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A fuzzy mathematical multi-period multi-echelon supply chain model based on extension principle

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Abstract. This paper deals with multi-echelon integrated purchase, production and distribution planning model in a supply chain system. The manufacturer procures raw material from suppliers then proceed to convert it as finished product, and finally delivers to the distribution centers in order to minimize the total cost of the chain, which faces imprecise and ill-known data, called fuzzy supply, process and demand of customers.

The model has been formulated as a fuzzy linear programming model. Here, the triangular fuzzy numbers are considered because the triangular form is the simplest type of fuzzy numbers and gives the most important information about a fuzzy number. The main objective of this paper is to solve fuzzy linear programming problems more efficiently.

In order to secularize the fuzzy linear programming model, an evaluation method is used wherein the proposed approach enables the decision maker to obtain alternative decision plans with different degrees of satisfaction. A noteworthy feature of this approach is that it is able to find the membership function of the fuzzy objective value and decision variables which is derived numerically by enumerating different values of α-cuts of the fuzzy triangular number. Finally, a numerical example is presented to clarify the features of the proposed approach.

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1. Introduction and literature review

A Supply Chain (SC) network comprises a number of facilities (e.g., suppliers, manufacturing plants, distribution centers, etc.) that perform a set of operations ranging from the acquisition of raw materials, transformation of these materials into final products, and transportation of the final products to distributors as shown in Fig.1. Huang et al. [12] describe a supply chain as a network of facilities that procure raw materials, transform them into intermediate goods and then final products, and deliver the products to customers through a distribution system.

Today, many companies operate and compete in a global environment. International business environment has forced many firms to focus on supply chain management. A well-structured supply chain is an important strategic competency that enables firms to be competitive in today's marketplace. Every product that reaches an end user represents the cumulative effort of multiple organizations.

Several authors have studied the modeling of purchase- production- distribution planning with different case studies and supply chain planning processes through mathematical programming models. In what follows, we review some papers that have dealt with uncertainty and different approaches toward purchase- production- distribution planning models.

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1.1. Purchase-production distribution planning within supply chain. Integrated Purchase, Production and Distribution Planning (PPDP) problem in a SC is a challenging problem as companies move towards higher collaborative and competitive environments. In real-world PPDP problems, the decision maker attempts to achieve the following: (1) set overall production levels for each product category for each source (producer) to meet fluctuating or uncertain demand for various destinations (distributors) over the planning horizon, and (2) make right strategies regarding production, subcontracting, back ordering, inventory and distribution levels, and thus determining appropriate resources to be used (see Cohen and Lee [8], Thomas and Griffin [28], Vidal and Goetschalckx [30], Gheidar-Kheljani et al. [11]).
Based on the characteristics of the problem, the most relevant and recent literature on PPDP are reviewd. Several authors have studied the modeling of PPDP in supply chains (see, for instance, Ereng, Simpson and Vakharia [9], Sarmiento and Nagi [26], Bilgen and Ozkarahan [5], Arshinder et al. [2], Peidro et al. [23], Chandra and Fisher [6], Mula et al. [19], Bhatnagar and Sohal [4] and Paksoya and Pehlivan [20]).
The articles reviewed include mainly classic operations research methods and consider one (or more than one) of the following aspects of Supply Chain Management (SCM) such as plant design, production scheduling, distribution and inventory management.

1.2. Fuzzy applications of supply chain planning problems. Some of the models stated above assume the parameters that influence the design decisions to be deterministic. However, the complex nature and dynamics of the relationships among the different actors of supply chains imply an important grade of uncertainty in the planning decisions. These challenges lead to an increased interest in stochastic programming and Fuzzy Mathematical Programming (FMP). A number of researchers have proposed stochastic SCM models that are closer to real situations. Most research has modeled the SC uncertainty (e.g., uncertain demand) by probability distribution that is usually predicted from historical data. However, whenever statistical data is unreliable or even unavailable, stochastic models may not be the best choice. Fuzzy set theory may provide an alternative approach for dealing with the SC uncertainty (see Lai and Hwang [13]). Fuzzy set theory was proposed by Zadeh [33, 34] and has been found extensive applications in various fields such as operations research, management science, control theory and artificial intelligence. Fuzzy sets theory has been implemented in mathematical programming since 1970 when Bellman and Zadeh [3] introduced the basic concepts of fuzzy goals, fuzzy constraints, and fuzzy decisions. A detailed discussion of the FMP procedures can be found in Lai and Hwang [13], Zimmermann [35]. A few studies have attempted to model integrated PPDP problem in a fuzzy environment. On the matter of integrated planning, the work of Chen and Chang [7] simultaneously handle multi-product, multi-echelon and multi-period SC model with fuzzy parameters and they propose a solution procedure that is able to calculate the fuzzy objective value of the fuzzy SC model. Aliev et al. [1] developed a fuzzy integrated multi-period and multi-product production and distribution model in SC, which was formulated in terms of fuzzy programming and the solution was provided by genetic algorithm. Torabi and Hassani [29] proposed a multi-objective possiblistic mixed integer linear programming model for integrating procurement, production and distribution planning by considering various conflicting objectives simultaneously as well as the imprecise nature of some critical parameters such as market demands, cost/time coefficients and capacity levels. Liang [16] developed a fuzzy multi-objective linear programming model to simultaneously minimize total costs and total delivery time in a supply chain, adopting the fuzzy goal programming method. Liang and Cheng [18] applied fuzzy sets to multi-objective manufacturing/distribution planning decision problems with multi-product and multi-time period in supply chains by considering time value of money for each of the operating categories. Peidro et al. [23] proposed a new mathematical programming model for supply chain planning under supply, process and demand uncertainty. The model has been formulated as a fuzzy mixed integer linear programming model where data are ill-known and modeled by triangular fuzzy numbers. Xu and Zhai [31] considered a two-stage SC coordination problem under fuzzy demand constraints. They investigated the optimization of the vertically integrated two-stage SC under perfect coordination and contrast with the non-coordination in case of the fuzzy demand. This paper proposes a practical, but tractable, fuzzy mathematical programming model under supply, capacity and demand uncertainty. It has been recognized that although there are many papers in the published literature on the optimization of the supply chain networks for different design of the chains, only few of them concern the solution approach. Most supply chain models in fuzzy environment deal with integer variables and cannot be solved by classical linear programming approaches. Different solution approachs are introduced. Approaches which the fuzzy integer model is transformed into an equivalent crisp model and the objective and variables are obtained as crisp values.

Also, many papers consider heuristic algorithms such as genetic algorithms for solving problems by integer variables [1, 10, 32]. Intuitively, in a fuzzy environment, a fuzzy decision should be made to meet the decision criteria and the objective and the variables must be obtaind fuzzy and based on the decision makers satisfication.

In this paper, a new fuzzy integrated PPDP problem in a SC is proposed which is able to handle the epistemic uncertainty in parameters in real cases results from unavailability or incompleteness and imprecise nature of input data. The uncertain input data are assumed to be triangular fuzzy numbers. This kind of fuzzy number gives the most important information about a fuzzy number: lower and upper bounds of the number and its most possible value. Moreover, many other types of fuzzy numbers can be expressed and estimated with this simple form of fuzzy number.

The resulting optimization problem has been solved using Zadeh,s extension principle [33] in order to calculate the upper and lower bounds of the objective value at possibility level α. The solution procedure is able to calculate the fuzzy objective value of the fuzzy PPDP problem, where at least one of the parameters is fuzzy numbers. In this solution the model is solved at different α levels to approximate the membership functions and provides more information for decision makers.

The remainder of this paper is arranged as follows. Section 2 describes the problem and formulates the original PPDP problems. Then in Section 3, a new Fuzzy Linear Programming (FLP) model for the PPDP problems under uncertainty is proposed. In Section 4, appropriate strategies for converting the fuzzy model into a pair of crisp linear programming models are applied. The proposed model is implemented for an numerical example and the computational results are reported in Section 5. Finally, the conclusions are provided in Section 6.

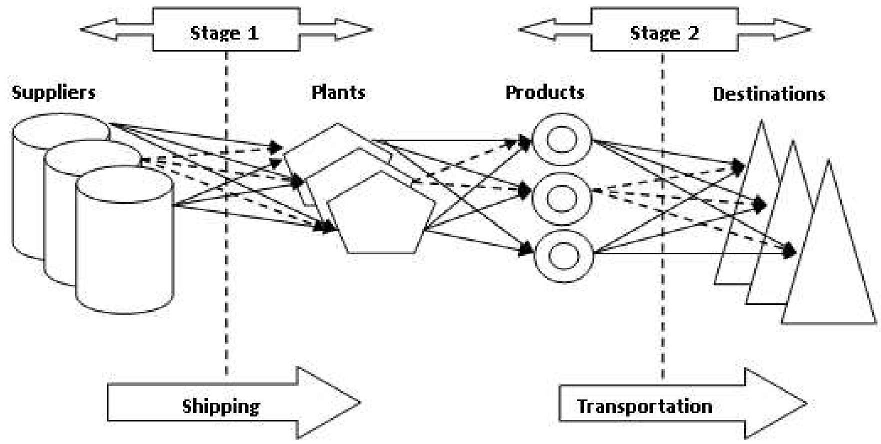

FIGURE 1. Two stage supply chain network

2. Problem description

The PPDP problems involves optimizing the transportation plan for allocating raw materials from a set of suppliers to a set of plants and goods and/or services from a set of plants to various destinations in a SC. In real-world SC network problems, environmental coefficients and related parameters, including supply, market demand and unit cost/time coefficients, available labor levels and machine capacity, are normally fuzzy/imprecise because of some information being incomplete and/or unobtainable over the intermediate planning horizon. It is critical that the satisfying goal value should normally be fuzzy/imprecise as the unit cost/time coefficients and parameters are vague and such imprecision always exists in real-world SC network problems in supply chains [21, 22]. The conventional solution methods and algorithms cannot solve all realistic problems in uncertain environments. This work focuses on developing a solution procedure that is able to calculate the fuzzy objective value, where at least one of the parameters is a fuzzy number. Fig.1 shows the integrated PPDP problems in a network form.

The mathematical programming model formulated here is based on the following assumptions:

1. The objective function is fuzzy with imprecise aspiration levels.
2. The objective function and constraints are linear equations.
3. The purchase costs from each source, production costs at each plant and distribution cost on a given route are directly proportional to the units shipped, manufactured and transferred capacity per truck, respectively.
4. The pattern of triangular distribution is adopted to represent all of the fuzzy/ imprecise numbers.
5. Supply, capacities and demands are fuzzy.

Assumption 1 relates to the fuzziness of the objective functions in practical SC optimization problems and incorporates the variations in the decision maker judgments regarding the solutions of fuzzy optimization problems in a framework of imprecise aspiration levels. Assumptions 2 and 3 indicate that the linearity and proportionality properties must be technically satisfied as a standard linear programming form. Assumption 4 concerns the simplicity and flexibility of the fuzzy arithmetic operations. Triangular distribution

is utilized to represent all of the fuzzy numbers and thus enhance the computational efficiency and facilitate data acquisition. Assumption 5 relates the imprecise data in real cases. In this section, we developed a mathematical model to quantify the relationship among all the decision variables involved in multi-echelon supply chain network. The problem of optimizing the supply chain configuration can be summarized in the following mathematical model.
Sets of indices, parameters and decision variables for the FLP model are defined in the the following:

- *Index sets*
  - i Index of plants $i = 1, 2, ..., I$
  - n Index for products $n = 1, 2, ..., N$
  - j Index for destinations $j = 1, 2, ..., J$
  - p Index for suppliers $p = 1, 2, ..., P$
  - m Index for raw materials $m = 1, ..., M$
  - h Index for periods $h = 1, ..., H$
- *Decision variables*

Q_{inh} Amount of product n produced in plant i at period h ($units$)
IN_{inh} Inventory level at plant i of product n at period h ($units$)
R_{injh} Quantity delivered from the plant i to destination j of product n at period h ($units$)
V_{inh} Subcontract of product n in the plant i at period h ($units$)
L_{mpih} Quantity of raw material m shipped from supplier p to plant i at period h ($units$)
IM_{mih} Inventory of raw material m at plant i at period h ($units$)
LS_{njh} The amount of lost sale for product n at destination j at period h ($units$)

- *Parameters*

λ_{inh} Production cost per unit of product n produced at plant i in period h ($\$\backslash unit$)
γ_{inh} Subcontracting cost per unit of product n at plant i in period h ($\$/unit$)
φ_{mih} Inventory cost for raw material m at plant i in period h ($\$/unit$)
η_{inh} Inventory cost per unit of product n at plant i in period h ($\$/unit$)
θ_{injh} Transport cost per unit of product n from plant i to destination j in period h ($\$/unit$)
μ_{mpih} Shipping cost per unit of raw material m from supplier p to plant i in period h ($\$/unit$)
ψ_{mph} Purchasing cost per unit of raw material m provided by supplier p in period h ($\$/unit$)
π_{nh} Lost sale cost per unit of product n in period h ($\$/unit$)
N_{mn} Raw material m needed by each product n
HL_{in} Hour of labor per unit of product n produced at plant i ($man - hour/unit$)
HM_{in} Hour of machine per unit of product n produced at plant i ($machine - hour/unit$)
W_{pn} Warehouse space required per unit of product n ($ft^2/unit$)
WM_m Warehouse space required per unit of raw material m ($ft^2/unit$)
MP_{ih} Final product storage capacity at plant i in period h ($ft^2/unit$)

S_{ih}	Raw material storage capacity at plant i in period h ($ft^2/unit$)
$\tilde{D}_{njh}$	Demand of product n at destination j in period h ($units$)
$\tilde{ML}_{ih}$	Maximum labor level of work for plant i in period h ($man - hour$)
$\tilde{MM}_{ih}$	Maximum machine capacity available for plant i in period h ($machine - hour$)
$\tilde{MS}_{mph}$	Maximum capacity of supplier p for raw material m in period h ($unit$)

In real-world situations, most PPDP problems minimize total costs, delivery time, and/or maximize profits. Furthermore, in most practical decisions involved in business PPDP problems the Decision Maker (DM) considers generally related operating costs, inventory and subcontracting levels, available resources and capacities, market demand, product life cycle, employment law and other factors, to minimize total costs, delivery time, and/or maximize profits. In particular, these objective functions are fuzzy in nature owing to incomplete and/or unavailable information over the planning. The proposed fuzzy single-objective linear programming model attempts to simultaneously minimize the total costs such as shipping cost, regular manufacture cost, subcontracting cost, inventory carrying cost and delivery cost. Accordingly, the achieving objective is simultaneously considered in formulating the original fuzzy integrated PPDP model, as follows:

- *Minimize total costs:*

$$\min Z \cong \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H}\lambda_{inh}Q_{inh} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H}\gamma_{inh}V_{inh} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H}\eta_{inh}IN_{inh} \tag{1}$$
$$+ \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H}\mu_{mpih}L_{mpih} + \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H}\psi_{mpih}L_{mpih}$$
$$+ \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H}\theta_{injh}R_{injh} + \sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{h=1}^{H}\phi_{mih}IM_{mih} + \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H}\pi_{nh}LS_{njh}$$

- *Constriants:*

(a_1) *Constraints on carrying inventory levels*

$$IN_{inh} = IN_{inh-1} + Q_{inh} + V_{inh} - \sum_{j=1}^{J}R_{injh} \quad \forall i,n,h \tag{2}$$

$$IM_{mih} = IM_{mih-1} + \sum_{p=1}^{P}L_{mpih} - \sum_{n=1}^{N}N_{mn}Q_{inh} \quad \forall i,m,h \tag{3}$$

(a_2) *Constraints on demand for each destination* j

$$\sum_{i=1}^{I}R_{injh} + LS_{njh} = \tilde{D}_{njh} \quad \forall n,j,h \tag{4}$$

(a_3) *Constraints on available capacity*

$$\sum_{n=1}^{N}HL_{in}Q_{inh} \le \tilde{ML}_{ih} \quad \forall i,h \tag{5}$$

$$\sum_{n=1}^{N}HM_{in}Q_{inh} \le \tilde{MM}_{ih} \quad \forall i,h \tag{6}$$

$$\sum_{n=1}^{N} WP_n IN_{inh} \leq MP_{ih} \quad \forall i, h \tag{7}$$

$$\sum_{m=1}^{M} WM_m IM_{mih} \leq S_{ih} \quad \forall i, h \tag{8}$$

$$\sum_{i=1}^{I} L_{mpih} \leq \tilde{M}S_{mph} \quad \forall p, m, h \tag{9}$$

(a_4) *Non-negativity constraints on decision variables*

$$Q_{inh}, V_{inh}, IN_{inh}, IM_{mih}, R_{injh}, L_{mpih}, LS_{njh} \geq 0, \quad \forall i, n, h, j, p, m \tag{10}$$

The symbol in Eq. 1 is the fuzzified form of and demonstrates the fuzzification of the aspiration levels. When the parameters in the problem are imprecise, the calculated objective value and the decision variables should be imprecise as well. It means that the DM has fuzzy objective. In practical situations, the market demand on right- hand sides of constraint 4 for each destination can never be determined exactly. Moreover, the available resources on the right-hand sides of constraints 5, 6 and 9 over the planning horizon, are often fuzzy/imprecise since they cannot be measured easily and they mainly imply human perception for their estimation. In practice, constraints 7 and 8, which respectively represent the limiting capacity of each plant and warehouse space, are normally certain.

We adopted the triangular fuzzy number to the PPDP network model to represent all of the fuzzy/imprecise numbers. The main advantages of the triangular fuzzy number are simplicity and flexibility of the fuzzy arithmetic operations [14, 25, 27, 35]. The distribution of a triangular fuzzy number $\tilde{A}_i = (A_i^p, A_i^m, A_i^o)$ is shown in Fig.2. Practically, the DM can construct the triangular distribution based on the following three prominent data [15, 17, 24]: The most pessimistic value is A_i^p; the most likely value is A_i^m; the most optimistic value is A_i^o.

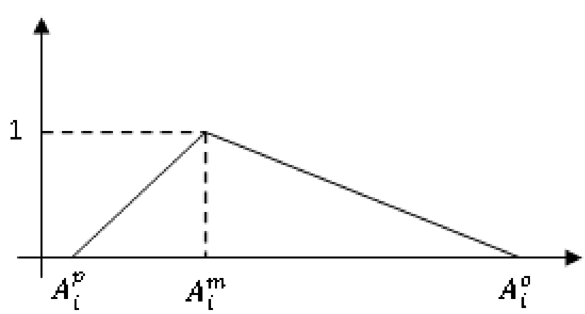

FIGURE 2. The distribution of triangular fuzzy number A_i^o

3. Treatment of the fuzzy constraints

As explained in the previous section, some parameters in the model are fuzzy in nature. Therefore, the decision variables and the objective function should be fuzzy as well and we are interested in deriving the membership functions of the objective function and the decision variables. In this section, we define an approach to transform the FLP model into an equivalent auxiliary crisp LP model for SC planning under supply, process and demand uncertainties. Recalling constraints 4, 5, 6 and 9 from the original fuzzy PPDP model formulated above, considers situations in which market demand, D_{njh}, labor levels, ML_{ih}, machine capacity, MM_{ih} and supplier capacity, MS_{mph}, are triangular fuzzy numbers with most and least likely values. We suppose that the approximately known parameters can be represented by the convex fuzzy sets $\tilde{D}_{njh}$, $\tilde{ML}_{ih}$, $\tilde{MM}_{ih}$ and $\tilde{MS}_{mph}$. Let $\mu_{\tilde{D}_{njh}}$, $\mu_{\tilde{ML}_{ih}}$, $\mu_{\tilde{MM}_{ih}}$ and $\mu_{\tilde{MS}_{mph}}$ denote their membership functions, respectively. We have

$$\tilde{D}_{njh} = \left\{ \left(D_{njh}, \mu_{\tilde{D}_{njh}}\left(D_{njh}\right) \right) | D_{njh} \in S\left(\tilde{D}_{njh}\right) \right\}$$
$$\tilde{ML}_{ih} = \left\{ \left(ML_{ih}, \mu_{\tilde{ML}_{ih}}\left(ML_{ih}\right) \right) | ML_{ih} \in S\left(\tilde{ML}_{ih}\right) \right\}$$
$$\tilde{MM}_{ih} = \left\{ \left(MM_{ih}, \mu_{\tilde{MM}_{ih}}\left(MM_{ih}\right) \right) | MM_{ih} \in S\left(\tilde{MM}_{ih}\right) \right\}$$
$$\tilde{MS}_{mph} = \left\{ \left(MS_{mph}, \mu_{\tilde{MS}_{mph}}\left(MS_{mph}\right) \right) | MS_{mph} \in S\left(\tilde{MS}_{mph}\right) \right\} \tag{11}$$

where $S\left(\tilde{D}_{njh}\right)$, $S\left(\tilde{ML}_{ih}\right)$, $S\left(\tilde{MM}_{ih}\right)$ and $S\left(\tilde{MS}_{mph}\right)$ are the supports of $\tilde{D}_{njh}$, $\tilde{ML}_{ih}$, $\tilde{MM}_{ih}$ and $\tilde{MS}_{mph}$ which denote the universe sets of the market demand, labor levels, machine capacity and supplier capacity, respectively. Without loss of generality, all $\tilde{D}_{njh}$, $\tilde{ML}_{ih}$, $\tilde{MM}_{ih}$ and $\tilde{MS}_{mph}$ are assumed to be convex fuzzy numbers, as crisp values can be represented by degenerated membership functions which only have one value in their domains. Denote the $\alpha-$cuts of $\tilde{D}_{njh}$, $\tilde{ML}_{ih}$, $\tilde{MM}_{ih}$ and $\tilde{MS}_{mph}$ as

$$\left(D_{njh}\right)_\alpha = \left[\left(D_{njh}\right)^L_\alpha, \left(D_{njh}\right)^U_\alpha \right] = [\min_{D_{njh}} \{ D_{njh} \in S\left(\tilde{D}_{njh}\right) | \mu_{\tilde{D}_{njh}}\left(D_{njh}\right) \geq \alpha \},$$
$$\max_{D_{njh}} \{ D_{njh} \in S\left(\tilde{D}_{njh}\right) | \mu_{\tilde{D}_{njh}}\left(D_{njh}\right) \geq \alpha \}].$$
$$\left(ML_{ih}\right)_\alpha = \left[\left(ML_{ih}\right)^L_\alpha, \left(ML_{ih}\right)^U_\alpha \right] = [\min_{ML_{ih}} \{ ML_{ih} \in S\left(\tilde{ML}_{ih}\right) | \mu_{\tilde{ML}_{ih}}\left(ML_{ih}\right) \geq \alpha \},$$
$$\max_{ML_{ih}} \{ ML_{ih} \in S\left(\tilde{ML}_{ih}\right) | \mu_{\tilde{ML}_{ih}}\left(ML_{ih}\right) \geq \alpha \}].$$
$$\left(MM_{ih}\right)_\alpha = \left[\left(MM_{ih}\right)^L_\alpha, \left(MM_{ih}\right)^U_\alpha \right] =$$
$$[\min_{MM_{ih}} \{ MM_{ih} \in S\left(\tilde{MM}_{ih}\right) | \mu_{\tilde{MM}_{ih}}\left(MM_{ih}\right) \geq \alpha \},$$
$$\max_{MM_{ih}} \{ MM_{ih} \in S\left(\tilde{MM}_{ih}\right) | \mu_{\tilde{MM}_{ih}}\left(MM_{ih}\right) \geq \alpha \}].$$
$$\left(MS_{mph}\right)_\alpha = \left[\left(MS_{mph}\right)^L_\alpha, \left(MS_{mph}\right)^U_\alpha \right] =$$
$$[\min_{MS_{mph}} \{ MS_{mph} \in S\left(\tilde{MS}_{mph}\right) | \mu_{\tilde{MS}_{mph}}\left(MS_{mph}\right) \geq \alpha \},$$
$$\max_{MS_{mph}} \{ MS_{mph} \in S\left(\tilde{MS}_{mph}\right) | \mu_{\tilde{MS}_{mph}}\left(MS_{mph}\right) \geq \alpha \}]. \tag{12}$$

These intervals indicate where the approximately known parameters lie at possibility level α. We are interested in deriving the membership function of the objective value $\tilde{Z}$. Since $\tilde{Z}$ is a fuzzy number rather than a crisp number, we apply Zadeh's extension principle [33] to transform the problem into a family of conventional programs to be solved. Based on the extension principle, the membership function $\mu_{\tilde{Z}}$ can be defined as

$$\mu_{\tilde{Z}}(z) = \sup_{a,b,c,d} \min\{ \mu_{\tilde{D}_{njh}}\left(D_{njh}\right), \mu_{\tilde{ML}_{ih}}\left(ML_{ih}\right), \mu_{\tilde{MM}_{ih}}\left(MM_{ih}\right),$$
$$\mu_{\tilde{MS}_{mph}}\left(MS_{mph}\right) \forall n,h,i,j,p,m | z = Z\left(a,b,c,d\right) \} \tag{13}$$

where $Z(a,b,c,d)$ is the function of the conventional linear program. In Eq.11, several membership functions are involved. To derive $\mu_{\tilde{Z}}$ in closed form is hardly possible. According to Eq. 13, $\mu_{\tilde{Z}}$ is the minimum of $\mu_{\tilde{D}_{njh}}$, $\mu_{\tilde{ML}_{ih}}$, $\mu_{\tilde{MM}_{ih}}$ and $\mu_{\tilde{MS}_{mph}}$, $\forall n,h,i,j,p,m$. It is needed $\mu_{\tilde{D}_{njh}}(D_{njh}) \geq \alpha$, $\mu_{\tilde{ML}_{ih}}(ML_{ih}) \geq \alpha$, $\mu_{\tilde{MM}_{ih}}(MM_{ih}) \geq \alpha$ and $\mu_{\tilde{MS}_{mph}}(MS_{mph}) \geq \alpha$ and at least one $\mu_{\tilde{D}_{njh}}(D_{njh})$, $\mu_{\tilde{ML}_{ih}}(ML_{ih})$, $\mu_{\tilde{MM}_{ih}}(MM_{ih})$ or $\mu_{\tilde{MS}_{mph}}(MS_{mph})$, $\forall n,h,i,j,p,m$ to be equal to α such that $z = Z(a,b,c,d)$ to satisfy $\mu_{\tilde{Z}}(z) = \alpha$. To find the membership function $\mu_{\tilde{Z}}$, it suffices to find the right shape function and left shape function of $\mu_{\tilde{Z}}$, which is equivalent to finding the upper bound of the objective value Z_α^U and lower bound of the objective Z_α^L at specific α level. Since Z_α^U is the maximum of $Z(a,b,c,d)$ and Z_α^L is the minimum of $Z(a,b,c,d)$, they can be expressed as the following:

$$\begin{aligned} Z_\alpha^U = \max\{ & Z(a,b,c,d) \,|\, (D_{njh})_\alpha^L \leq D_{njh} \leq (D_{njh})_\alpha^U, (ML_{ih})_\alpha^L \leq ML_{ih} \leq (ML_{ih})_\alpha^U, \\ & (MM_{ih})_\alpha^L \leq MM_{ih} \leq (MM_{ih})_\alpha^U, (MS_{mph})_\alpha^L \leq MS_{mph} \leq \\ & (MS_{mph})_\alpha^U, \forall n,h,i,j,p,m\} \end{aligned} \tag{14}$$

$$\begin{aligned} Z_\alpha^L = \min\{ & Z(a,b,c,d) \,|\, (D_{njh})_\alpha^L \leq D_{njh} \leq (D_{njh})_\alpha^U, (ML_{ih})_\alpha^L \leq ML_{ih} \leq (ML_{ih})_\alpha^U, \\ & (MM_{ih})_\alpha^L \leq MM_{ih} \leq (MM_{ih})_\alpha^U, (MS_{mph})_\alpha^L \leq MS_{mph} \leq \\ & (MS_{mph})_\alpha^U, \forall n,h,i,j,p,m\} \end{aligned} \tag{15}$$

which can be reformulated as the following pair of two-level mathematical programs:

$$Z_\alpha^U = \max_{\substack{(D_{njh})_\alpha^L \leq D_{njh} \leq (D_{njh})_\alpha^U \\ (ML_{ih})_\alpha^L \leq ML_{ih} \leq (ML_{ih})_\alpha^U \\ (MM_{ih})_\alpha^L \leq MM_{ih} \leq (MM_{ih})_\alpha^U \\ (MS_{mph})_\alpha^L \leq MS_{mph} \leq (MS_{mph})_\alpha^U}} \left\{ \begin{array}{l} \min \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \lambda_{inh} Q_{inh} \\ + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \gamma_{inh} V_{inh} \\ + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \eta_{inh} IN_{inh} \\ + \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H} \mu_{mpih} L_{mpih} \\ + \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H} \psi_{mpih} L_{mpih} \\ + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H} \theta_{injh} R_{injh} \\ + \sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{h=1}^{H} \phi_{mih} IM_{mih} \\ + \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H} \pi_{nh} LS_{njh} \\ s.t. \\ \left\{ \begin{array}{l} IN_{inh} = IN_{inh-1} + Q_{inh} + V_{inh} - \sum_{j=1}^{J} R_{injh}, \\ \forall i,n,h \\ IM_{mih} = IM_{mih-1} + \sum_{p=1}^{P} L_{mpih} \\ \qquad - \sum_{n=1}^{N} N_{mn} Q_{inh}, \forall i,m,h \\ \sum_{i=1}^{I} R_{injh} + LS_{njh} = D_{njh} \quad \forall n,j,h \\ \sum_{n=1}^{N} HL_{in} Q_{inh} \leq ML_{ih} \quad \forall i,h \\ \sum_{n=1}^{N} HM_{in} Q_{inh} \leq MM_{ih} \quad \forall i,h \\ \sum_{n=1}^{N} WP_n IN_{inh} \leq MP_{ih} \quad \forall i,h \\ \sum_{m=1}^{M} WM_m IM_{mih} \leq S_{ih} \quad \forall i,h \\ \sum_{i=1}^{I} L_{mpih} \leq MS_{mph} \quad \forall p,m,h \\ Q_{inh}, V_{inh}, IN_{inh}, IM_{mih}, R_{injh}, L_{mpih}, \\ LS_{njh} \geq 0, \\ \forall i,n,h,j,p,m \end{array} \right. \end{array} \right. \tag{16}$$

$$Z_{\alpha}^{L}=\min_{\substack{(D_{njh})_{\alpha}^{L}\leq D_{njh}\leq (D_{njh})_{\alpha}^{U}\\ (ML_{ih})_{\alpha}^{L}\leq ML_{ih}\leq (ML_{ih})_{\alpha}^{U}\\ (MM_{ih})_{\alpha}^{L}\leq MM_{ih}\leq (MM_{ih})_{\alpha}^{U}\\ (MS_{mph})_{\alpha}^{L}\leq MS_{mph}\leq (MS_{mph})_{\alpha}^{U}}}\begin{cases}\min \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H}\lambda_{inh}Q_{inh}\\ +\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H}\gamma_{inh}V_{inh}\\ +\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H}\eta_{inh}IN_{inh}\\ +\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H}\mu_{mpih}L_{mpih}\\ +\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H}\psi_{mpih}L_{mpih}\\ +\sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H}\theta_{injh}R_{injh}\\ +\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{h=1}^{H}\phi_{mih}IM_{mih}\\ +\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H}\pi_{nh}LS_{njh}\\ s.t.\\ \begin{cases}IN_{inh}=IN_{inh-1}+Q_{inh}+V_{inh}-\sum_{j=1}^{J}R_{injh},\\ \forall i,n,h\\ IM_{mih}=IM_{mih-1}+\sum_{p=1}^{P}L_{mpih}\\ \qquad -\sum_{n=1}^{N}N_{mn}Q_{inh},\forall i,m,h\\ \sum_{i=1}^{I}R_{injh}+LS_{njh}=D_{njh}\quad \forall n,j,h\\ \sum_{n=1}^{N}HL_{in}Q_{inh}\leq ML_{ih}\quad \forall i,h\\ \sum_{n=1}^{N}HM_{in}Q_{inh}\leq MM_{ih}\quad \forall i,h\\ \sum_{n=1}^{N}WP_{n}IN_{inh}\leq MP_{ih}\quad \forall i,h\\ \sum_{m=1}^{M}WM_{m}IM_{mih}\leq S_{ih}\quad \forall i,h\\ \sum_{i=1}^{I}L_{mpih}\leq MS_{mph}\quad \forall p,m,h\\ Q_{inh},V_{inh},IN_{inh},IM_{mih},R_{injh},L_{mpih},\\ LS_{njh}\geq 0,\\ \forall i,n,h,j,p,m\end{cases}\end{cases} \tag{17}$$

The inner program in Model 16 calculates the objective value for each set of $(D_{njh}, ML_{ih}, MM_{ih}, MS_{mph})$ defined by the outer program, while the outer program determines the set of $(D_{njh}, ML_{ih}, MM_{ih}, MS_{mph})$ that derives the largest objective value. Likewise, in Model 17 the inner program calculates the objective value for each given set of $(D_{njh}, ML_{ih}, MM_{ih}, MS_{mph})$, while the outer program determines the set of $(D_{njh}, ML_{ih}, MM_{ih}, MS_{mph})$ that produces the smallest objective value. In the next section a solution method to transform Models 16 and 17 into one-level LP programs is developed.

4. Solution procedure

As explained in the previous section, to find the membership function of the optimal objective function, the right and the left shape functions of the objective functions membership function should be found which is equivalent to finding the upper bound and the lower bound of the objective function at different α levels.

4.1. Obtaining upper bound. In Model 16 the aim is to find a set of $(D_{njh}, ML_{ih}, MM_{ih}, MS_{mph})$ that derive the maximal objective value. To obtain the upper bounds of the optimal objective function, parameters D_{njh}, ML_{ih}, MM_{ih} and MS_{mph} must be set correctly in their bounds.

Since the outer and inner program are in different directions, Chen and Chang [7] proposed a solution approach which was able to tackel it and then optain the upper bounds of the objective function. In their proposed approach the dual of the inner program is formulated and then the constraints of the outer program are inserted into the inner program and finally the two-level mathematical program is transfered to an LP program. Forasmuch as the proposed model is linear and the variables assume the nonnegetavity conditions, in order to derive the upper bounds of the optimal objective function, parameters D_{njh}, ML_{ih}, MM_{ih} and MS_{mph} must be set correctly in their bounds and there is no need to obtain the dual form of the program. Since the outer program perform maximization operation, all D_{njh}, ML_{ih}, MM_{ih} and MS_{mph} can be set to their upper bounds $(D_{njh})^U$, $(ML_{ih})^U$, $(MM_{ih})^U$ and $(MS_{mph})^U$, respectively. Consequently, the two-level mathematical program in 16 can be simplified to the following LP program:

$$Z_\alpha^U = \min \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \lambda_{inh} Q_{inh} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \gamma_{inh} V_{inh} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \eta_{inh} IN_{inh} \quad (18)$$
$$+ \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H} \mu_{mpih} L_{mpih} + \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H} \psi_{mpih} L_{mpih} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H} \theta_{injh} R_{injh}$$
$$+ \sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{h=1}^{H} \phi_{mih} IM_{mih} + \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H} \pi_{nh} LS_{njh}$$

$$s.t. \begin{cases} IN_{inh} = IN_{inh-1} + Q_{inh} + V_{inh} - \sum_{j=1}^{J} R_{injh} & \forall i,n,h \\ IM_{mih} = IM_{mih-1} + \sum_{p=1}^{P} L_{mpih} - \sum_{n=1}^{N} N_{mn} Q_{inh} & \forall i,m,h \\ \sum_{i=1}^{I} R_{injh} + LS_{njh} = (D_{njh})_\alpha^U & \forall n,j,h \\ \sum_{n=1}^{N} HL_{in} Q_{inh} \le (ML_{ih})_\alpha^U & \forall i,h \\ \sum_{n=1}^{N} HM_{in} Q_{inh} \le (MM_{ih})_\alpha^U & \forall i,h \\ \sum_{n=1}^{N} WP_n IN_{inh} \le MP_{ih} & \forall i,h \\ \sum_{m=1}^{M} WM_m IM_{mih} \le S_{ih} & \forall i,h \\ \sum_{i=1}^{I} L_{mpih} \le (MS_{mph})_\alpha^U & \forall p,m,h \\ Q_{inh}, V_{inh}, IN_{inh}, IM_{mih}, R_{injh}, L_{mpih}, LS_{njh} \ge 0, & \forall i,n,h,j,p,m \end{cases}$$

The upper bound of the objective value is then obtained by solving Model 18.

4.2. Obtaining lower bound. Similarly, to derive the lower bound of the objective value in Model 17, the D_{njh}, ML_{ih}, MM_{ih} and MS_{mph} parameters are set to their lower bounds in the objective function. Therefore, the values of D_{njh}, ML_{ih}, MM_{ih} and MS_{mph} should, respectively, set to their lower bounds D_{njh}^L, ML_{ih}^L, MM_{ih}^L and MS_{mph}^L. Hence, Model 17 can be rewritten as the following mathematical program:

$$Z_\alpha^L = \min \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \lambda_{inh} Q_{inh} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \gamma_{inh} V_{inh} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{h=1}^{H} \eta_{inh} IN_{inh} \quad (19)$$
$$+ \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H} \mu_{mpih} L_{mpih} + \sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{i=1}^{I}\sum_{h=1}^{H} \psi_{mpih} L_{mpih} + \sum_{i=1}^{I}\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H} \theta_{injh} R_{injh}$$
$$+ \sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{h=1}^{H} \phi_{mih} IM_{mih} + \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{h=1}^{H} \pi_{nh} LS_{njh}$$

$$
s.t. \begin{cases}
IN_{inh} = IN_{inh-1} + Q_{inh} + V_{inh} - \sum_{j=1}^{J} R_{injh} \quad \forall i,n,h \\
IM_{mih} = IM_{mih-1} + \sum_{p=1}^{P} L_{mpih} - \sum_{n=1}^{N} N_{mn} Q_{inh} \quad \forall i,m,h \\
\sum_{i=1}^{I} R_{injh} + LS_{njh} = (D_{njh})_{\alpha}^{L} \quad \forall n,j,h \\
\sum_{n=1}^{N} HL_{in} Q_{inh} \leq (ML_{ih})_{\alpha}^{L} \quad \forall i,h \\
\sum_{n=1}^{N} HM_{in} Q_{inh} \leq (MM_{ih})_{\alpha}^{L} \quad \forall i,h \\
\sum_{n=1}^{N} WP_{n} IN_{inh} \leq MP_{ih} \quad \forall i,h \\
\sum_{m=1}^{M} WM_{m} IM_{mih} \leq S_{ih} \quad \forall i,h \\
\sum_{i=1}^{I} L_{mpih} \leq (MS_{mph})_{\alpha}^{L} \quad \forall p,m,h \\
Q_{inh}, V_{inh}, IN_{inh}, IM_{mih}, R_{injh}, L_{mpih}, LS_{njh} \geq 0, \quad \forall i,n,h,j,p,m
\end{cases}
$$

Then this program is solved easily and the lower bound of the objective value Z_{α}^{L} are obtained. Together with Z_{α}^{U} solved from Model 18, $\left[Z_{\alpha}^{L}, Z_{\alpha}^{U}\right]$ constitutes the interval that the objective value lies. For two possibility levels α_1 and α_2 such that $0 < \alpha_2 < \alpha_1$, the feasible regions defined by α_1 in Models 18 and 19 are smaller than those defined by α_2. Consequently, the left shape function $L(z)$ is nondecreasing and the right shape function $R(z)$ is nonincreasing. This property assures the convexity of $\tilde{Z}$. From $L(z)$ and $R(z)$, the membership function $\mu_{\tilde{Z}}$ is constructed as:

$$
\mu_{\tilde{Z}} = \begin{cases}
L(z), & Z_{\alpha=0}^{L} \leq z \leq Z_{\alpha=1}^{L} \\
1, & Z_{\alpha=1}^{L} \leq z \leq Z_{\alpha=1}^{L} \\
R(z), & Z_{\alpha=1}^{L} \leq z \leq Z_{\alpha=0}^{L}
\end{cases} \tag{20}
$$

The numerical solutions for Z_{α}^{L} and Z_{α}^{U} at different possibility level α can be collected to approximate the shapes of $L(z)$ and $R(z)$.

5. Supply chain network example

5.1. Application of the model. In this section we present a numerical hypothetical example to illustrate the solution approach given in Section 4. The supply chain network of the company includes suppliers which supply eight raw materials by importing from three different foreign countries. Five products are ordered to satisfy market demand from four distribution centers with production based at two plants. The planning horizon is three months from June to August.

Tables 1-9 summarize the related supplier, manufacture, distribution and demand data, respectively, for the coming three months. Capacity per truck from each source to various destinations is fixed to carry 100 tones. Notably, the market demand for each destination is an imprecise value with a triangular distribution, and has been estimated based on historical data and the experience and knowledge of DM. The available fuzzy labor levels in each period are (620, 700, 850) man-hours for the first factory and (750, 850, 970) man-hours for the second factory and available fuzzy machine capacities in each period are (750, 800, 950) machine-hours for the first factory and (770, 850, 980) machine-hours for the second factory. Also the available plant space for final product in first factory and second factory is 200 and 250 ($\$/unit$), respectivly and the available plant space for raw material in both factories are equal to 120 ($\$/unit$).

The multi-product problem with fuzzy objective focuses on optimizing the plan in a fuzzy environment. Its aim is to minimize total costs with reference to plant capacity, inventory and subcontracting levels, available labor levels and machine capacity at each plant, as well as market demand and maximum warehouse space available at each destination.

5.2. Solution procedure for the example. The solution procedure of the proposed model is indicated as follows. First, the original PPDP model is formulated for solving supply chain network under fuzzy parameters. Second, the fuzzy objective function is

TABLE 1. Summarized lost sales data of the example

	Product				
Period	1	2	3	4	5
1.June	500	550	590	500	580
2.July	510	500	595	505	585
3.August	520	560	600	510	590

TABLE 2. Summarized data for two plants of the example

Plant	Period	Product	λ_{inh}	γ_{inh}	η_{inh}	HL_{in}	HM_{in}	WP_n
1	1.June	1	10	200	0.28	0.5	0.8	6
		2	15	185	0.3	0.4	0.3	9
		3	13	170	0.31	0.7	0.5	4
		4	12	190	0.24	0.6	0.5	12
		5	9	193	0.5	0.5	0.7	8
	2.July	1	10	153	0.15	0.5	0.8	6
		2	14	171	0.25	0.4	0.3	9
		3	13	192	0.43	0.7	0.5	4
		4	12	181	0.4	0.6	0.5	12
		5	10	167	0.3	0.5	0.7	8
	3.August	1	11	190	0.11	0.5	0.8	6
		2	14	193	0.21	0.4	0.3	9
		3	13	195	0.17	0.7	0.5	4
		4	12	177	0.5	0.6	0.5	12
		5	11	177	0.42	0.5	0.7	8
2	1.June	1	9	101	0.3	0.5	0.8	6
		2	15	186	0.32	0.35	0.25	9
		3	12	171	0.3	0.65	0.45	4
		4	12	192	0.4	0.6	0.5	12
		5	10	194	0.5	0.5	0.7	8
	2.July	1	10	156	0.15	0.5	0.8	6
		2	14	170	0.25	0.35	0.25	9
		3	12	191	0.18	0.65	0.45	4
		4	15	186	0.42	0.6	0.5	12
		5	11	163	0.5	0.5	0.7	8
	3.August	1	12	198	0.14	0.5	0.8	6
		2	18	191	0.46	0.35	0.25	9
		3	19	195	0.15	0.65	0.45	4
		4	19	173	0.4	0.6	0.5	12
		5	17	176	0.51	0.5	0.7	8

TABLE 3. Summarized distribution data (transport cost (\$/*unit*)) for the first period*

		Destination			
Plant	Product	1	2	3	4
1	1	2.8	2.5	2.3	2.7
	2	2.8	2.5	2.3	2.7
	3	2.8	2.5	2.3	2.7
	4	2.8	2.5	2.3	2.7
	5	2.8	2.5	2.3	2.7
2	1	2.5	2.7	2.9	2.0
	2	2.5	2.7	2.9	2.0
	3	2.5	2.7	2.9	2.0
	4	2.5	2.7	2.9	2.0
	5	2.5	2.7	2.9	2.0

*For periods 2 and 3 the estimations are multiplied by 1.1 and 1.2, respectively.

TABLE 4. Summarized market demand data for the destinations of the example (*units*)

		Product				
Destination	Period	1	2	3	4	5
1	1.June	(71,80,92)	(68,70,85)	(64,75,89)	(41,45,53)	(0,0,0)
	2.July	(50,60,70)	(42,50,87)	(40,43,52)	(79,85,94)	(65,80,98)
	3.August	(55,90,102)	(70,75,87)	(55,60,69)	(55,70,84)	(62,75,89)
2	1.June	(47,65,76)	(60,75,84)	(53,60,72)	(45,60,68)	(35,45,53)
	2.July	(42,50,67)	(43,55,60)	(60,79,85)	(0,0,0)	(40,50,75)
	3.August	(40,60,74)	(82,90,100)	(70,75,95)	(35,50,65)	(27,30,53)
3	1.June	(0,0,0)	(84,90,105)	(42,50,72)	(0,0,0)	(72,90,103)
	2.July	(45,50,84)	(45,50,61)	(19,25,29)	(70,90,116)	(19,24,33)
	3.August	(100,115,120)	(42,58,67)	(85,89,98)	(0,0,0)	(80,96,108)
4	1.June	(42,50,67)	(71,80,95)	(80,98,120)	(23,28,33)	(84,90,110)
	2.July	(67,80,95)	(37,40,67)	(42,55,67)	(69,85,87)	(82,90,100)
	3.August	(60,70,80)	(38,50,62)	(40,43,51)	(43,45,64)	(48,60,73)

TABLE 5. Bill of material matrix

	Raw material							
Product	1	2	3	4	5	6	7	8
1	1	0	1	0	1	0	2	0
2	0	1	0	2	0	1	0	1
3	2	0	1	0	1	0	1	0
4	0	2	0	1	0	2	0	1
5	1	0	1	0	2	0	1	0

TABLE 6. Summarized shipping data of the example ($\$/unit$)*

		Raw material							
Supplier	Plant	1	2	3	4	5	6	7	8
1	1	2.0	2.1	2.3	2.3	1.8	2.6	2.1	2.0
	2	1.6	1.6	1.7	1.9	1.6	2.9	1.9	1.9
2	1	2.5	2.4	1.9	2.7	2.4	2.3	2.4	2.3
	2	2.3	2.0	2.5	2.6	2.3	2.5	2.5	1.8
3	1	2.6	2.5	2.0	2.8	1.7	2.6	2.7	2.1
	2	2.8	2.8	2.8	2.6	1.9	2.8	2.5	1.6

*For periods 2 and 3 the estimations are multiplied by 1.1 and 1.2, respectively.

TABLE 7. Summarized data for purchasing cost

		Raw material							
Supplier	Period	1	2	3	4	5	6	7	8
1	1.June	4	7	9	8	5	10	11	4
	2.July	4.5	7	10	11	7	9	8	7
	3.August	3	8	8	9	6	8	7	6
2	1.June	6	8	4	3	3	7	6	7
	2.July	7	9	5	6	11	9	6	9
	3.August	6	8	4	5	14	11	3	5
3	1.June	3	10	5	6	6	6	4	6
	2.July	6	8	10	9	7	9	9	11
	3.August	5	7	9	10	10	12	10	8
Ware house space for raw material $ft^2/unit$		2	2.4	1.8	2.3	1.7	2	3	2.4

TABLE 8. Summarized data for inventory cost of raw materials

		Raw material							
Plant	Period	1	2	3	4	5	6	7	8
1	1.June	0.26	0.19	0.21	0.15	0.24	0.17	0.12	0.22
	2.July	0.26	0.19	0.21	0.15	0.24	0.17	0.12	0.22
	3.August	0.26	0.19	0.21	0.15	0.24	0.17	0.12	0.22
2	1.June	0.12	0.15	0.17	0.16	0.21	0.19	0.13	0.20
	2.July	0.12	0.15	0.17	0.16	0.21	0.19	0.13	0.20
	3.August	0.12	0.15	0.17	0.16	0.21	0.19	0.13	0.20

derived by finding the upper bound and the lower bound of the objective function at different α levels according to 4.1 and 4.2. The original PPDP problem for solving is formulated according to Models 18 and 19 and is determined by the value of possibility level α. Table 10 lists the $\alpha-$cuts of the objective value and decision variables at 11 distinct α values: 0, 0.1, 0.2, . . . , 1.0.

Note that the upper and lower bound of some decision variables are shown in Table 10 which others can be derived. At $\alpha-$level= 0, the value of Z_α^U is 242275.3 and the value of Z_α^L is 168132. At $\alpha-$ level= 1, the value of Z_α^U is 200923.9. The α value indicates the level of possibility and the degree of uncertainty of the obtained information which is assumped by the DM. The greater the α value, the greater the level of possibility and the lower the degree of uncertainty is.

Since the fuzzy objective value lies in a range, different $\alpha-$cuts shows the different intervals and the uncertainty level of the objective value. Specifically, $\alpha = 0$ has the widest interval indicating that the objective value will definitely fall into this range. At the other extreme end, the possibility level $\alpha = 1$ is the most possible value of the objective value. In this example, the objective value is impossible to exceed 242275.3 or fall below 168132 and its most possible value is 200923.9. When the uncertain parameters are represented by crisp values, the objective value is believed to be a single value of 200923.9, rather than an interval estimation in the range of 168132 and 242275.3.

5.3. Computational analysis. Several consequential content for the practical application of the proposed model to solve integrated supply chain network problems in a fuzzy environment include the following.

First, the deterministic models are not appropriate methods of obtaining an efficient solution, due to the ambiguous and imprecise information relating to the decision parameters in real-world integrated PPDP problems. Traditional mathematical programming techniques, obviously, cannot solve all fuzzy programming problems. Because of this, various models and solution approaches are developed. Most of the developed approachs transform the model to an equivalent crisp model and obtain certain values but the proposed solution approach calculates the objective in a fuzzy manner and a satisfactory solution is obtained due to the DM's preferance by different $\alpha-$cuts. Second, the proposed approach yields an effective solution for fuzzy supply chain models which deal with integer variables. As mentioned in the literature, models with integer variables cannot be solved by classical approaches so different solution approaches are introduced [16, 35, 37,38]. The proposed approach derives an interval for each variable and the objective, so it is no need to be considered integer.

Beyond these implications, the most important advantage of the proposed model is that the model can be solved in a level that the DM is satisfied.

TABLE 9. Summarized capacity data for raw material

		Raw material							
Supplier	Period	1	2	3	4	5	6	7	8
1	1	(350,370,392)	(360,400,510)	(450,550,670)	(400,450,490)	(250,300,410)	(300,350,432)	(550,600,740)	(350,400,470)
	2	(900,950,1100)	(410,500,564)	(300,400,530)	(300,400,510)	(200,250,300)	(250,300,410)	(600,650,710)	(400,500,560)
	3	(300,350,384)	(310,400,520)	(350,450,550)	(350,400,480)	(250,300,370)	(150,200,230)	(500,550,620)	(300,400,540)
2	1	(600,650,710)	(650,700,870)	(610,650,680)	(650,700,870)	(400,450,510)	(450,500,567)	(800,850,932)	(600,700,830)
	2	(300,350,395)	(320,400,560)	(350,400,510)	(400,500,650)	(100,200,280)	(250,300,411)	(500,550,670)	(300,400,470)
	3	(700,720,780)	(730,800,970)	(720,800,870)	(750,800,920)	(500,550,620)	(550,600,740)	(900,950,1140)	(700,800,910)
3	1	(700,850,920)	(600,750,850)	(820,900,980)	(650,800,950)	(900,950,1050)	(550,700,870)	(820,950,970)	(700,800,890)
	2	(620,700,850)	(600,750,850)	(700,850,940)	(700,850,970)	(900,950,1025)	(870,1000,1150)	(820,900,980)	(850,1000,1100)
	3	(850,900,980)	(750,900,1150)	(580,700,850)	(650,800,970)	(900,950,1200)	(750,900,1010)	(850,920,1050)	(800,900,970)

TABLE 10. The α-cuts of the objective value and decision variables at 11 α-values for example

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Z_α^U	242275.3	239695.1	235364.2	231329.3	227083.6	222335.4	218467.6	214758.6	210231.6	206454.4	200923.9
Z_α^L	168132	169849.4	173640.2	176196.7	180369.5	183720.1	186568.4	189834.3	193297.7	196181.5	200923.9
Q_{131}^U	122	120	118	116	114	111	109	107	105	103	100
Q_{131}^L	92	92	93	94	95	96	96	97	98	99	100
V_{131}^U	0	0	0	0	0	0	0	0	0	0	0
V_{131}^L	0	0	0	0	0	0	0	0	0	0	0
IN_{131}^U	50	50	50	50	50	50	50	50	50	50	50
IN_{131}^L	50	50	50	50	50	50	50	50	50	50	50
IM_{311}^U	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67
IM_{311}^L	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67
R_{1331}^U	72	70	68	66	64	61	59	57	55	53	50
R_{1331}^L	42	42	43	44	45	46	46	47	48	49	50
L_{3311}^U	419.83	410.83	399.83	387.83	376.83	360.83	350.83	339.83	327.83	316.83	298.83
L_{3311}^L	225.83	226.83	235.83	243.83	251.83	260.83	263.83	271.83	279.83	283.83	298.83
LS_{331}^U	0	0	0	0	0	0	0	0	0	0	0
LS_{331}^L	0	0	0	0	0	0	0	0	0	0	0

6. Conclusions

Supply chain environments imply the production planning decisions have to be made under conditions of uncertainty in parameters as important as demand. This study proposes a fuzzy mathematical programming model in a multi-echelon supply chain network with multiple suppliers, multiple plants, and multiple destinations. In real world applications, the parameters in the PPDP may not be known precisely due to insufficient information. When some parameters are only approximately known, the averages or the most likely values are used to find a point solution. Since only one point value is obtained, much valuable information is lost. Despite the past research works, this paper develops a method that is able to find the fuzzy objective value when the market demand and available resources are fuzzy numbers. The idea is based on Zadehs extension principle to transform the fuzzy problem to a pair of two-level mathematical programs. Solving the programs produces the upper bound and lower bound of the objective value at specific α level. The membership function is approximated via different $\alpha-$levels of the objective values. This could help end up the solution procedure more rapidly by solving less problems. The proposed PPDP problem is solved using Lingo 9.0. The illustrated example shows that the technique proposed in this paper is effective and easy to apply. With the additional ability of calculating fuzzy objective value developed in this paper, it might help lead to wider applications in the future.

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