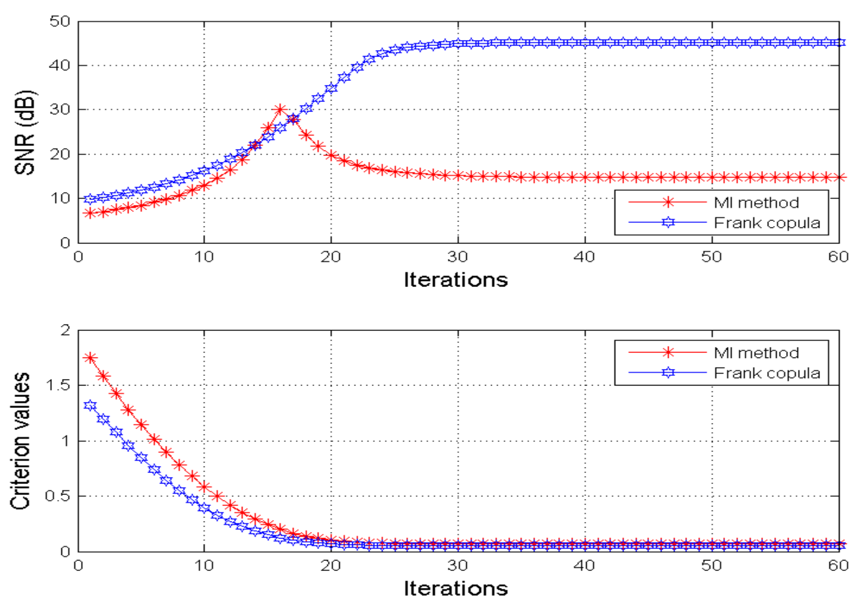

FIGURE 13. Average output SNRs versus iteration number: Bpsk dependent sources from Frank-copula.

be mentioned that our proposed algorithms based on copula densities, rather than the classical ones based on probability densities, are more time consuming, since we estimate both copulas density of the vector and the marginal distribution function of each component. The present approach can be extended to deal with convolutive mixtures, that will be addressed in future communications.

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