Applied Mathematics Letters 116 (2021) 107027

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

High perturbations of critical fractional Kirchhoff equations with logarithmic nonlinearity

Sihua Liang [a,b], Hongling Pu [a], Vicenţiu D. Rădulescu [c,d,e,*]

[a] College of Mathematics, Changchun Normal University, Changchun 130032, Jilin, PR China
[b] College of Mathematics and Informatics, Fujian Normal University, Qishan Campus, Fuzhou 350108, PR China
[c] Faculty of Applied Mathematics, AGH University of Science and Technology, 30-059 Kraków, Poland
[d] Department of Mathematics, University of Craiova, 200585 Craiova, Romania
[e] Institute of Mathematics "Simion Stoilow" of the Romanian Academy, P.O. Box 1-764, 014700 Bucharest, Romania

ARTICLE INFO

Article history:
Received 19 November 2020
Accepted 4 January 2021
Available online 14 January 2021

Keywords:
Kirchhoff equation
Critical growth
Logarithmic nonlinearity
Degenerate problem
Concentration–compactness principle

ABSTRACT

This paper deals with the study of combined effects of logarithmic and critical nonlinearities for the following class of fractional p-Kirchhoff equations:

$$\begin{cases} M([u]_{s,p}^p)(-\Delta)_p^s u = \lambda |u|^{q-2}u\ln|u|^2 + |u|^{p_s^*-2}u & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary, $N > sp$ with $s \in (0,1)$, $p \geq 2$, $p_s^* = Np/(N-ps)$ is the fractional critical Sobolev exponent, and λ is a positive parameter. The main result establishes the existence of nontrivial solutions in the case of high perturbations of the logarithmic nonlinearity (large values of λ). The features of this paper are the following: (i) the presence of a logarithmic nonlinearity; (ii) the lack of compactness due to the critical term; (iii) our analysis includes the degenerate case, which corresponds to the Kirchhoff term M vanishing at zero.

1. Introduction and the main result

Consider the following fractional p-Kirchhoff equations with logarithmic and critical nonlinearity:

$$\begin{cases} M([u]_{s,p}^p)(-\Delta)_p^s u = \lambda |u|^{q-2}u\ln|u|^2 + |u|^{p_s^*-2}u & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.1}$$

* Corresponding author at: Faculty of Applied Mathematics, AGH University of Science and Technology, 30-059 Kraków, Poland.

E-mail addresses: liangsihua@163.com (S. Liang), pauline_phl@163.com (H.L. Pu), radulescu@inf.ucv.ro (V.D. Rădulescu).

https://doi.org/10.1016/j.aml.2021.107027
0893-9659/

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary, $N > sp$ with $s \in (0,1)$, $p \geq 2$, $p_s^* = Np/(N-ps)$ is the fractional critical Sobolev exponent, $p\sigma < q < p_s^*$ and σ will be given by condition (M_2), λ is a positive parameter, and

$$[u]_{s,p}^p = \iint_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}} dxdy.$$

Here, $(-\Delta)_p^s$ is the fractional p-Laplace operator which, up to a normalization constant, is defined as

$$(-\Delta)_p^s \varphi(x) = 2 \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{|\varphi(x)-\varphi(y)|^{p-2}(\varphi(x)-\varphi(y))}{|x-y|^{N+ps}}\, dy, \quad x \in \mathbb{R}^N,$$

for all $\varphi \in C_0^\infty(\mathbb{R}^N)$. Henceforward, $B_\varepsilon(x)$ denotes the open ball of $\mathbb{R}^N$ centered at $x \in \mathbb{R}^N$ with radius $\varepsilon > 0$. The Kirchhoff function $M : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is assumed to be continuous, nondecreasing and satisfying

(M_1) For any $\tau > 0$, there exists $m_0 = m_0(\tau) > 0$ such that $M(t) \geq m_0$ for all $t \geq \tau$.
(M_2) There exists $\sigma \in [1, p_s^*/p)$ such that $\sigma \widehat{M}(t) \geq M(t)t$ for all $t \geq 0$, where $\widehat{M}(t) = \int_0^t M(s)ds$.
(M_3) There exists $m_1 > 0$ such that $M(t) \geq m_1 t^{\sigma-1}$ for all $t \in \mathbb{R}^+$ and $M(0) = 0$.

An example is given by $M(t) = a + b\,t^{\sigma-1}$ for $t \in \mathbb{R}_0^+$, where $a \in \mathbb{R}_0^+$, $b \in \mathbb{R}_0^+$ and $a + b > 0$. When M is of this type, problem (1.1) is said to be *non-degenerate* if $a > 0$, while it is called *degenerate* if $a = 0$.

Clearly, assumptions (M_1)–(M_3) cover the degenerate case. It is worth mentioning that the degenerate case is rather interesting and is treated in well-known papers in Kirchhoff theory, see [1]. In [2], condition (M_3) was applied to investigate the existence of entire solutions for the stationary Kirchhoff type equations driven by the fractional p-Laplace operator in $\mathbb{R}^N$. In the literature on degenerate Kirchhoff problems, the transverse oscillations of a stretched string, with nonlocal flexural rigidity, depend continuously on the Sobolev deflection norm of u via $M(\|u\|_s^2)$. From a physical point of view, the fact that $M(0) = 0$ means that the base tension of the string is zero, a very realistic model. Non-degenerate Kirchhoff-type problems are treated in [3] while the degenerate case is considered in [4–6]. We also refer to [7] for logarithmic Hartree problems. There are very few papers that deal with the existence and multiplicity of solutions for fractional problems involving logarithmic nonlinearity. Xiang, Hu and Yang [8] considered the following Kirchhoff problems in the non-degenerate case:

$$\begin{cases} M([u]_{s,p}^p)(-\Delta)_p^s u = h(x)|u|^{\theta p-2}u \ln|u| + \lambda |u|^{q-2}u & x \in \Omega, \\ u = 0 & x \in \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $s \in (0,1)$, $1 < p < N/s$, Ω is a bounded domain in $\mathbb{R}^N$ with Lipschitz boundary, $q \in (1, p_s^*)$ and h is a sign-changing function. When λ is sufficiently small (that is, *low perturbations*), they obtained two nonnegative local least energy solutions by using Nehari manifold analysis. However, to the best of our knowledge, there are no results concerning the existence of solutions for fractional p-Kirchhoff equations with logarithmic and critical nonlinearity in the *degenerate* case.

Our main result establishes the existence of solutions in the case of *high perturbations* of the logarithmic nonlinearity.

Theorem 1.1. *Let the conditions (M_1)–(M_3) be satisfied. Then there exists $\lambda^* > 0$ such that for any $\lambda \geq \lambda^*$ problem (1.1) has a nontrivial solution.*

Finally, we point out the lack of compactness of Sobolev embedding due to the presence of the critical nonlinearity. That is why we use the Concentration–compactness principle to prove that the $(PS)_c$ condition holds. In addition, we would like to stress that the extension from the case $p = 2$ to the case $1 < p < \infty$ is not trivial. We believe that this paper is the first contribution to study the existence of solutions for the fractional p-Kirchhoff equations with logarithmic and critical nonlinearity in the degenerate case.

2. Auxiliary results and proof of Theorem 1.1

Let S_r denote the best constant for the compact embedding $W_0^{s,p}(\Omega) \hookrightarrow L^r(\Omega)$ $(p < r < p_s^*)$, hence $S_r|u|_r \le \|u\|$ for all $u \in W_0^{s,p}(\Omega)$. If S is the best constant for the embedding $W_0^{s,p}(\Omega) \hookrightarrow L^{p_s^*}(\Omega)$, then

$$S = \inf_{u\in W_0^{s,p}(\Omega)\setminus\{0\}} \frac{\iint_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}} dxdy}{\left(\int_\Omega |u|^{p_s^*}\, dx\right)^{\frac{p}{p_s^*}}}. \tag{2.1}$$

For each $\lambda > 0$, we define the C^1-functional $\mathcal{J}_\lambda : W_0^{s,p}(\Omega) \to \mathbb{R}$ by

$$\mathcal{J}_\lambda(u) = \frac{1}{p}\widehat{M}\left([u]_{s,p}^p\right) + \frac{2\lambda}{q^2}\int_\Omega |u|^q dx - \frac{\lambda}{q}\int_\Omega |u|^q \ln|u|^2 dx - \frac{1}{p_s^*}\int_\Omega |u|^{p_s^*}\, dx.$$

Since $p\sigma < q < p_s^*$, we have $\lim_{t\to 0} \frac{|t|^{q-1}\ln|t|^2}{|t|^{p\sigma-1}} = 0$ and $\lim_{t\to\infty} \frac{|t|^{q-1}\ln|t|^2}{|t|^{r-1}} = 0$ for all $r \in (q, p_s^*)$. Then for any $\varepsilon > 0$, there exists $C_\varepsilon > 0$ such that

$$|t|^{q-1}\ln|t|^2 \le \varepsilon|t|^{p\sigma-1} + C_\varepsilon|t|^{r-1}. \tag{2.2}$$

On the one hand, the Vitali convergence theorem yields

$$\lim_{n\to\infty}\int_\Omega |u_n|^q \ln|u_n|^2 dx \to \int_\Omega |u|^q \ln|u|^2 dx. \tag{2.3}$$

On the other hand, since $u_n \to u$ in $L^q(\Omega)$, we have

$$\lim_{n\to\infty}\int_\Omega |u_n|^q dx \to \int_\Omega |u|^q dx. \tag{2.4}$$

In order to prove the Palais–Smale condition, we use the fractional version of Lions' Concentration–compactness principle [9] in the framework of fractional Sobolev spaces, see [10, Theorem 2.5].

Lemma 2.1. *Assume that hypotheses (M_1)–(M_3) hold. Then the functional $\mathcal{J}_\lambda$ satisfies the $(PS)_c$ condition for* $c \in \left(0, \left(\frac{1}{q} - \frac{1}{p_s^*}\right)(m_1 S^\theta)^{\frac{p_s^*}{p_s^*-p\theta}}\right)$.

Proof. If $\inf_{n\ge 1}\|u_n\| = 0$, then there exists a subsequence of $\{u_n\}_n$ still denoted by $\{u_n\}_n$ such that $u_n \to 0$ in $W_0^{s,p}(\Omega)$ as $n \to \infty$. Thus, we assume that $d := \inf_{n\ge 1}\|u_n\| > 0$. Let $\{u_n\}_n$ be a $(PS)_c$ sequence. Then $\mathcal{J}_\lambda(u_n) \to c$ and $\mathcal{J}'_\lambda(u_n) \to 0$ as $n \to \infty$. It follows from (M_1) and (M_3) that

$$\begin{aligned} c + o(1)\|u_n\| &= \mathcal{J}_\lambda(u_n) - \frac{1}{q}\langle \mathcal{J}'_\lambda(u_n), u_n\rangle \\ &\ge \left(\frac{1}{p\sigma} - \frac{1}{q}\right) m_1\|u\|^{p\sigma} + \frac{2\lambda}{q^2}\int_\Omega |u|^q dx + \left(\frac{1}{q} - \frac{1}{p_s^*}\right)\int_\Omega |u|^{p_s^*} dx \\ &\ge \left(\frac{1}{p\sigma} - \frac{1}{q}\right) m_1\|u\|^{p\sigma}. \end{aligned} \tag{2.5}$$

Thus, by $2 \le p < p\sigma$, we deduce that $\{u_n\}_n$ is bounded in $W_0^{s,p}(\Omega)$. Passing to the limit in (2.5) we obtain $c \ge 0$. So, up to a subsequence, $u_n \rightharpoonup u$ in $W_0^{s,p}(\Omega)$. We claim that

$$\int_\Omega |u_n|^{p_s^*} dx \to \int_\Omega |u|^{p_s^*} dx \quad \text{as } n \to \infty. \tag{2.6}$$

In fact, it follows from [10, Theorem 2.5] that either $u_n \to u$ in $L_{loc}^{p_s^*}(\Omega)$ or $\nu = |u|^{p_s^*} + \sum_{j\in\Lambda}\delta_{x_j}\nu_j$, as $n \to \infty$, where Λ is a countable set, $\{\nu_j\}_j \subset [0,\infty)$, $\{x_j\}_j \subset \Omega$. Take $\phi \in C_0^\infty(\Omega)$ such that $0 \le \phi \le 1$;

$\phi \equiv 1$ in $B(x_j,\rho)$, $\phi(x)=0$ in $\Omega\setminus B(x_j,2\rho)$. For any $\rho>0$, define $\phi_\rho^j=\phi\left(\frac{x-x_j}{\rho}\right)$, where $j\in\Lambda$. It follows that $\{u_n\phi_\rho^j\}_n$ is bounded in $W_0^{s,p}(\Omega)$ since $\{u_n\}_n$ is bounded in $W_0^{s,p}(\Omega)$. Then $\langle \mathcal{J}'_\lambda(u_n), u_n\phi_\rho^j\rangle\to 0$, which implies

$$\begin{aligned}&M\left([u_n]_{s,p}^p\right)\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)-u_n(y)|^p\phi_\rho^j(y)}{|x-y|^{N+ps}}dxdy+M\left([u_n]_{s,p}^p\right)L_p(u_n,u_n\phi_\rho^j)\\&=\lambda\int_\Omega|u_n|^q\phi_\rho^j\ln|u|^2dx+\int_\Omega|u|^{p_s^*}\phi_\rho^j\,dx+o_n(1),\end{aligned}\tag{2.7}$$

where

$$L_p(u_n,u_n\phi_\rho^j)=\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)-u_n(y)|^{p-2}(u_n(x)-u_n(y))u_n(x)(\phi_\rho^j(x)-\phi_\rho^j(y))}{|x-y|^{N+ps}}dxdy.$$

It is easy to verify that

$$\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)-u_n(y)|^p\,\phi_\rho^j(y)}{|x-y|^{N+ps}}dxdy\to\int_{\mathbb{R}^N}\phi_\rho^jd\mu$$

as $n\to\infty$ and $\int_{\mathbb{R}^N}\phi_\rho^jd\mu\to\mu(\{x_j\})$ as $\rho\to 0$. Note that the Hölder inequality implies

$$\begin{aligned}\left|M\left([u_n]_{s,p}^p\right)L_p(u_n,u_n\phi_\rho^j)\right|&\le C\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)-u_n(y)|^{p-1}|\phi_\rho^j(x)-\phi_\rho^j(y)\parallel u_n(x)|}{|x-y|^{N+ps}}dxdy\\&\le C\left(\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)|^p|\phi_\rho^j(x)-\phi_\rho^j(y)|^p}{|x-y|^{N+ps}}dxdy\right)^{1/p}.\end{aligned}\tag{2.8}$$

With the same arguments as in the proof of Lemma 3.4 in [11], we have

$$\lim_{\rho\to0}\lim_{n\to\infty}\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)|^p|\phi_\rho^j(x)-\phi_\rho^j(y)|^p}{|x-y|^{N+ps}}dxdy=0.\tag{2.9}$$

It follows that

$$\lim_{\rho\to0}\lim_{n\to\infty}M\left([u_n]_{s,p}^p\right)L_p(u_n,u_n\phi_\rho^j)=0.\tag{2.10}$$

Note that by (M_3), we have

$$M\left([u_n]_{s,p}^p\right)\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)-u_n(y)|^p\phi_\rho^j(y)}{|x-y|^{N+ps}}dxdy\ge m_1\left(\iint_{\mathbb{R}^{2N}}\frac{|u_n(x)-u_n(y)|^p\phi_\rho^j(y)}{|x-y|^{N+ps}}dxdy\right)^\theta.$$

Letting $\rho\to0$ in (2.7) and using the standard theory of Radon measures, we conclude that $\nu_j\ge m_1\mu_j^\theta$. Using [10, Theorem 2.5] we have that $\nu_j=0$ or $(m_1S^\theta)^{\frac{p_s^*}{p_s^*-p\theta}}\le\nu_j$ for all $j\in\Lambda$. Let us assume that $(m_1S^\theta)^{\frac{p_s^*}{p_s^*-p\theta}}\le\nu_{j_0}$ for some $j_0\in\Lambda$. Thus, it follows that

$$\begin{aligned}c&=\lim_{n\to\infty}\left(\mathcal{J}_\lambda(u_n)-\frac{1}{q}\langle\mathcal{J}'_\lambda(u_n),u_n\rangle\right)\ge\left(\frac1q-\frac1{p_s^*}\right)\int_\Omega|u_n|^{p_s^*}dx\\&\ge\left(\frac1q-\frac1{p_s^*}\right)\int_\Omega\phi_\rho^{j_0}|u_n|^{p_s^*}dx\ge\left(\frac1q-\frac1{p_s^*}\right)\nu_{j_0}>\left(\frac1q-\frac1{p_s^*}\right)(m_1S^\theta)^{\frac{p_s^*}{p_s^*-p\theta}}.\end{aligned}$$

This is impossible. Then $\Lambda=\emptyset$, and hence (2.6) holds.

Now, we are ready to show that $\{u_n\}$ converges strongly to u in $W_0^{s,p}(\Omega)$ as $n\to\infty$. Indeed, using (2.3), the weak lower semicontinuity of the norm and the Brezis–Lieb lemma [12], we obtain

$$\begin{aligned}o_n(1)=\mathcal{J}'_\lambda(u_n)u_n&=M\left([u_n]_{s,p}^p\right)[u_n]_{s,p}^p-\lambda\int_\Omega|u_n|^q\ln|u_n|^2dx-\int_\Omega|u_n|^{p_s^*}dx\\&\ge M\left([u]_{s,p}^p\right)[u_n-u]_{s,p}^p+M\left([u]_{s,p}^p\right)[u]_{s,p}^p-\lambda\int_\Omega|u|^q\ln|u|^2dx-\int_\Omega|u|^{p_s^*}dx\end{aligned}$$

$$\begin{aligned}&\geq m_0[u_n-u]_{s,p}^p+M\left([u]_{s,p}^p\right)[u]_{s,p}^p-\lambda\int_\Omega |u|^q\ln|u|^2dx-\int_\Omega |u|^{p_s^*}\,dx\\&\geq m_0\|u_n-u\|^p+\mathcal{J}_\lambda'(u)u+o_n(1),\end{aligned}$$

since $\mathcal{J}_\lambda'(u)=0$. Thus, $\{u_n\}$ converges strongly to u in $W_0^{s,p}(\Omega)$. This completes the proof. □

Lemma 2.2. *The functional $\mathcal{J}_\lambda$ has a mountain pass geometry.*

Proof. From (M_3), (2.2) and Sobolev embedding inequality, we have

$$\begin{aligned}\mathcal{J}_\lambda(u)&\geq \frac{m_1}{p\sigma}\|u\|^{p\sigma}-\frac{\lambda}{q}\varepsilon\int_\Omega |u|^{p\sigma}dx-\frac{\lambda}{q}C_\varepsilon\int_\Omega |u|^r dx-\frac{1}{p_s^*}\int_\Omega |u|^{p_s^*}\,dx\\&\geq\left(\frac{m_1}{p\sigma}-\frac{\lambda}{q}\varepsilon C_1\right)\|u\|^{p\sigma}-\frac{\lambda}{q}C_\varepsilon C_2\|u\|^r-\frac{1}{p_s^*}C_3\|u\|^{p_s^*},\end{aligned}$$

where C_1, C_2 and C_3 are some positive constants. Choose $\varepsilon>0$ such that $\left(\frac{m_1}{p\sigma}-\frac{\lambda}{q}\varepsilon C_1\right)>0$. Since $r,p_s^*>p$, there exists $\rho,\alpha>0$ such that $\mathcal{J}_\lambda(u)\geq\alpha$ for $\|u\|=\rho$ and $\lambda>0$. We first observe that

$$2\tau^q-q\tau^q\ln|\tau|^2\leq 2\quad\text{for all }\tau\in(0,\infty).\tag{2.11}$$

On the other hand, by integrating (M_2), we obtain

$$\widehat{M}(s)\leq\frac{\widehat{M}(s_0)}{s_0^\sigma}s^\sigma=C_0s^\sigma\quad\text{for all}\quad s\geq s_0>0.\tag{2.12}$$

Let $\nu\in W_0^{s,p}(\Omega)$ with $\nu\neq0$. Thus $\mathcal{J}_\lambda(t\nu)\leq\frac{C_0}{p}t^{p\sigma}\|\nu\|^{p\sigma}+\frac{2\lambda}{q^2}|\Omega|-\frac{1}{p_s^*}t^{p_s^*}|\nu|_{p_s^*}^{p_s^*}$. By $p_s^*>\frac{p}{\sigma}$, we deduce that $\mathcal{J}_\lambda(t_0\nu)<0$ and $t_0\|\nu\|>\rho$ for t_0 large enough. Set $\omega=t_0\nu$. This completes the proof. □

Next, we claim that

$$c_\lambda=\inf_{\gamma\in\Gamma}\max_{t\in[0,1]}\mathcal{J}_\lambda(\gamma(t))<\left(\frac{1}{q}-\frac{1}{p_s^*}\right)\left(m_1S^\theta\right)^{\frac{p_s^*}{p_s^*-p\theta}}.\tag{2.13}$$

Assuming that (2.13) holds true, then Lemmas 2.1, 2.2 and the mountain pass theorem give the existence of nontrivial critical points of $\mathcal{J}_\lambda$. To prove (2.13), we choose $v_0\in W_0^{s,p}(\Omega)$ such that $\|v_0\|=1$ and $\lim_{t\to\infty}\mathcal{J}_\lambda(tv_0)=-\infty$. Then $\sup_{t\geq0}\mathcal{J}_\lambda(tv_0)=\mathcal{J}_\lambda(t_\lambda v_0)$ for some $t_\lambda>0$. Hence t_λ satisfies

$$M\left(t_\lambda^p\right)t_\lambda^p=\lambda\int_\Omega |t_\lambda v_0|^q\ln|t_\lambda v_0|^2dx+t_\lambda^{p_s^*}\int_\Omega |v_0|^{p_s^*}\,dx.\tag{2.14}$$

Furthermore, by (2.12), (2.14) and (M_2), we obtain $\sigma C_0t_\lambda^{p\sigma}\geq\sigma\widehat{M}(t_\lambda)\geq M\left(t_\lambda^p\right)t_\lambda^p\geq t_\lambda^{p_s^*}\int_\Omega|v_0|^{p_s^*}dx$, hence $\{t_\lambda\}_\lambda$ is bounded since $\frac{p}{\sigma}<p_s^*$.

We claim that $t_\lambda\to0$ as $\lambda\to\infty$. Arguing by contradiction, we can assume that there exists $t_0>0$ and a sequence λ_n with $\lambda_n\to\infty$ such that $t_{\lambda_n}\to t_0$ as $n\to\infty$. By Lebesgue's dominated convergence theorem, we have $\lim_{n\to\infty}\int_\Omega|t_{\lambda_n}v_0|^q\ln|t_{\lambda_n}v_0|^2dx\to\int_\Omega|t_0v_0|^q\ln|t_0v_0|^2dx$ as $n\to\infty$. It follows that $\lambda_n\int_\Omega|t_0v_0|^q\ln|t_0v_0|^2dx\to\infty$ as $n\to\infty$. Hence, (2.14) implies that $M\left(t_\lambda^p\right)t_\lambda^p=\infty$ which is absurd. Therefore, $t_\lambda\to0$ as $\lambda_n\to\infty$. Furthermore, we deduce from (2.14) that $\lim_{\lambda\to\infty}\lambda\int_\Omega|t_\lambda v_0|^q\ln|t_\lambda v_0|^2dx=0$ and $\lim_{\lambda\to\infty}\lambda\int_\Omega|t_\lambda v_0|^qdx=0$. From this, $t_\lambda\to0$ as $\lambda\to\infty$ and the definition of $\mathcal{J}_\lambda$, we get $\lim_{\lambda\to\infty}\left(\sup_{t\geq0}\mathcal{J}_\lambda(tv_0)\right)=\lim_{\lambda\to\infty}\mathcal{J}_\lambda(t_\lambda v_0)=0$. Then there exists $\lambda_*>0$ such that for any $\lambda\geq\lambda_*$, we have $\sup_{t\geq0}\mathcal{J}_\lambda(tv_0)<\left(\frac{1}{q}-\frac{1}{p_s^*}\right)\left(m_1S^\theta\right)^{\frac{p_s^*}{p_s^*-p\theta}}$. If we take $\omega=Tv_0$, with T large enough to verify $\mathcal{J}_\lambda(\omega)<0$, then we obtain $c_\lambda\leq\max_{t\in[0,1]}\mathcal{J}_\lambda(\gamma(t))$ by taking $\gamma(t)=tTv_0$. Therefore, our claim (2.13) holds true for λ large enough. The proof of Theorem 1.1 is now complete. □

Acknowledgments

S. Liang was supported by the Foundation for China Postdoctoral Science Foundation (Grant no. 2019M662220), Scientific research projects for Department of Education of Jilin Province, China (JJKH20210874KJ). H. Pu was supported by the Graduate Scientific Research Project of Changchun Normal University (SGSRPCNU).

References

[1] P. D'Ancona, S. Spagnolo, Global solvability for the degenerate Kirchhoff equation with real analytic data, Invent. Math. 108 (1992) 247–262.
[2] P. Pucci, M. Xiang, B. Zhang, Existence and multiplicity of entire solutions for fractional p–Kirchhoff equations, Adv. Nonlinear Anal. 5 (2016) 27–55.
[3] M. Xiang, B. Zhang, V.D. Rădulescu, Existence of solutions for perturbed fractional p–Laplacian equations, J. Differential Equations 260 (2016) 1392–1413.
[4] G. Autuori, A. Fiscella, P. Pucci, Stationary Kirchhoff problems involving a fractional elliptic operator and a critical nonlinearity, Nonlinear Anal. 125 (2015) 699–714.
[5] M. Caponi, P. Pucci, Existence theorems for entire solutions of stationary Kirchhoff fractional p–Laplacian equations, Ann. Mat. Pura Appl. 195 (2016) 2099–2129.
[6] M. Xiang, B. Zhang, V.D. Rădulescu, Multiplicity of solutions for a class of quasilinear Kirchhoff system involving the fractional p–Laplacian, Nonlinearity 29 (2016) 3186–3205.
[7] F. Bernini, D. Mugnai, On a logarithmic Hartree equation, Adv. Nonlinear Anal. 9 (1) (2020) 850–865.
[8] M. Xiang, D. Hu, D. Yang, Least energy solutions for fractional Kirchhoff problems with logarithmic nonlinearity, Nonlinear Anal. 198 (2020) 111899.
[9] P.L. Lions, The concentration compactness principle in the calculus of variations. The locally compact case. I–II, Ann. Inst. H. Poincaré Anal. Non. Linéaire 1 (1984) 109–145, and 223–283.
[10] S. Mosconi, M. Squassina, Nonlocal problems at nearly critical growth, Nonlinear Anal. 136 (2016) 84–101.
[11] X. Zhang, B. Zhang, D. Repovš, Existence and symmetry of solutions for critical fractional Schrödinger equations with bounded potentials, Nonlinear Anal. 142 (2016) 48–68.
[12] H. Brezis, E. Lieb, A relation between pointwise convergence of functions and convergence of functionals, Proc. Amer. Math. Soc. 88 (1983) 437–477.