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Numerical simulation of turbulence effects on the spectral matrix elements

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Abstract. We present the results of a numerical simulation to specify quantitatively the effect of turbulence on the treatment of antenna. This simulation is based on the interspectral analysis of signals from a linear antenna. The decorrelation of waves along the antenna is properly predicted from the parabolic approximation and a Von Karman model for the spectrum of velocity fluctuations.

Key words and phrases. Interspectral matrix, coherence, turbulence, parabolic approximation.

1. Effect of the turbulence on the propagation of the waves

The acoustic or electromagnetic waves which propagate in a turbulent environment undergo great disturbance due to the random fluctuations in the environment. Thus the turbulent state of the atmosphere along the path depends essentially on some specific characteristics of the fluctuations in speed [see Figure 1] and of temperature.

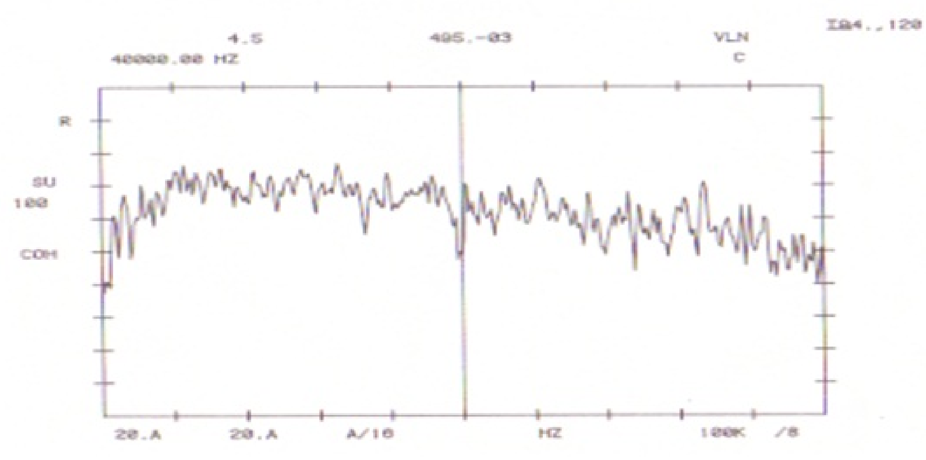

Figure 1. Evolution of the coherence according to the frequency.

Yet, numerous theoretical or experimental studies have shown that the presence of the turbulence (of kinematic or thermal origin) along the path of propagation causes a partial decorrelation of the waves, we thus have to expect that in real conditions;

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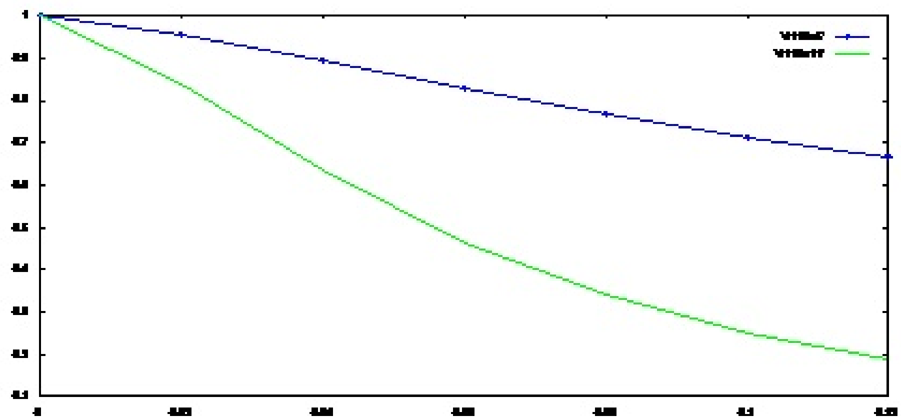

FIGURE 2. Spatial coherence in perturbed environment.

especially in atmosphere, the power of resolution of superresolvents methods used in treating of an antenna is limited by the turbulence, which acts in a multiplicative way on the signals.

We have addressed in this study this aspect of degradation of functioning of the method of MUSIC due to the turbulence. In the field of high frequencies, the distribution of the waves by the turbulence is executed essentially for the main part in the vicinity of the axis of propagation. Thus if we are interested in the deformation of the wave in the vicinity of the direction of average propagation (x), we obtain then a parabolic equation as follows:

$$2ik_0 \frac{\partial}{\partial x}\psi(x, \overrightarrow{\rho}) + \Delta \overrightarrow{\rho}\psi(x, \overrightarrow{\rho}) + K_0^2 \varepsilon(x, \overrightarrow{\rho})\psi(x, \overrightarrow{\rho}) = 0, \tag{1}$$

where x is the coordinate following the direction of propagation, $\overrightarrow{\rho}$ and is the one which is in a perpendicular plane. $\varepsilon(x)$ represents the index fluctuations of the environment caused by turbulent. On the basis of equation (1), it is possible to determine the coherent wave $< \Psi >$ and correlation function $\Gamma(x, \overrightarrow{\rho_1}, \overrightarrow{\rho_2})$ using the order time two in two points of a plane perpendicular to the direction of propagation.

1.1. Loss of consistency along the antenna. Figure 2 shows the route of the coherence function calculated for a transmitter located on the axis. Two values of the performed average speed performed 6m/s and 12m/s. We notice that in assessing satisfactorily the loss of spatial coherence of the transmitted field, it is necessary to calculate the function Γ with Von Karman spectrum.

The mistake made by limiting the spectrum $S_\varepsilon(K)$ to the inertial range and especially greater than the efficient value of velocity fluctuations is small and the distance between sensors is great.

1.2. Antenna treatment in turbulent environment. The imaging of the noise sources, that is the determination of the direction of the sources and their relative

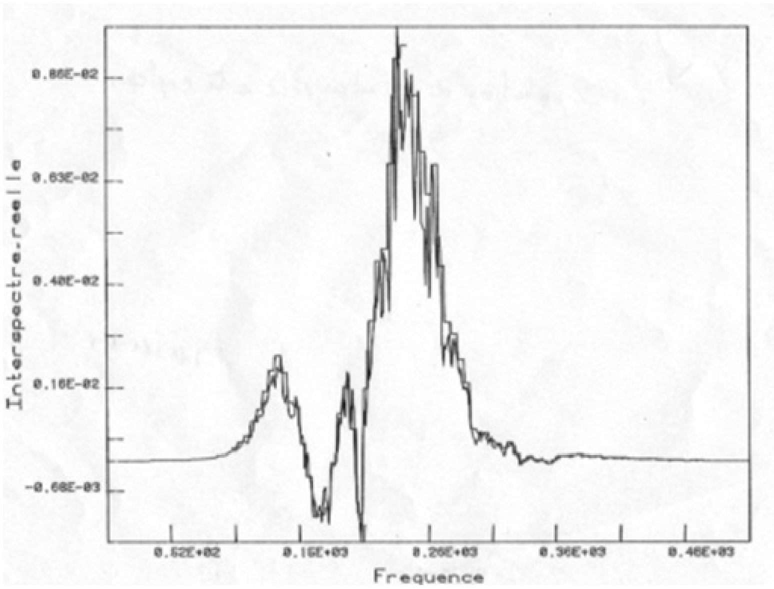

FIGURE 3. Evolution of the interspectrum between sensors.

intensities according to the frequency, is a problem which has numerous applications in telecommunication (radar, sonar).

If we want to locate very close issuers whose angular deviation is lower at the threshold of Rayleigh $\Delta\Theta = \frac{\lambda}{L}$ where λ is the length of the wave and L the length of antenna, we can use methods known as superresolution using priori hypotheses about sources? field and the environment of propagation.

1.2.1. *Interspectral analysis of multidimensional signals.* The interspectral matrix that forms the basis of the imaging technique is constituted by interspectrums between the sensors of the antenna. See Figure 3.

$$\underline{\underline{\gamma}}(f) = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{1N} \\ . & . & . \\ \gamma_{N1} & \gamma_{N2} & \gamma_{NN} \end{bmatrix}. \tag{2}$$

Each term γ_{kl} the matrix can be put in the form:

$$\gamma_{kl}(f) = \mid \gamma_{kl} \mid e^{-j\phi_{kl}}, \tag{3}$$

where the module $\mid \gamma_{kl} \mid$ And the phase ϕ_{kl} of the interspectrum between the sensors k and l ap. This matrix is hermitian and the diagonal terms γ_{kk} et γ_{ll} represent respectively autospectrum stemming from sensors k and l of the antenna.

As a result of the decorrelation of the signals emitted by the noises, the interspectral matrix decomposes as $\gamma(f)$ decomposes as follows:

$$\underline{\underline{\gamma}}(f) = \underline{\underline{\gamma_s}}(f) + \underline{\underline{\gamma_B}}(f), \tag{4}$$

$\underline{\underline{\gamma_s}}(f)$ is the interspectral matrix of the only sources and $\underline{\underline{\gamma_B}}(f)$ is the matrix of noise. When a single source does exists, it is simple to verify that the interspectral matrix can be written as follows:

$$\underline{\underline{\gamma}}(f) = \underline{S^j}.\underline{S}^{j*} \tag{5}$$

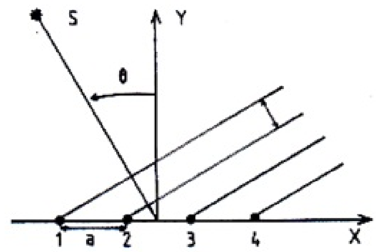

FIGURE 4. Estimation of the appropriate values and the estimated direction of sources in quiet and turbulent environment.

$\underline{S^j}$: With Sj: vector source is visible when the source j emits alone.

An example of vector source is indicated in Figure 4 for a linear antenna formed of equidistant sensors, and a source emitting plane waves which propagate according to the direction Θ_j compared to the normal of the antenna.

$$S^j(f) = \sqrt{\gamma^j}\begin{bmatrix} 1 \\ e^{j\varphi} \\ e^{2j\varphi} \\ e^{3j\varphi} \end{bmatrix}, \tag{6}$$

γ^j : spectral density of the transmitter;
φj : being the phase shift and which depends on the incidence angle of the transmitter, compared to the axis of the antenna, the frequency and the wavelength λ.

1.2.2. *Principle of the underspace noise.* MUSIC technique exploits the orthogonality property underspace sound

$$\underline{V_k^*}(f).\underline{S^j}(f) = 0 \quad \text{for} \quad k \in [p+1, n] \quad \text{and} \quad j \in [1, p], \tag{7}$$

with $\underline{\underline{V_k}}(f) = [V^{p+1}(f), ..., V^n(f)]$ appropriate vectors associated to the smallest appropriate values.

In practice, we use the appropriate vectors of the estimated interspectral matrix. There will be thus no cancelation of the expression (7) but only the minimums. Therefore, the equation (7) for a given frequency can be simply written as:

$$Q(f,\theta) = [\sum_{i=p+1}^{N} \mid \underline{V_i^*}(f).\underline{S}(f,\theta) \mid^2]^{-1} \tag{8}$$

$\underline{V_i^*}(f)$: appropriate transposed vectors from the estimated interspectral matrix.

2. Results obtained

First, we have controlled the functioning of MUSIC method in the absence of turbulence in order to show the limits introduced by the conditions of propagation. In Table 1 below, we present the obtained results of normalized appropriate values of coherence matrix for an angular gap between sources $S^{\Delta\theta=4}$, with a long antenna ($L = 14\lambda$)), in a quiet environment and in the presence of a turbulent flow which presents fluctuations in speed. Figure 5 shows the appropriate values of the interspectral matrix (always brought back to λ_1 for an angular gap between sources of $S^{\delta\theta=4}$ with an antenna ($L = 14\lambda$). We notice that the appropriate values λ_3 and λ_4 are

TABLE 1. Estimation of the appropriate values and the estimated direction of sources in quiet and turbulent environment.

	Normalized eigenvalues			Directions of sources	
Speed flow	λ_1	λ_2	λ_3	θ_1	θ_2
Quiet environment	0,651	0	0	0	4,45
U = 5m/s	0 ,708	0,090	0,028	0,05	4,41
U = 10m/s	0,861	0,325	0,129	0,2	4,25
U = 15m/s	0,971	0,567	0,318	0,45	4,01

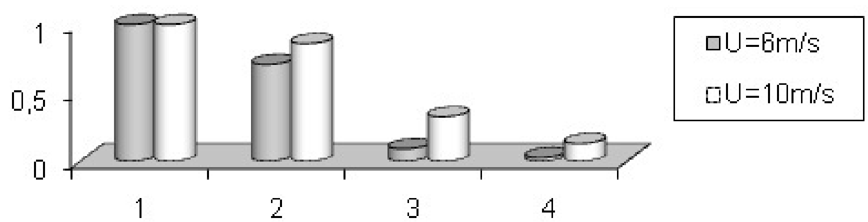

FIGURE 5. Distribution of the appropriate values normalized in turbulent environment.

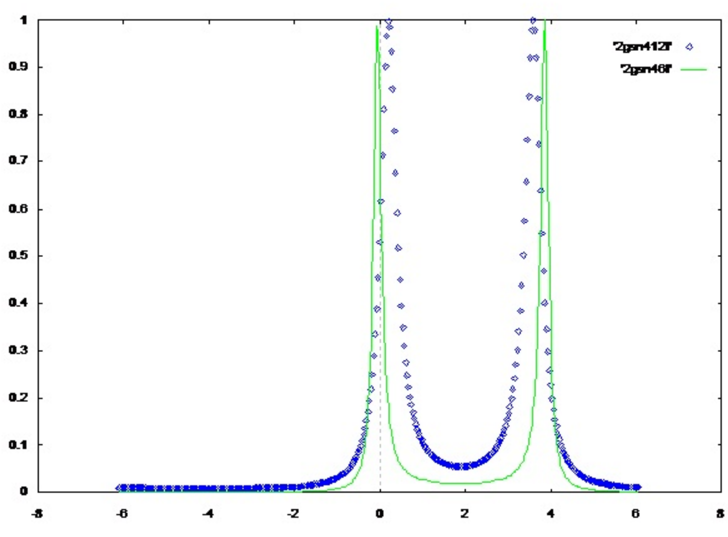

FIGURE 6. Localization of 2 sources in perturbed environment.

very low in an undisturbed environment. The evolution of λ_2 with the frequency are slightly modified by the turbulence. On the contrary, the appropriate values associated with the noise take significant values which can make it difficult to identify transmitters. The results of localization of sources are given in Figure 6, for a space$S^{\delta\theta=4}$. We notice that the positions of transmitters are perfectly determined in quiet environment. These results are still degraded when the turbulence is stronger.

It is necessary to note that the detection of transmitter is easier with a short antenna ($L = 7\lambda$), since the decorrelation introduced by the turbulence is the increasing function of the space between noise. However, in terms of the localization of sources, the effects are similar with both antennas for the same factor of superresolution. Moreover, this approach allows some parametric analyzes which are impossible to be undertaken experimentally such as the study of the localization of errors according to the angular gap between transmitters, which makes it possible to predict the role of the turbulence in a real site of observation.

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