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Robust multi-frame super-resolution with non-parametric deformations using diffusion registration

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Abstract. In this paper, we present a new approach of super-resolution. Since almost all super-resolution problems suffer from the motion and blur estimations, new techniques are considered to improve the registration and restoration steps. The proposed method consists of a non parametric registration based on diffusion regularisation and a total variation restoration in the reconstitution step, since super-resolution reconstruction is actually an ill-posed problem. We consider that the deformation is not parametric and differs from one image to another. We also prove the existence of a solution to the two well posed problems (registration and debluring). Simulation results show the effectiveness and robustness of our algorithm against small deformations compared to other existing methods.

Key words and phrases. Robust, super resolution, diffusion registration, ML estimator, MAP estimator, image restoration, regularization.

1. Introduction

Currently, image super-resolution (SR) [28, 29, 27, 14] reconstruction is a relevant research topic in image processing. The aim of this technique is to reconstruct a high-resolution (HR) image from a set of low-resolution (LR) ones that are noisy, blurred, deformed and down-sampled [22, 31] in two steps (finding a blurred HR image from the LR measurements estimating and finally deblurring this HR image) [10, 35, 17, 15]. The SR is used in many applications, such as video surveillance [30], medical diagnostics [11] and image satellite [22], ...etc.

The primary aim of SR algorithm is using motion information [5, 34] to enhance the quality of the image sequence. A crucial step that guarantees the success of the SR algorithm is the registration part. Since we deal with small deformed images, the diffusion registration is used [23, 12, 24, 18]. In other hand, the exact selection of image prior function in the deblurring step is very important for the image reconstruction accuracy and on the computational cost of the algorithm, since some prior functions are much more expensive to evaluate than others. To avoid the ill-posedness of the restoration step we use the popular total variation (TV) function [1, 2] since it preserve edge. Inspired by the efficiency of diffusion regularisation in the ill-registration problem [1, 21] and the TV regularization in the deblurring step. We propose a novel improved SR reconstruction specified at low resolution images with small deformations to avoid different and annoying artifacts such as blur, jagged edges, and ringing.

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2. Problem formulation

The observed images of a real scene are usually in low resolution due to some degradation operators. In practice, the acquired image is corrupted by noise, blur and decimation. In almost cases the degradation is generated by inappropriate camera parameters or configuration, in addition we have also the effects of atmospheric turbulence. All these facts corrupt the resolution of the image, therefore improvement of resolution techniques is desired in those cases. We assume that the LR images are taken under the same environmental conditions using the same sensor. The relationship between an ideal HR image X and a LR images Y_k (represented by a column vector of size M) can be described by this formula

$$Y_k = DF_kHX + E_k \qquad \forall k = 1, 2, ..., n. \tag{1}$$

where E_k represents the additive noise for each image, H: the blurring operator of size $N \times N$, D represents the decimation matrix of size $M \times N$, F_k: is a geometric warp matrix representing a diffusion transformation that differs in all frames.

In the presence of different operators of degradation, the problem becomes very sensitive. We use the same approach in [10] that suggest to separate it in three steps.

(1) Computing the warp matrix F_k for each image.
(2) Fusing the low-resolution images Y_k into a blurred HR version $B = HX$.
(3) Finding the estimation of the HR image X from the blurring one B.

3. The warp matrix F_k

We choose arbitrarily one image Y_i from Y_k as a reference image and we look for the deformations u_k between Y_i and the other images, such that

$$Y_i(x) = Y_k(u_k(x)) \qquad \text{for } k \neq i \quad \text{and} \quad \forall x \in \Omega. \tag{2}$$

To find the deformations u_k, we minimize the distance between each image. Since this problem is ill-posed we propose to use the diffusion regularisation [24].

The registration problem is now well defined in (3).

$$\min_{u_k} \mathcal{J}_{diff}(u_k), \tag{3}$$

with:

$$\mathcal{J}_{diff}(u_k) = \mathcal{D}_{SSD}(Y_i, Y_k, u_k) + \beta S_{diff}(u_k), \tag{4}$$

where

$$\mathcal{D}_{SSD}(Y_i, Y_k, u_k) = \int_\Omega (Y_k(u_k(x)) - Y_i(x))^2 \, dx,$$

and

$$S_{diff}(u_k) = \frac{1}{2} \sum_{l=1}^{2} \int_\Omega \langle \nabla u_{k_l}, \nabla u_{k_l} \rangle dx.$$

β: is the regularization parameter of the registration problem.

The first question that may be asked is about the existence of a solution to the problem (3). The choice of the functional space is very important to demonstrate the ellipticity of the functional $\mathcal{J}$. A natural choice is the Sobolev space [4] defined as

$$\mathcal{T} = \{u_k \in H^1(\Omega) \, \text{and} \, u_k = 0 \, \text{on} \, \partial\Omega\}. \tag{5}$$

Theorem 3.1. *Let Ω be a regular bounded open subset of $\mathbb{R}^2$ and f be in $\mathrm{L}^2(\Omega)$. Then the minimization problem*

$$\min_{u_k \in \mathcal{T}} \mathcal{J}_{diff}(u_k) \tag{6}$$

Admits an unique solution.

Proof. Step 1: Existence.
To demonstrate this theorem, we have to prove that $\mathcal{J}_{diff}$ is elliptic and weakly sequentially l.s.c.
Ellipticity:
For the ellipticity, we have to prove that

$$\lim_{\|u_k\|_{H^1(\Omega)} \to +\infty} \mathcal{J}_{diff}(u_k) = +\infty,$$

Let $u_k \in \mathcal{T}$, then

$$\begin{aligned} \mathcal{J}_{diff}(u_k) &= \frac{1}{2}\int_\Omega |\nabla u_k(x)|^2 + \langle f(x), u_k(x)\rangle dx \\ &\geq \frac{1}{2}\int_\Omega |\nabla u_k(x)|^2 dx - \|f\|_{L^2(\Omega)}\|u_k\|_{H^1(\Omega)}, \end{aligned} \tag{7}$$

where

$$\|f\|_{L^2(\Omega)} = \|Y_i\|_{L^\infty(\Omega)} \times \|Y_k\|_{L^2(\Omega)}.$$

Since $u_k \in \mathcal{T}$, using the Poincaré inequality, the norm $\|.\|_{H^1(\Omega)}$ is equivalent to the norm $u_k \to \int_\Omega |\nabla u_k|^2$, thus, there exists a constant C such as

$$\int_\Omega |\nabla u_k|^2 \geq C\|u_k\|_{H^1(\Omega)},$$

then

$$\mathcal{J}_{diff}(u_k) \geq \frac{C}{2}\|u_k\|^2_{H^1(\Omega)} - \|f\|_{L^2(\Omega)}\|u_k\|_{H^1(\Omega)}.$$

Using the Young inequality

$$\begin{aligned} \mathcal{J}_{diff}(u_k) &\geq \frac{C}{2}\|u_k\|^2_{H^1(\Omega)} - C(\epsilon)\|f\|^2_{L^2(\Omega)} - \frac{\epsilon}{2}\|u_k\|^2_{H^1(\Omega)} \\ &\geq (\frac{C}{2} - \frac{\epsilon}{2})\|u_k\|^2_{H^1(\Omega)} - C(\epsilon)\|f\|^2_{L^2(\Omega)}, \end{aligned} \tag{8}$$

where ε is chosen such as $\frac{C-\varepsilon}{2} > 0$.

We get finally $\mathcal{J}_{diff}(u_k) \to +\infty$ if $\|u_k\|_{H^1(\Omega)} \to +\infty$, we obtain that $\mathcal{J}_{diff}$ is elliptic.
Weak sequentially l.s.c:
We have to prove that $\mathcal{J}_{diff}$ is continuous and convex.

The continuity :

Since S_{diff} is a bilinear function on u_k, it's easy to check there continuity.
Indeed lets firstly define the bilinear form S_{diff}

$$S_{diff}(u_k, v_k) = \frac{1}{2}\sum_{l=1}^{2}\int_\Omega \langle \nabla u_{k_l}, \nabla v_{k_l}\rangle dx.$$

We have then

$$
\begin{aligned}
|S_{diff}(u_k, v_k)| &= |\frac{1}{2}\sum_{l=1}^{2}\int_{\Omega}\langle \nabla u_{k_l}, \nabla v_{k_l}\rangle dx| \\
&\leq \frac{1}{2}\|\nabla u_k\|_{\mathrm{L}^2(\Omega)}\|\nabla v_k\|_{\mathrm{L}^2(\Omega)} \\
&\leq \frac{1}{2}\|\nabla u_k\|_{H^1(\Omega)}\|\nabla v_k\|_{H^1(\Omega)}.
\end{aligned} \tag{9}
$$

Concerning the linear part $\mathcal{D}_{SSD}$, we have also

$$
\begin{aligned}
|\mathcal{D}_{SSD}(Y_i, Y_k, u_k)| &= |\int_{\Omega}\left(Y_k(u_k(x)) - Y_i(x)\right)^2 dx| \\
&\leq \|f\|_{\mathrm{L}^2(\Omega)}\|u_k\|_{\mathrm{L}^2(\Omega)} \\
&\leq \|f\|_{\mathrm{L}^2(\Omega)}\|u_k\|_{H^1(\Omega)}.
\end{aligned} \tag{10}
$$

Where f is defined above.
From (9) and (10) we can deduce that $\mathcal{J}_{diff}$ is continuous.

The convexity:
It is easy to demonstrate the convexity of the function $\mathcal{J}_{diff}$, since $u_k \to S_{diff}(u_k, u_k)$ is strictly convex (because the norm $\|.\|_{\mathrm{L}^2(\Omega)}$ is strictly convex). In addition, since $\mathcal{D}_{SSD}$ is linear its also strictly convex. Then the function $\mathcal{J}_{diff}$ is strictly convex. Which concludes the proof. □

4. The fusion of Y_k

To compute the blurred HR image B from the LR frames Y_k, we use the Maximum Likelihood (ML) estimator. Finally we obtain

$$
\widehat{B} \quad = \quad \underset{B}{\operatorname{argmin}} \sum_{k=1}^{n} \|Y_k - DF_kB\|_{L^2}^2 \tag{11}
$$

5. The de-blurring step

Finding the HR image $\widehat{X}$ is equivalent to solve the minimization problem (10), using the Maximum a posteriori (MAP) [10, 16].

$$
\widehat{X}_{MAP} \quad = \quad \underset{X}{\operatorname{argmin}}\{-log(p(\widehat{B}/X)) - log(p(X))\} \tag{12}
$$

where $p(\widehat{B}/X)$ represents the likelihood term.
$p(X)$: denotes the prior knowledge on the high-resolution image.
To solve this problem we need to describe the prior Gibbs function (PDF) p, we use the generalized TV [1] as we know that it tends to preserve edges in the reconstruction, as it does not severely penalize steep local gradients.

$$
p(X) = \exp\{\alpha\|\varphi(|\nabla X|)\|_1\} \tag{13}
$$

where:
α: the regularization parameter;
φ: is a strictly convex, non-decreasing function from $\mathbb{R}^+$to $\mathbb{R}^+$, with $\varphi(0) = 0$ (without a loss of generality)

$$
\lim_{X \to +\infty} \varphi(X) = +\infty.
$$

The norm $|.|$ is defined like that

$$|X| = \sqrt{X_1^2 + X_2^2} \quad \forall (X_1, X_2) \in \mathbb{R}^2.$$

A typical choice of φ is the so-called hyper-surface minimal function defined as:

$$\varphi(X) = \sqrt{1 + X^2}.$$

5.1. Resolution of the MAP estimator problem. Since we have defined the operators F_k and the prior function p, we deduce the equation of the MAP estimator

$$\widehat{X}_{MAP} = \underset{X}{\operatorname{argmin}} \{ \sum_{x \in \Omega} \|HX(x) - \widehat{B}(x)\|_1 + \alpha \|\varphi(|\nabla X|)\|_1 \}, \tag{14}$$

where Ω contains all the pixels on the HR grid X.
The norm $\|HX - \widehat{B}\|_1$ is used because it's very robust against outliers [10].
Before solving this minimisation problem, we have to prove the existence and the uniqueness of the solution. In order to use the classical method of the calculus of variations, we have to assume another hypothesis on φ. We suppose that φ grows at most linearly i.e.: $\exists c > 0$ and $b \geq 0$ such that

$$cx - b \leq \varphi(x) \leq cx + b. \tag{15}$$

According to (15), a natural choice of the functional space on which we can seek a solution is the Sobolev Space $W^{1,1}(\Omega)$, defined as:

$$W^{1,1}(\Omega) = \{X \in \mathrm{L}^1(\Omega), \quad \nabla X \in [\mathrm{L}^1(\Omega)]^2\}.$$

Unfortunately, this space is not reflexive. In this case, it is classical to use the relaxed function on the space of bounded variation $BV(\Omega)$.

$$BV(\Omega) = \{X \in \mathrm{L}^1(\Omega)\,;\, \int_\Omega |DX| < +\infty\},$$

where

$$\int_\Omega |DX| = sup\{\int_\Omega X\, div\, \varphi\, dx\,;\, \varphi \in \mathcal{C}_0^1(\Omega)^N, ||\varphi||_\infty \leq 1\}$$

For the reason that every bounded sequence in $W^{1,1}(\Omega)$ is also bounded in $BV(\Omega)$, we use the characteristics of the $BV - \omega*$ topology to deduce the existence of a subsequence that converge $BV - \omega*$. We define the relaxed function by the same way in [1] and we use the same technique to demonstrate the existence of the solution. We cannot say anything about the uniqueness since the norm $\|HX - \widehat{B}\|_1$ is not strictly convex.

To minimise the problem (14) we use the classical steepest descent algorithm. We finally compute the HR image $\widehat{X}$ as follows.

$$\widehat{X}_{n+1}(x) = \widehat{X}_n(x) - \eta \left\{ H^\intercal \mathrm{sing}(H\widehat{X}_n(x) - \widehat{B}(x)) + \alpha\, \mathrm{div} \left(\frac{\varphi^{'}(|\nabla X_n|)}{|\nabla X_n|} \nabla X_n \right) \right\}, \tag{16}$$

where α is the steepest descent parameter.
div: is the divergence operator defined by the adjoint operator of ∇ as $\mathrm{div} = -\nabla^*$.
The second part of the problem is used as a discrete part. We will denote by $X_{i,j}$, $i,j = 1,...N$ a discrete image and $M = \mathbb{R}^{N^2}$: the set of all discrete images. The discretization of the operators ∇ and div is given by

$$(\nabla X)^1_{i,j} = \begin{cases} X_{i+1,j} - X_{i,j} & \text{if} \quad i < N \\ 0 & \text{if} \quad i = N \end{cases},$$

$$(\nabla X)^2_{i,j} = \begin{cases} X_{i,j+1} - X_{i,j} & \text{if} \quad j < N \\ 0 & \text{if} \quad j = N \end{cases},$$

and

$$(\mathrm{div})_{i,j} = (\mathrm{div})^1_{i,j} + (\mathrm{div})^2_{i,j},$$

where

$$(\mathrm{div}\, p)^1_{i,j} = \begin{cases} p^1_{i,j} - p^1_{i-1,j} & \text{if} \quad 1 < i < N \\ p^1_{i,j} & \text{if} \quad i = 1 \\ 0 & \text{if} \quad i = N \end{cases},$$

$$(\mathrm{div}\, p)^2_{i,j} = \begin{cases} p^2_{i,j} - p^2_{i,j-1} & \text{if} \quad 1 < j < N \\ p^2_{i,j} & \text{if} \quad j = 1 \\ -p^2_{i,j-1} & \text{if} \quad j = N \end{cases},$$

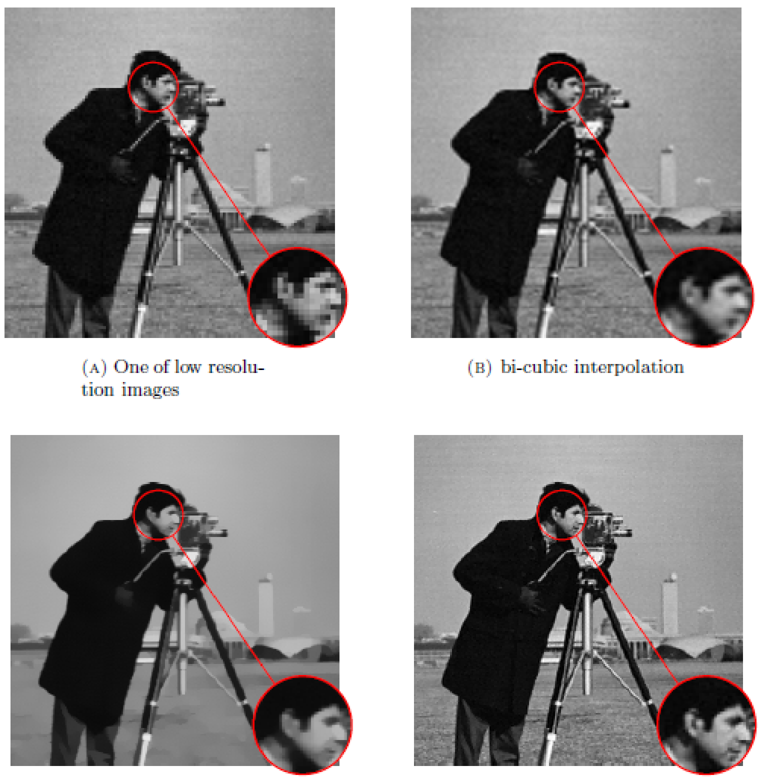
6. Numerical result

In this section, we present simulation results for the proposed method. We dealt with slightly deformed low resolution images. We evaluate the performance of the proposed algorithm using the peak-signal-to-noise ratio (PSNR) criterion. We construct a synthetic LR images in the example 1 and 2, to test our algorithm and compare it with classical method such as the bi-cubic interpolation and robust super resolution algorithms (RSR) [10]. The bi-cubic interpolation is used after a diffusion registration, while we use the same data for the RSR resolution and our proposed method. We choose in example 1 the famous Cameraman as an original image in (1) of size 256×256.

We illustrate in Figure 1 one of the $N = 8$ input low-resolution frames chosen arbitrary and blurring with 5×5 Gaussian blur kernel with standard deviation equal to 3, and sub-sampling by a factor of 2. In addition we add a noise E_k arbitrary in each frame. The parameters chosen for our algorithm are $\eta = 0.1$, $\alpha = 0.4$ and $maxiter = 100$ iteration for the steepest descent, finally we choose $\beta = 0.1$ in the registration step.

In Figure 2 (B), the result is obtained by the bi-cubic interpolation after a diffusion registration, in the Figure 2 (C) the RSR result and finally the Figure 2 (D) illustrate the obtained image by the proposed method, we can clearly see the efficiency of our algorithm.

FIGURE 1. The original image of Cameraman.



(A) One of low resolution images

(B) bi-cubic interpolation

(C) RSR resolution (BTV regulizer)

(D) The proposed method

FIGURE 2. Comparison between classical method and the proposed algorithm for the example 1.

In the example 2, we take an image document as an original (3) of size 187×182. We keep the same parameters using above. The Figure 4 illustrates a comparison between our algorithm and the two classical methods (bi-cubic interpolation and RSR).

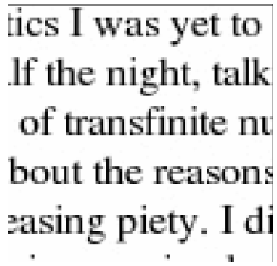
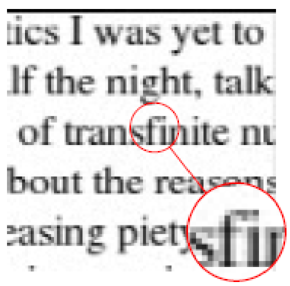
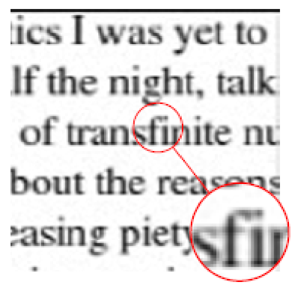


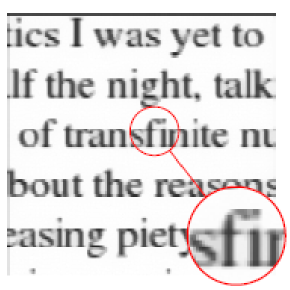
FIGURE 3. The original image of text.



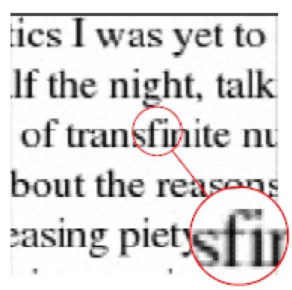
(A) One of low resolution images



(B) bi-cubic interpolation



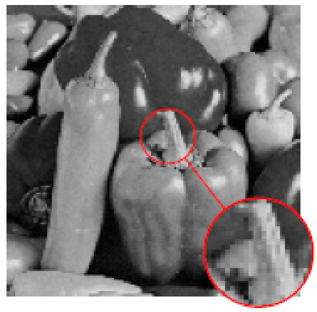
(C) RSR resolution (BTV regulizer)



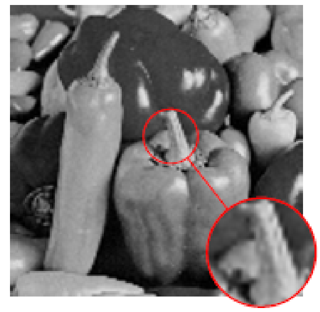
(D) The proposed method

FIGURE 4. Comparison between classical method and the proposed algorithm for the example 2.

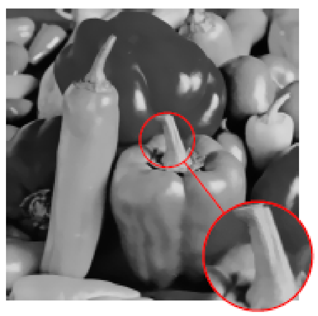
In the last example we take a 40 LR images of size 126×126, and we consider that the transformations between all LR frames are not parametric. In the Figure 5, we compare the result obtained with a resolution augmentation factor $r = 4$.



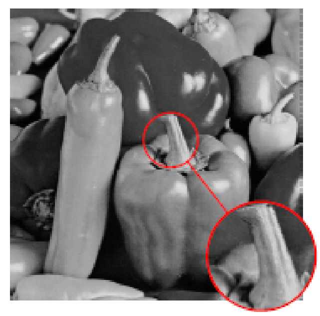
(A) One of low resolution images



(B) bi-cubic interpolation



(C) RSR resolution (BTV regulizer)



(D) The proposed method

FIGURE 5. Comparison between classical method and the proposed algorithm for the example 3.

The algorithm used	Noise $\sigma = 3$	Noise $\sigma = 5$
Exemple 1 (Cameraman)		
bi-cubic interpolation	19.18	17.66
RSR resolution (BTV regulizer)	27.33	26.11
The proposed method	**28.03**	**27.77**
Exemple 2 (Image document)		
bi-cubic interpolation	17	16.07
RSR resolution (BTV regulizer)	**26.88**	25.91
The proposed method	26.44	**26.36**
Exemple 3 (Papers)		
bi-cubic interpolation	18.25	17.88
RSR resolution (BTV regulizer)	**29.658**	**28.881**
The proposed method	29.5078	28.777

FIGURE 6. The PSNR table.

In the table in Figure 6, we measure the quality of the reconstruction using the PSNR criterion for the three examples. In this table, the bold numbers represent the best results.

7. CONCLUSION

Recently, the registration and the regularization approaches are considered the most and recent techniques that can be used in solving the SR reconstruction problem. Thus, we present a new approach to the SR image reconstruction problem based on diffusion registration and a generalized TV restoration. The proposed algorithm differs from the others in the registration and deblurring steps. We prove existence and uniqueness of minimizers of the diffusion registration and we assure also the existence of the solution of the final SR problem. Numerical results show the robustness of our approach compared with other methods in the literature.

References

[1] G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing Partial Differential Equations and the Calculus of Variations Second Edition*, Springer Science, 2006.

[2] J.F. Aujol, *Calculus of variations in image processing*, CMLA, ENS Cachan, CNRS, UniverSud, 2007.

[3] A. Baudes, B. Coll, and J.M. Morel, On image denoising method, *SIAM Multiscale Modeling and Simulation* **42** (2005), 490–530.

[4] H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2011.

[5] D. Capel and A. Zisserman, Computer Vision Applied to Super-resolution, *IEEE Signal Process. Mag* **20** (2003), 75–86.

[6] D. Datsenko and M. Elad, Robust Statistical Modeling Using the t Distribution, *Journal of the American Statistical Association* **84** (1989), 881–896.

[7] D. Datsenko and M. Elad, Example-based single document image super-resolution: a global MAP approach with outlier rejection, *Multidimensional Systems and Signal Processing* **18** (2007), 103–121.

[8] K. Donaldson and G.K. Myers, Bayesian Super-Resolution of Text in Video with a Text-Specific Bimodal Prior, *International Journal of Document Analysis and Recognition (IJDAR)* **7** (2001), 159–167.

[9] M. Elad, On the Origin of the Bilateral Filter and Ways to Improve It, *IEEE Trans. Image Processing*, **11** (2002), 1141–1151.

[10] S. Farsiu, M.D. Robinson, M. Elad, and P. Milanfar, Fast and Robust Multiframe Super Resolution, *IEEE Trans. on Image Processing* **13** (2004), no. 10, 1327–1344.

[11] D.L.G. Hill, P.G. Batchelor, M. Holden, and D.J. Hawkes, *Medical Image Registration*, Physics in medicine and biology **46** (2001), no. 3.

[12] A.A. Goshtasby, *Image Registration: Principles, Tools and Methods, Advances in Computer Vision and Pattern Recognition*, Springer, 2012.

[13] C.O. Horgan, KORN'S inequalities and their applications in continum mechanics, *SIAM Review* **37** (1995), 491–511.

[14] C. Jung and A. Gu, Curvature preserving image super-resolution with gradient-consistency-anisotropic-regularization prior, *Signal Processing: Image Communication* **29** (2014), 1211–1222.

[15] A. Laghrib, A. Hakim, S. Raghay, and M. El Rhabi, Robust Super Resolution of Images with Non-parametric Deformations Using an Elastic Registration, *Applied Mathematical Sciences* **8**(2014), 8897–8907.

[16] A. Laghrib, A. Hakim, and S. Raghay, *A combined total variation and bilateral filter approach for image robust super resolution*, EURASIP J Image Video Process **2015:19** (2015).

[17] A. Laghrib, A. Hakim, S. Raghay, and M. El Rhabi, A robust multi-frame super resolution based on curvature registration and second order variational regularization, *International Journal of Tomography and Simulation* **28** (2015), 63–71.

[18] T.M. Lehmann, C. Gönner, and K. Spitzer, Survey: Interpolation Methods in Medical Image Processing, *IEEE Transactions On Medical Imaging* **18** (1999), 1049–1075.

[19] X. Li, Y. Hu, X. Gao, D. Tao, and B. Ning, A multi-frame image super resolution method, *Signal Processing* **90** (2010), 405–414.

[20] C. Liu, D. B. Rubin, and Y. N. Wu, Parameter Expansion to Accelerate EM: The PX-EM Algorithm, *Biometrika* **85** (1998), no. 4, 755–770.
[21] E.H.W. Meijering, W.J. Niessen, and M.A. Viergever, Quantitative Evaluation of Convolution-Based Methods for Medical Image Interpolation, *IEEE Transactions On Medical Imaging* **18** (2001), 111–126.
[22] P. Milanfar, *Super-Resolution Imaging, Digital Imaging and Computer Vision*, CRC Press, 2010.
[23] J. Modersitzki, *Numerical Methods for Image Registration*, Oxford University Press, USA, 2007.
[24] J. Modersitzki, *Fair: Flexible Algorithms for Image Registration, Fundamentals of Algorithms*, SIAM, 2009.
[25] N.C. Nguyen, A Note on Tikhonov Regularization of Linear Ill-Posed Problems, *Massachusetts Institute of Technologyl* (2006).
[26] J. Nocedal and S. Wright, *Numerical Optimization, Springer Series in Operations Research and Financial Engineering*, Springer; 2nd edition 2006.
[27] S.C. Park, M.K. Park, and M. G. Kang, Super-resolution image reconstruction: a technical overview, *IEEE Signal Processing Magazine* **3** (2003), 21–36.
[28] T. Peleg and M. Elad, A statistical prediction model based on sparse representations for single image super-resolution *IEEE Trans. on Image Processing* **23**, (2014), 2569–2582.
[29] L.C. Pickup, *Machine Learning in Multi-frame Image Super-resolution*, Ph.D. thesis, University of Oxford, 2007.
[30] M. Protter and M. Elad, Super Resolution With Probabilistic Motion Estimation, *IEEE Transactions on Image Processing* **18** (2009), no. 8, 1899 - 1904.
[31] J.M. Sanches and J.S. Marques, A MAP Estimation Algorithm Using IIR Recursive Filters, *Lecture Notes in Computer Science*, no. 2683 (2003), 436–449.
[32] H. Shen, L. Zhang, B. Huang, and P. Li, A MAP approach for joint motion estimation segmentation and super resolution, *IEEE Transaction on Image Processing* **16** (2007), 479–490.
[33] P.D. Thouin and C.I. Chang, A method for restoration of low-resolution document images, *IJDAR* **2** (2000), no. 4, 200-210.
[34] R.Y. Tsai and T. S. Huang, Multiframe Image Restoration and Registration, *Advances in Computer Vision and Image Processing, chap. 7, JAI Press. Greenwich, Conn, USA* (1984), 317–339.
[35] A. Zomet, A. Rav-Acha and S. Peleg, Robust super-resolution, *Computer Vision and Pattern Recognition, CVPR* **1**, I-645–I-650.

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