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On a numerical approximation of a highly nonlinear parabolic inverse problem in hydrology

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Abstract. In this paper, we consider an inverse problem in hydrology governed by a highly nonlinear parabolic equation called Richards equation. This inverse problem consists to determine a set of hydrological parameters describing the flow of water in porous media, from some additional observations on pressure. We propose an approximation method of this problem based on its optimal control formulation and a temporal discretization of its state problem. The obtained discrete nonlinear state problem is approached by the finite difference method and solved by Picard's method. Then, for the resolution of the discrete associated optimization problem, we opt for an evolutionary algorithm. Finally, we give some numerical results showing the efficiency of the proposed approach.

Key words and phrases. inverse problem, hydrology, Richards equation, optimal control, evolutionary algorithm.

1. Introduction

The parameter identification problems are an important type of inverse problems, that are connected to a variety of phenomena in various scientific sectors[10]. Motivated by the various applications of flow in porous media in many fields of engineering, agricultural and chemical sciences, we are interested here by an identification inverse problem in hydrology. This problem consists in determining hydrological parameters, from additional measurements on the observed pressure. Indeed, It's well known that the flow of water in the soil is characterized by parameters, which take into account the initial condition, the boundary conditions and the soil type. However, these parameters are, in most cases, unknowns or badly measured. It is then necessary to identify all these parameters.

The study of inverse problems has several difficulties related to their nonlinearity and the fact that they are generally ill-posed in the sense of Hadamard [8]. This give a primordial importance of its formulation and requires a good knowledge of the direct problem (regularity and a priori estimation of the solution, . . .). Beside these difficulties, our considered inverse problem is governed by a highly nonlinear parabolic equation called Richards equation. At this stage, it must be mentioned that there are various numerical and theoretical investigations concerned by a direct problem governed by Richards equation. More precisely, several papers deal with the existence and uniqueness, regularity of the weak solution and analytical solution of one dimensional Richards equation (see for examples [2, 3, 4, 5, 13, 18]). Moreover, various methods for the numerical treatment of the Richards direct problem have been done in [9, 11]. Unlike to this direct problem, mathematical works concerned by the investigation of the associated inverse problem, are few in number [15], without citing

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some attempts by geophysicists, who try to implement some algorithms by using the finite difference method or finite element method or some softwares like HYDRUS [7, 17]. However, in most of all these works the proposed technics of parameters identification does not allow us to identify all the unknown parameters simultaneously, but they try to identify a single parameter by assuming that the others are known. The approximation method that we propose in this work presents the advantage of identifying all parameters simultaneously.

This paper is organized as follows. In section 2, we present the physical modeling related to the problem and its formulation. In section 3, we give an optimal control formulation of our inverse problem. The section 4 is devoted to the numerical approximation of the optimal control problem. More precisely, after a temporal discretization, the Picard's method is summarized in section 4.1. The numerical algorithm used to solve the optimization problem is given in section 4.2. Section 5 is devoted to computational aspects and numerical results of model examples.

2. Mathematical model

We consider the one-dimensional problem of the flow of water through porous media. From a physical point of view, the flow in a saturated-unsaturated porous media can be described by the equation of Richards [16], which combines the equation of mass conservation (called continuity equation) [14] and Darcy's law [19].

There are generally three main forms of Richards equation present in the literature namely the mixed formulation, the ψ-based formulation and θ-based formulation, where ψ is the weight-based pressure potential and θ is the volumetric water content. We present all these forms in the following.

Since, we consider the one-dimensional infiltration of water in vertical direction of unsaturated soil, the continuity equation and Darcy's law are then given by equations (1) and (2) respectively:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} \tag{1}$$

and

$$q = -K(\theta)\frac{\partial H}{\partial z}, \tag{2}$$

where q is the volumetric flow density, t is the time, z is the depth measured positively downward, K is the hydraulic conductivity which depends on θ and H is the hydraulic head.

The first form of Richards equation (called mixed formulation) obtained by combining the equation (1) and (2) is given by:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}[K(\theta)(\frac{\partial \psi}{\partial z} - 1)]. \tag{3}$$

Introducing the diffusivity coefficient

$$D(\theta) = K(\theta)\frac{\partial \psi}{\partial \theta}, \tag{4}$$

the second form of Richards equation (θ-based formulation) is stated as follows:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}[D(\theta)\frac{\partial \theta}{\partial z} - K(\theta)], \tag{5}$$

this equation can be used only in saturated conditions [15].

For a unsaturated environment, the equation (3) is expressed as a function of the actual pressure ψ, by introducing the concept of capillary capacity

$$C(\psi) = \frac{\partial \theta}{\partial \psi}, \tag{6}$$

the third form of Richards equation (ψ-based formulation) is given by:

$$C(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}[k(\psi)(\frac{\partial \psi}{\partial z} - 1)]. \tag{7}$$

As we see the three forms of Richards equation depends on parameters $K(\theta)$, $D(\theta)$ and $C(\psi)$. These parameters are in general empirically defined. This explain the wide and different expressions of hydrological models [15]. In order to get a complete description of infiltration process, all these hydrological parameters must be known. It is therefore necessary to develop an efficient strategy to determine these parameters based on the measure taken on the soil.

In this paper, we are interested by the flow problem described by the equation (7). Unfortunately, we can not use equation (3) which is more simpler than (7), since it does not allow us to identify all the parameters, especially the parameters involved in the capillary capacity.

Then the aim of this problem is to find the pressure ψ, the hydraulic conductivity $K(\psi)$ and the capillary capacity $C(\psi)$ solution of:

$$(P)\begin{cases} C(\psi)\dfrac{\partial \psi}{\partial t} - \dfrac{\partial}{\partial z}(k(\psi)(\dfrac{\partial \psi}{\partial z} - 1)) = f(z,t), & (z,t) \in]0,L[\times(0,T), \\ \psi(0,t) = \psi_{min}, \quad t \in (0,T), \\ \psi(L,t) = \psi_{max}, \qquad t \in (0,T), \\ \psi(z,0) = \psi_0(z), \qquad z \in]0,1[, \end{cases} \tag{8}$$

from the following additional observations on the pressure taken in N_{obs} points in the soil:

$$\psi(t,z_i) = \psi_{obs}(t,z_i) \ \ \forall t \in [0,T], \ i = 1,\dots,N_{obs} \tag{9}$$

where ψ_{min}, ψ_{max}, and ψ_0 are given functions.

3. Optimal control formulation

In the following, we will reformulate the problem (8)-(9) into an optimal control problem. For this, we consider the cost functional J defined by:

$$J(P(\psi)) = \frac{1}{2}\sum_{i=1}^{N_{obs}} \int_0^T |\psi(z,t;P(\psi)) - \delta(z,z_i)\psi_{obs}(t)|^2\, dt, \tag{10}$$

where $\psi(z,t)$ is the solution of (8) associated to $P(\psi) = (C(\psi),K(\psi))$ and $\delta(z,z_i)$ is the Dirac mass concentrated at the point z_i.
Then the optimal control problem consists to find $P^*(\psi) = [C^*(\psi),K^*(\psi)]$ solution of:

$$J(P^*(\psi)) = \inf_{P(\psi)\in\Theta_{ad}} J(P(\psi))$$

where ψ is the solution of (8) and Θ_{ad} is the set of admissible controls, defined as follows:

$$\Theta_{ad} = \{P(\psi) = (C(\psi),K(\psi)) \text{ continuous functions defined from } \mathbb{R} \text{ to } \mathbb{R} \text{ and there exist, } c_0,c_1,k_0,k_1 \geq 0 \text{ such that } c_0 \leq C(\psi) \leq c_1,\ \ k_0 \leq K(\psi) \leq k_1 \text{ a.e. in } \mathbb{R}\}.$$

We note that, for the choice of Θ_{ad}, we assume that $C(\psi)$ and $K(\psi)$ are continuous and bounded functions. In fact, this assumption is not restrictive. Indeed the capillary capacity and the hydraulic conductivity are continuous positive and bounded with respect to ψ, and independent on the characteristic of the soil [15].

Hence, our optimal control problem is summarized as follows:

$$(PO)\begin{cases}\inf\limits_{P\in\Theta_{ad}} J(\psi(P);P(\psi))\\ \text{Where } \psi(P) \text{ is the solution of (8).}\end{cases} \tag{11}$$

4. Numerical approach

In order to approximate the direct problem, we propose a time and space discretization of equation (8). The time discretization is done by using an implicit Euler scheme, while the space discretization is performed using finite difference progressive scheme. Denote by i (respectively j) the space index discretization (respectively time index discretization), Δz (respectively Δt) the space mesh size (respectively time mesh size), Then we have $z_i = i\Delta z$, for $i = 1, \ldots, N$ and $t_j = j\Delta t$, for $j = 1, \ldots, M$. We consider then an approximation of problem (8) by the discrete problem (Pd) stated as follows :

$$(Pd)\begin{cases}C_i^{j+1}\dfrac{\psi_i^{j+1}-\psi_i^j}{\Delta t} - \dfrac{1}{\Delta z}(k_{i+1}^{j+1}(\dfrac{\psi_{i+1}^{j+1}-\psi_i^{j+1}}{\Delta z}-1) - k_i^{j+1}(\dfrac{\psi_i^{j+1}-\psi_{i-1}^{j+1}}{\Delta z}-1)) = f_i^{j+1},\\ \psi_0^{j+1} = \psi_{min},\\ \psi_{N+1}^{j+1} = \psi_{max},\\ \psi_i^0 = \psi_0,\end{cases} \tag{12}$$

for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, where C_i^j, K_i^j, ψ_i^j and f_i^j are respectively, the approximates functions of $C(\psi)$, $K(\psi)$, ψ and f in the point $(z_i,\, t_j)$. Hence, the matrix form of this problem is given by

$$A(\psi^{j+1})\psi^{j+1} = B(\psi^{j+1}),$$

where ψ^{j+1} is the water pressure vector at time step $j+1$. Moreover, the matrix $A(\psi^{j+1})$ is symmetric tridiagonal whose elements are given by:

$$A_{i,i+1} = A_{i+1,i} = -K_{i+1}^{j+1}$$

$$A_{i,i} = K_i^{j+1} + K_{i+1}^{j+1} + \frac{\Delta z^2}{\Delta t}C_i^{j+1}$$

and $B(\psi^{j+1})$ is the global vector whose elements are functions of the hydraulic conductivity and capillary capacity,

$$B(i) = \frac{\Delta z^2}{\Delta t}C_i^{j+1}\psi_i^j + (K_i^{j+1} - K_{i+1}^{j+1})\Delta z + \Delta z^2 f_i^{j+1}.$$

4.1. Picard's method. The successive iteration of Picard is a linearization method widely used by several authors (see for example [1]). This process gives a sequence of functions, which converges asymptotically to the solution. This method consists, at each time step $j+1$, to construct a sequence $(\psi^{j+1,m})_m$, from a given $\psi^{j+1,0}$, such that

$$A(\psi^{j+1,m})\psi^{j+1,m+1} = B(\psi^{j+1,m}).$$

By introducing the pressure vector $\Delta\psi^{j+1,m+1} = \psi^{j+1,m+1} - \psi^{j+1,m}$, the previous system is written as follows:

$$A(\psi^{j+1,m})\Delta\psi^{j+1,m+1} = B(\psi^{j+1,m}) - A(\psi^{j+1,m})\psi^{j+1,m}. \tag{13}$$

Then the considered Picard algorithm is summarized as follows: (for a given precision ε)

Step 1 : Put $\psi^{j+1,0} = \psi^j$ the initial vector where ψ^j is the pressure at time t_j.

Step 2 : For $m = 0, 1, 2, ..., N$ do

- Build the system (13).
- Solve the system (13) for $\Delta\psi^{j+1,m+1}$.
- Calculate the new solution $\psi^{j+1,m+1} = \Delta\psi^{j+1,m+1} + \psi^{j+1,m}$.
- Test if $|\Delta\psi^{j+1,m}| < \varepsilon$, then $\psi^{j+1,m+1}$ is the approximate solution at time step $j+1$,
  else $m = m+1$ and go to step 2.

4.2. Numerical algorithms. For the discretization of the optimal control problem, let us define a family of discrete admissible controls

$$\Theta^h_{ad} = \{(C_h, K_h) \in C([0,1])^2 \mid C_h|_{[z_m,z_{m+1}]} \text{ and } K_h|_{[z_m,z_{m+1}]} \in \mathbb{P}_1([z_m, z_{m+1}]), \\ \forall m = 1, \dots, N\} \cap \Theta_{ad},$$

and we approach the cost functional by the following discrete one

$$J_h(P_h(\psi_h)) = \frac{1}{2}\sum_{m=1}^{N}\sum_{j=0}^{M}\int_{t_j}^{t_{j+1}} (\psi_m(t) - \psi_{obs,m}(t))^2\, dt,$$

where $\psi_m(t)$ is the solution of (12) in all time in the point z_m and h is the space mesh size.

We state our discrete optimal shape problem as follows

$$(PO)\begin{cases} \inf\limits_{P_h\in\Theta^h_{ad}} J_h(\psi_h(P_h); P_h(\psi_h)) \\ \text{where } \psi_h(P_h) \text{ is the solution of (12).} \end{cases} \tag{14}$$

with $\psi_h = (\psi^j_m)_{1\le m\le N,\,1\le j\le M}$.

To solve this discrete optimal problem (14), we developed a numerical algorithm based on a genetic algorithm procedure [12]. Genetic algorithms (GA), primarily developed by Holland [6], have been successfully applied to various optimization problems. It is essentially a searching method based on the Darwinian principles of biological evolution. In GA a new generation of individuals is produced using the simulated genetic operations crossover and mutation. The probability of survival of generated individuals depends of their fitness: the best ones survive with the high probability, the worst die rapidly. This procedure can be summarized in the following algorithm see [12].

(1) Iteration $k = 0$, generate randomly an admissible population.
(2) Solve (8) for each individual of population.
(3) Evaluate the fitness (10) for each individual of population.
(4) If the termination criteria is hold $J < \epsilon$, then stop.
    Else set $k = k+1$ and go to step 5.
(5) Roulette wheel selection
(6) Applied to the selected individuals, the barycenter crossover procedure.
(7) Select randomly some individual, and applied to them the mutation.
(8) Go to step 2

5. Numerical results

In this section, we present some numerical results in which the exact functions $C(\psi)$ and $K(\psi)$ are known. For different configurations, one is asked to reconstruct the exact functions by using the optimal control formulation (14). In all the following numerical examples, we solve the identification problem considering the domain $\Omega = [0, 1]$.

5.1. Validation of the method against an exact solution. We consider an example in which the exact solution is known analytically. Indeed, for these given data

$$\psi_0 = z, \quad \psi_{min}(t) = t, \quad \psi_{max}(t) = 1 + t \quad \text{and } f = 0,$$
$$C(\psi) = 2\psi \qquad K(\psi) = \psi^2,$$

the exact solution is:

$$\psi_{exacte} = z + t.$$

For all following numerical examples, we take these numerical data

$$N = 11, \quad M = 10, \quad \epsilon = 10^{-5}, \quad N_{obs} = 1, \quad z_{obs} = 0.5$$

For the evolutionary algorithm, after several numerical tests, the optimal numerical data allowing us to get a better solution with a reasonable computational cost are:

population size = 10, crossover probability = 0.6, probability of mutation = 5%

In order to study the numerical behavior and show the efficiency of our approach, different configurations are investigated. Through some numerical examples, we show the convergence and the performance of our approach.

First configuration:

case (a): In this case, we suppose that the hydraulic conductivity $K(\psi)$ is known, and capillary capacity $C(\psi)$ is a linear function.

$$C(\psi) = \alpha\psi \quad \text{and} \quad K(\psi) = \psi^2$$

The identification problem in this case, is reduced to find the parameter α.

case (b): In this new case, the hydraulic conductivity is the same as in the case (a), but the capillary capacity is supposed to be an affine function.

$$C(\psi) = 2\psi + \beta \quad \text{and} \quad K(\psi) = \psi^2,$$

we look to identify the parameter β. In the Figure 1, we illustrate the decrease

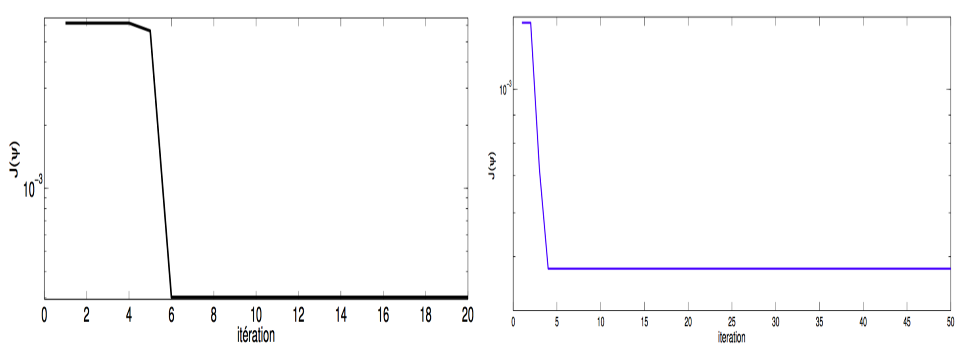

FIGURE 1. Cost functional.

of the cost functional J_h with the iteration numbers. We note that, for these two cases, the cost function decreases quickly, in about four order of magnitude occurring over 5 or 6 iterations. The approached values of α and β are:

$$\alpha_{app} = 2.01973 \quad \text{and} \quad \beta_{app} = 0.03122;$$

It can be seen that we have obtained a good approximations of the parameters α and β.

case (c): We keep the same data as in the case (b), and we look to identify the both parameters α and β simultaneously

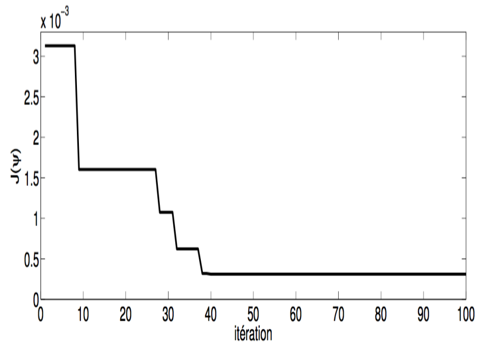

FIGURE 2. Cost functional.

In Figure 2, we present the variation of the cost function with respect to the iteration number. In this case, we note that at the first five iterations, the cost function decreases quickly to reach a precision 1.510^{-2}, then the cost function continues to decrease until it reaches a precision less than 5.10^{-3} in 40 iterations. The approached values of α and β are:

$$\alpha_{app} = 2.197 \quad \text{and} \quad \beta_{app} = 0.0223;$$

We note that in this case the accuracy is not better than the previous cases, in fact, this is due to the difficulty of approximating both of the parameters simultaneously.

Second configuration:

case (a): In this case, we suppose that the capillary capacity $C(\psi)$ is known, and hydraulic conductivity $K(\psi)$ is a quadratic function of ψ.

$$C(\psi) = 2\psi \quad \text{and} \quad K(\psi) = \lambda\psi^2.$$

The identification problem, in this case, is reduced to find the parameter λ.

case (b): We maintain the same function $C\psi)$ as in the previous case, and we introduce another parameter σ in the expression of hydraulic conductivity $K(\psi)$.

$$C(\psi) = 2\psi \quad \text{and} \quad K(\psi) = \psi^2 + \sigma,$$

we seek in this stage, to identify the parameter σ.

In the Figure 3, we illustrate the decrease of the cost functional J_h with the iteration number. It is noted that in these two cases, the functional J_h doesn't decreases quickly as in the first configuration, which justifies the difficulty related to the determination of hydraulic conductivity. The approached values of λ and σ are:

$$\lambda_{app} = 0.9873 \quad \text{and} \quad \sigma_{app} = 0.03.$$

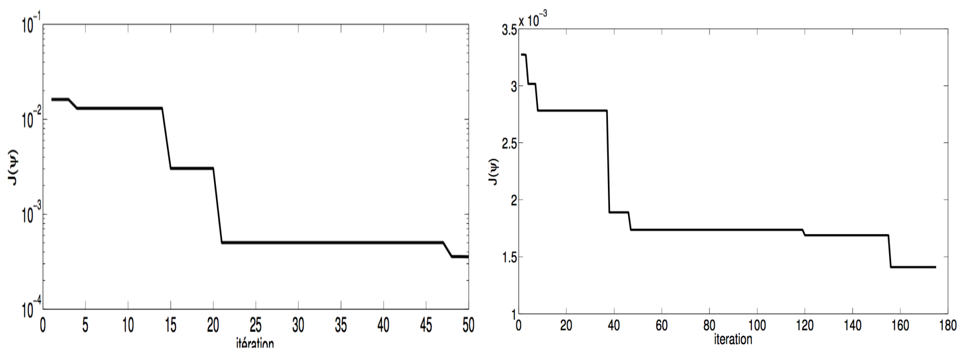

FIGURE 3. Cost functional.

also, in this case we obtains an excellent approximation of approached values.

case (c): As in the case (c) of the first configuration, we keep the same functions for $C(\psi)$ and $K(\psi)$, and we are interested in identifying the parameters λ and σ simultaneously.

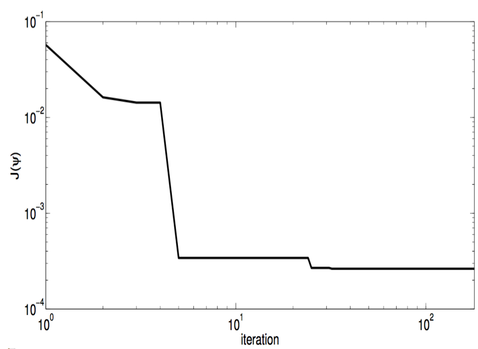

FIGURE 4. Cost functional.

The Figure (4) shows the decrease of the cost function with respect to iteration number, which decreases slowly compared to the case (c) of the first configuration. Then, the functional continues to decreased until it reaches the precision 2.10^{-3} at iteration 50. The approached values of λ and σ are:

$$\lambda_{app} = 0.9642 \quad \text{and} \quad \sigma_{app} = 0.0324.$$

The accuracy in this case is great, in fact, this case demonstrates the complex nature of this problem.

Third configuration:

: In this case, the previous configurations will be mixed, we assume that the capillary capacity is linear and the hydraulic conductivity is quadratic.

$$C(\psi) = \alpha\psi + \beta \quad \text{and} \quad K(\psi) = \lambda\psi^2 + \sigma,$$

Here, we are looking for the identification of these four parameters simultaneously. In the Figure 5, we give the variation of the cost functional with respect to the number of iteration, we note that for this configuration, the cost functional decrease slowly compared to the previous configurations. We note that in this case, despite the complicated nature of the problem due to to the fact that we

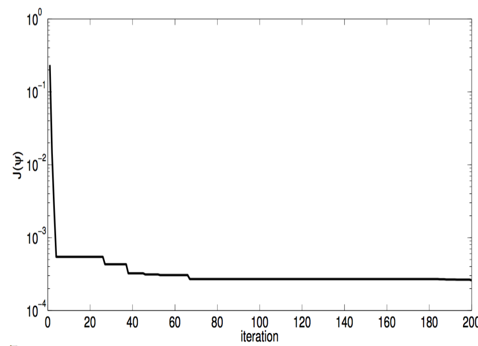

FIGURE 5. Cost functional.

have to identify four parameters from a single observation, the numerical results remain good. The approached values in this case are:

$$\alpha_{app} = 2.134, \quad \beta_{app} = 0.052, \quad \lambda_{app} = 0.9342 \quad \text{and } \sigma_{app} = 0.042.$$

Conclusion

In this work, we have proposed an effective method for the approximation of an inverse problem governed by a highly nonlinear equation. This method is based on its formulation on an optimal control problem and its discretization by a finite difference method combined with the Picard's iteration. The discrete associate optimization problem is solved by an evolutionary algorithm. This method has the advantage of identifying all parameters simultaneously with a reasonable computational cost, even if we take only a single observation data ($N_{obs} = 1$).

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