I, Mathematician II

Further Introspections on the Mathematical Life

Peter Casazza, Steven G. Krantz, and Randi D. Ruden

COMAP

The Consortium for Mathematics and Its Applications

I, Mathematician II,

Further Introspections on the Mathematical Life

The Consortium for Mathematics and Its Applications, Inc.
170 Middlesex Tpk., Suite 3B, Bedford, MA 01730

Electronic edition ISBN: 1-933223-98-7
Print edition ISBN: 1-933223-99-5

Cover Design by George Ward, COMAP

Printed in the United States of America

I, Mathematician II,

Further Introspections on the Mathematical Life

Edited by

Peter Casazza
University of Missouri-Columbia

Steven G. Krantz
Washington University

Randi D. Ruden
University City, Missouri

Published and Distributed by

COMAP

The Consortium for Mathematics and Its Applications
170 Middlesex Tpk., Suite 3B, Bedford, MA 01730
www.comap.com

Contents

Preface

What This Book is About

The mathematical life is a rewarding, satisfying, and fulfilling one. But it has its trials. As a clinical psychologist has observed, mathematicians and oboe players have a lot in common: they both do something very difficult that other people do not appreciate.

We have all had our fill of the shopworm lament, frequently encountered in social situations, to the effect that "I was never any good at mathematics." Or, perhaps worse, "I was very good at mathematics until we got to the stuff with the letters–like algebra."

Fortunately, there is evidence that the tiresome observations in the last paragraph are not really indicative of the general public feeling about mathematics. Just as an instance, the popularity of the movie A Beautiful Mind and the television show Numb3rs is an indication of some public fascination with mathematics. The public responded strongly and enthusiastically to Andrew Wiles's solution of Fermat's last problem. There was genuine interest and curiosity about Grisha Perelman's proof of the Poincaré conjecture. Most anyone would like to know more about the shape of the universe. People respect mathematics; they are just intimidated by it.

At the MAA Summer MathFest in 2007, Peter Casazza, Steven Krantz, and Frank Morgan were invited to organize a special session entitled "The Psychology of the Mathematician.'' The avowed purpose of that event was to discuss what mathematicians think of themselves and what others think of us. This was a well-attended session, with lively and heartfelt discussions. The speakers went into some detail, and were often quite emotional. As a consequence, MAA Editor Don Albers invited us to produce a volume inspired by our special session. Frank Morgan withdrew from that particular project, and Randi Diane Ruden joined it. So we now have a volume edited by three scholars with diverse interests.

We invited two types of articles for this volume. The plenary articles, generally by well-known mathematical figures, are putatively about the theme of the session: "What Do People Think About Mathematicians?" The secondary articles, rather more brief, are about "Why I Became a Mathematician." Taken together, the two collections of articles paint a varied and multifaceted panorama of ways to think about our profession, our subject, and those who people it.

Among our plenary contributors are Tom Apostol, Bob Strichartz, Daniele Struppa, Christian Wenzel, Jim Milgram, and Vincentiu Radulescu. Certainly a distinguished and varied group. Several of the plenary writers have devoted a good part of their lives to the teaching and communicating of mathematics. These include Cliff Pickover, Reece Harris, H. O. Pollak, and Dean Simonton.

The secondary contributors number among them some notable mathematicians–including Marco Abate, Joseph Cima, Loredana Lanzani, Cathleen Morawetz, and Marjorie Senechal. Many of the articles are quite personal. Almost all the authors made a point of telling us how cathartic they found it to write for this volume.

The plenary articles are quite broadly distributed. The writers focused on the question at hand in a variety of ways. The results are fascinating, and will be of particular interest to budding mathematicians, budding math teachers, budding math communicators, and in turn their teachers. In order to provide some context for readers, we have divided the plenary articles into two types:

- Who Are Mathematicians?
- On Becoming a Mathematician

These are just rough guidelines, as many articles do not fit squarely into either category. But they will give the reader a hint of what the reading will entail. And they group together like-minded pieces.

What We are About

The job of a computer scientist is to find algorithms that will accomplish certain tasks. The job of an engineer is to make things that work. The job of a mathematician is to develop new theories and establish new ideas and new truths and to teach and communicate them.

And we are obsessive about it. If mathematicians seem to be other-worldly, seem to be hopeless nerds, seem to be excessively compulsive, it is because we are. Trivial worldly matters are of no interest. Nothing measures up to the discovery and establishing of a new mathematical truth, and there is no joy like communicating those truths to our students and colleagues. Nothing has the timelessness, the enduring value, the pure pleasure of mathematical learning and discovery and teaching. This is what we are about.

But one upshot of these considerations is that mathematicians can appear to be isolated. We have trouble communicating with the rest of the world, and the rest of the world has trouble communicating with us. We are perceived to be in an ivory tower, and–God bless us–we may as well stay there. And do no harm to anyone else.

But there is a price to pay for this isolation, and that price is frequently not very pretty. Even in the context of the college or university, we often do not fit in. We frequently are unable to make a good case for our just rewards. We often find ourselves passed over for more trendy or more broadly appealing intellectual pursuits such as genetic engineering or computer visualization or biotechnology. Whereas a geneticist can speak of gene cloning and DNA matching, a computer scientist can speak of bits and bytes and megapixels, a biologist can speak of species verification and evolutionary differentiation, a mathematician has a tough time explaining what we are about. We feel disconnected and unappreciated.
This volume is an effort to reach out. It gives a significant number of mathematicians the

opportunity to speak about who they are, where they come from, and what they do. There are also essays by non-mathematicians–ones who know mathematicians intimately–explaining how they see the matter. How do they interact with mathematicians and what do they get from that interaction? Does this relationship enrich their lives? What have they learned in the process?

It is important to note that this is the second volume of a two-volume work. The first volume was published by the Mathematical Association of America. This second volume is published by COMAP. It has been a pleasure to work with both of these publishers. The resulting two volumes form a coherent whole with considerable weight and substance.

There are many good people who helped to make this book happen. The most important person to thank is Sol Garfunkel, who invited us to publish this volume with COMAP, and who did everything to make it happen with dispatch. George Ward did a splendid job of handling all the copy editing tasks and turning this jumble of contributions into a coherent whole.

The production of this book has been rewarding for all of us. It has been an opportunity to ruminate and introspect. We have all taken this opportunity to re-live segments of our lives and see what we have learned. We hope that, in the process, we have produced a book that will speak to young people hoping to become mathematicians, math teachers, math communicators, or mathematical scientists. Our view is that this is a glimpse into the personal side of the mathematical equation, one which has been infrequently explored in the past. It should prove to be a productive adventure.

Peter Casazza, Columbia, Missouri

Steven G. Krantz, St. Louis, Missouri

Randi D. Ruden, University City, Missouri

A 21st Century Mathematical Renaissance

Teodora–Liliana Rădulescu* & Vicenţiu Rădulescu†

In a 2009 analysis developed for the leading and very influential American newspaper *The Wall Street Journal* [WSJ], a study evaluated 200 professions "to determine the best and worst according to five criteria inherent to every job: environment, income, employment outlook, physical demands, and stress." Mathematicians placed in the first position. According to this study, *mathematician* is the top job in the U.S. in terms of good environment, income, employment outlook, and physical demands. *Actuary* and *statistician*, two related professions, rank second and third, respectively. This important sociological analysis provides a good answer to the young student's question *Why do mathematics?* The analysis evidences that mathematicians are in demand in terms of job prospects. Should anyone be wondering whether interest in mathematics has slowed down, such a study shows that undoubtedly the answer is **NO**! The role of mathematicians, both pure and applied, in the development of our society is as important as ever. Some arguments related to the importance of the job of the mathematician and of mathematics education at all levels follow.

1. Has Progress in Mathematics Slowed Down?

> When introduced at the wrong time or place, good logic may be the worst enemy of good teaching.
>
> George Pólya (1887–1985)

The title of this section is borrowed from a celebrated paper [HALMOS] by Paul Halmos (1916–2006). Halmos's paper was translated into Bulgarian and Czech and presented in a lecture at the Seventy-fifth Anniversary Celebration of the Founding of the Mathematics

* Teodora–Liliana Rădulescu, Department of Mathematics, "Fraţii Buzeşti," National College, 200352 Craiova, Romania, e-mail: teodoraradulescu@yahoo.com

† Vicenţiu Rădulescu, Institute of Mathematics "Simion Stoilow" of the Romanian Academy, P.O. Box 1-764, 014700 Bucharest, Romania, Department of Mathematics, University of Craiova, 200585 Craiova, Romania, e-mail: vicentiu.radulescu@imar.ro vicentiu.radulescu@math.cnrs.fr

Association of America. In his report Halmos lists 22 subjects that are particularly active in mathematics today as follows: 9 concepts, 2 explosions, and 11developments.

♦ 9 CONCEPTS: Moore-Smith limits, distributions, the Monte Carlo method, categories, *K*-theory, the fast Fourier transform, nonstandard analysis, catastrophes, and chaos.

The first concept mentioned above is related to a general theory of limits. The notion of *Moore-Smith sequence* (also known as a *net*) is a generalization of the notion of *sequence, w*hich is one of the central concepts in analysis. The concept of *distributions* is due to the renowned French mathematician Laurent Schwartz[1] (1915–2002) and generalizes the basic notion of *function* that arises in all branches of mathematics. The theory of distributions has played a huge role in the modern theory of partial differential equations. Basic distributions are the *Dirac* δ *function* and the *Heaviside step function*. The concept of *Monte Carlo methods* refers to a class of computational algorithms that are intensively applied when simulating various physical and mathematical systems. In a general way, *category theory* is concerned with mathematical structures and relationships between them. Categories were first introduced by Samuel Eilenberg (1913–1998) and Saunders Mac Lane (1909–2005) in the 1940s and appear in many fields of mathematics, including theoretical computer science and mathematical physics. The concept of *K-theory* was defined by the celebrated French mathematician Alexandre Grothendieck[2] (1928–2014) in 1957. *K*-theory is used in several mathematical disciplines, such as algebra, algebraic topology, algebraic geometry, and operator algebras. The *fast Fourier transform* is an efficient algorithm to compute both the discrete Fourier transform and its inverse. This concept has many important applications, from simple complex-number arithmetic to group theory and number theory. Nonstandard analysis was introduced in the 1960s and is related to the classical differential and integral calculus, first developed by Newton and Leibniz. *Catastrophe theory* was created by the French mathematician René Thom[3] (1923–2002). This theory has many applications to the study of dynamical systems, in strong relationship with bifurcation theory. A closely related concept is that of *chaos*, which points to the high sensitivity of dynamical systems with respect to their initial conditions. This sensitivity is frequently referred to as the *butterfly effect*: "Small variations of the initial condition of a dynamical system may produce large variations in the long term behavior of the system."

♦ 2 EXPLOSIONS: The four-color theorem and Mordell's conjecture.

The *four color problem* was an old conjecture in mathematics. It states that, given any map (with countries) of the plane, its regions can be colored using at most four colors so that no two adjacent regions have the same color. This conjecture was first proposed in 1852 by Francis Guthrie (1831–1899) while trying to color the map of counties in England. Guthrie communicated his conjecture to De Morgan and the first printed reference on this problem is due to Cayley in 1878. A major breakthrough in this concern is due to Appel and Haken [3], but there remains some skepticism regarding the validity of their proof of the four color theorem. The first rigorous proof of the four-color theorem was found in 1996 and is due to Neil Robertson, Daniel P. Sanders, Paul Seymour, and Robin Thomas [13].

---

[1] Schwartz was awarded the Fields Medal, often described as the "Nobel Prize of Mathematics," in 1950.

[2] Grothendieck was awarded the Fields Medal in 1966.

[3] Thom was awarded the Fields Medal in 1958.

The *Mordell conjecture* states a basic result regarding the rational number solutions of Diophantine equations. The Mordell conjecture was proved by the German mathematician Gerd Faltings[4] in 1983.

By an *explosion*, Halmos means "a piece of mathematical progress that is genuine mathematics, so recognized by the whole profession, but that is hot news not only for the *Transactions*, but also for the *Times* for a week, and for student mathematics clubs for many months." In Halmos's vision, this class "consists of the deep and in some cases even breathtaking developments (but not explosions) of the kind that may not make the *Times*, but could possibly get Fields Medals for their discoverers." However, we point out that significant contributions that shaped mathematics and computing during the last century are related to the huge impact of the two Gödel's incompleteness theorems, see [6].

♦ 11 DEVELOPMENTS: Ergodic theory, transcendental numbers, the continuum hypothesis, Lie groups, simple groups, the Atiyah-Singer index theorem, Fourier series, Diophantine equations, Banach bases, manifolds, and the Bieberbach conjecture.

The above "developments" are strongly connected to the important progress of pure and applied mathematics in the 20th century. A description of the main contributions in the above 11 fields would coincide with an overview of the most renowned discoveries in mathematics during the last 80 to 100 years. We just mention that Sir Michael Francis Atiyah and Isador M. Singer received the very prestigious Abel[5] Prize in 2004 "for their discovery and proof of the index theorem, bringing together topology, geometry, and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics."

After a careful and thorough analysis of modern research findings in the above subjects, Halmos concludes that the answer to the question *Has progress in mathematics slowed down?* "is clearly and decisively *no*."

2. Toward Excellence in Mathematical Education

Whoso neglects learning in his youth, Loses the past and is dead for the future.

Phrixus, Euripides (484 BC–406 BC)

A principal reason why mathematics is healthy today is the breakdown of barriers within the field. At first glance, the full span of mathematics (an enormous body of concepts, conjectures, hypotheses, and theorems amassed over more than 2,000 years) seems to defy the possibility of unity. Gone are the days when a single giant (as Euler, Gauss, or Poincaré) could command its entirety. With the rapid development of subfields after World War II, mathematics became so specialized that practitioners had difficulty communicating with anyone outside their own specialty. And today these specialists are commonly scattered among Paris, Princeton, Canberra, and Tokyo, to name just a few world centers.

Mathematics has a dual nature: it is both an independent discipline valued for precision and intrinsic beauty, and it is a rich source of tools for the world of applications. The two

[4] Faltings (b. 1954) was awarded the Fields Medal in 1986.

[5] Niels Henrik Abel (1802–1829), Norwegian mathematician who proved the impossibility of solving the quintic equation in radicals. As remarked by A.L. Crelle in 1829, "Abel is one of those rare beings that nature produces barely once a century."

parts of this duality are intimately connected. Mathematicians rely heavily on aesthetics as well as intuition. But, with respect to utility, the argument in mathematicians' favor is a strong one. To mention just a few examples, the modern computer would not exist without Leibnitz's binary number code, Einstein could not have formulated his theory of relativity without the development of Riemannian geometry, and the edifices of quantum mechanics, crystallography, and communications technology all rest firmly on the platform of group theory.

Mathematicians have always carried their discoveries into adjacent fields where they have produced new insights and whole new subfields. Francis Bacon (1561–1626) prefigured this principle of integrative science with an apt image: "No perfect discovery can be made upon a flat or a level: neither is it possible to discover the more remote or deeper parts of any science, if you stand but upon the level of the same science and ascend not to a higher science."

Mathematics is a science undergoing continuous change and development. Mathematics is being used increasingly in new and various areas of society. Meanwhile the subject of mathematics, as taught in the school, is too often and incorrectly regarded as fixed and complete. Research, developmental work, and evaluation of learning and teaching in mathematics has been extensive in the last few decades, but findings in these areas have reached the classroom only to a limited extent. There is a need to challenge tradition, develop the contents of teaching and show that there are different approaches to changing attitudes, stimulating development, and increasing interest in mathematics.

Felix Klein (1849–1925) was the first president of the International Commission on the Teaching of Mathematics, which was created in Rome in 1908. The ICTM invited countries to undertake studies of the state of mathematics teaching and education. The French journal *Enseignement Mathématique*, founded in 1899, became the commission's official journal and recorded the growing world-wide interest in how mathematics is taught and learned. European mathematicians of the time contributed to our thinking about mathematics education, including Henri Poincaré (1854–1912), Jacques Hadamard (1865–1963), Giuseppe Peano (1858–1932), Samuel Dickstein (185–1939), and John Perry (1850–1920).

Today the opening line of the book *Toward Excellence: Leading a Doctoral Mathematics Department in the 21st Century* (see [4]) by American Mathematical Society's John Ewing reads: "We have a simple message: To ensure their institution's commitment to excellence in mathematics research, doctoral departments must pursue excellence in their instructional programs. No single issue is more important than undergraduate instruction in determining whether research universities, especially public research universities, will receive strong support from alumni, legislatures, business leaders and the general public. We can debate endlessly whether the criticism that higher education has been getting is fair, but the fact remains that universities do not have the public support they once had and that they certainly need." Obviously, all is not well in mathematics education today. This is a matter we must take to heart. The situation in high schools and elementary schools is serious and although there is much travail already on this problem, any solution necessitates long-term plans. On the other hand, as university professors who really do understand what math is and how powerful and beautiful it is, *there are things we can do*. We could make significant changes this very Fall if we have the courage to do so.

The recommendations in *Toward Excellence* are a wise response to a fundamental shift in society that has altered our nation's socio-economic ethos and higher education's relationship to it. These forces will change over time, but the change will be evolutionary and on the time scale of decades, not years.

3. Research and Mathematics Education

Life is good for only two things, discovering mathematics and teaching mathematics.
Siméon Poisson (1781–1840)

Thus the core of this essay is not devoted to trends and problems of research in contemporary mathematics, which is very lively as we have seen, but on considerations having to do with mathematics education. In the *Presentation* [1] of the Conference *The Future of Mathematics Education in Europe*[6] it is stated that "there is substantial evidence that, in developed countries, the interest in mathematics among teens has declined sharply in the last 20 years." Unfortunately, the same tendency holds in developing countries, too. More precisely, recent data shows the following:

♦ this phenomenon extends to many other countries, such as former communist countries in Eastern Europe;

♦ a significant decline has been remarked in overall enrollment in the overwhelming majority of mathematics departments of universities throughout the world.

It will mean trouble for mathematics if the trend continues much longer. At the turn of a new millennium and in this era of constrained resources and increased accountability, there is only one alternative: either enrollment will go up or the number of faculty positions will go down. Fortunately, the outlook is not all bad since mathematics has deeper, useful, easily identifiable assets. Beyond the breakdown of internal barriers, mathematics has become much more interactive with other sciences that research various phenomena and models arising in finance, business, management, security, or decision-making. Mathematics continues to have (undoubtedly, it always will have) the potential for strong connections to applied disciplines, especially in the physical sciences, biological and social sciences, and technology. These disciplines, in turn, provide interesting new types of problems, which then lead to new applications. We will give two examples.

(i) One of the major and most active areas of scientific study today is related to theoretical computer science, founded more than a half century ago, when Alan Turing (1912–1954) defined rigorously in [15] the concept of "computation" and studied its powers and limits. These questions led to the practical construction by John von Neumann (1903–1957) and Herman Goldstine (1913–2004) of the first stored-program computer, followed by the computer revolution we are witnessing today.

(ii) Key questions have been raised by recent advances in evolutionary biology, cf. [5, 10]. How does evolution really work for biological systems? What drives it? What is the notion of fitness landscapes? Evolutionary theory has existed for 80 or 100 years, going back to Sir Ronald Aylmer Fisher (1890–1962), Sewall Green Wright (1889–1988), and other researchers of the early 20th century. The amazing variety we see in living systems

[6] Held in Lisbon, December 16-18, 2007.

cannot be explained by a static sequence which is relatively short. It suffices to point out that the human genome has less than 3 billion base pairs.

Maintaining the strength of mathematics should be one of the main educational trends in this century. This issue is strongly connected to progress in mathematics research, but should be not disconnected from the context in which research occurs. The success of research depends both on the quality of the people doing the research and on the degree to which it receives sustained support from society. Meanwhile, mathematics applications are what guarantees the sustained support from society.

4. Supporting Continuous Mathematical Education

> The teachers must train their students to answer little fragmentary questions quite well, and they give them model answers that are often veritable masterpieces and that leave no room for criticism. To achieve this, the teachers isolate each question from the whole of mathematics and create for this question alone a perfect language without bothering about its relationships to other questions. Mathematics is no longer a monument but a heap.
>
> Henri Lebesgue (1875–1941)

As stated in [9], the Government of Sweden decided in 2003 to set up a mathematics delegation to survey the whole field of mathematics education from kindergarten to the university level. The tasks of this committee were very wide and the action plan covered the whole of the school system from pre-school to higher education. The delegation was concerned with the competence of teachers, viewed as the most important single factor determining results achieved by pupils. Another major issue was how to create the good learning environments necessary to stimulate pupils' and students' learning, that is, how to encourage teachers to acquire a deeper knowledge of mathematics and keep up their interest in the subject.

The action plan of the Swedish delegation focused on four areas that are all of the utmost importance for the development of mathematics education. The conclusions are the following:

1.Qualified teachers in mathematics at all levels, from pre-school to the university. To implement this point, stricter requirements regarding mathematics in teacher education and an extensive program for competence development of in-service teachers has been proposed. This program also applies to university-level education, where lecturers traditionally have too little time for research and competence development.

2. Development of local activities in mathematics as a subject from pre-school to the university. Local and regional activities should be encouraged in an extensive mathematics initiative where funding may be applied for at all schools and institutions of higher learning.

A very important problem for mathematics education as a whole is that children often lose interest in mathematics around the age of eleven or twelve, although it seems they learn mathematics cheerfully up to that age. After that, many students try to avoid mathematics and do not enroll in voluntary courses. In the view of the Swedish commission, a profound change in the subject area as well as in the methods of teaching is necessary. This should be implemented in locally and regionally based programs. The involvement of schools and institutions of higher learning is expected in this effort.

3. Development of mathematics as an educational subject from pre-school to the university. A point of special concern in this area is the role of mathematics in the education of engineers and the integration of mathematics and scientific computing with technical subjects in engineering curricula.

4. The interest for mathematics and understanding of the value, role, and importance of mathematics and mathematics education in society at large. These are strongly related both to education in general and, specifically, to mathematics education. These values in mathematics education are the deep affective qualities which education allegedly aims to develop and strengthen through the teaching of any subject. Mathematics has its own role and values in society.

5. Motivating Interest in Mathematics

To Thales the primary question was not what do we know, but how do we know it.
Aristotle (384 BC–322 BC)

How to encourage a young talent to learn mathematics? In this respect, Paul Halmos [7] said: "*The best way to learn is to do; the worst way to teach is to talk. The best way to teach is to make students ask, and do. Don't preach facts–stimulate acts*."[7] Science and technology developed in an impressive way before, during, and immediately after World War II, which attracted post-war students to careers in research. This trend received a powerful stimulus in the sixth and seventh decade of the 20th century, together with the adventure of the space age: launch of the Soviet satellite Sputnik (1957), Gagarin's first human travel into space (1961), and the first manned mission to land on the Moon (Neil A. Armstrong and Edwin E. Aldrin, Apollo 11, 1969). The impact was huge. Science was recognized for the political and economic power it could generate. In those golden years, research became as important to society as it was fascinating to practitioners. A few decades later, society's interest in many areas of research appears to have diminished in most countries around the world. With some exception in biomedical areas, the importance of mathematics and science in society is not as widely recognized as it once was. Mathematics and the sciences are no longer perceived as offering desirable career opportunities. It is striking to remark that a general tendency in our days is that, in both developed and developing countries, many brilliant students who once would have chosen careers in mathematics are not doing so. They are choosing applied information science, business, economics, biomedical engineering, genetics, software engineering, or other areas where the future looks more promising. There appears to be a fundamental lack of appreciation for the richness and relevance of mathematics itself.

It is important at an early stage in schooling to strengthen pupils' confidence in their own ability and assist them in developing positive attitudes towards learning and education. In this context mathematics occupies a special position. Students who have experienced

[7] Halmos said this in 1975, so 39 years ago. In 1929 Alfred North Whitehead [16] also said this and much more. Both were wise and respected scholars. And there is general agreement that what they said is correct and important. But the problem seems as bad as ever. Surely that suggests that we need a new agenda. Perhaps we need to look for the reason change is so difficult, and try to do something first about that.

difficulties in mathematics early in their schooling naturally do not choose educational programs in the upper secondary school or later that contain a high dose of mathematics. Difficulties experienced in learning mathematics early on represent for many people a substantial obstacle to taking up further studies in adult life in many subject areas. It is ironic that students' interest is low at a time when career opportunities for professional mathematicians should be greater and more diverse. This is true both for the traditional disciplinary areas, which are rich with new developments and challenging problems (such as the *Millennium Problems*, cf. [2][8]), and in the applied fields and other areas of science, where demand for mathematicians with proper training will continue to grow rapidly in the immediate future.

An apparent reason for students' disinterest is that we are not communicating a full picture of mathematics as a field where one may choose among many intellectually rewarding and challenging careers. The people best positioned to effect this communication are high school teachers, college educators, and fellow students. However, these groups can describe current opportunities and fast-growing fields only if they, in turn, are informed by those in the profession. Thus the mathematics community, as a whole, faces a critical challenge: to foster more interaction at every level of teaching, and to widen the channels of communication with the students who will eventually replace us and extend, diversify, and perfect our work into the future.

Closely allied with our educational needs is the opportunity to better communicate with and educate the public about mathematical issues. Mathematicians clearly understand the purpose and value of what they do, but many people and decision makers in governments, business, and even education do not share this understanding. If mathematicians expect their research at universities to be supported by public funds, we must present to the public a more vivid picture of that research and its power. This will deepen understanding of mathematics and mathematicians and improve the outlook of our profession.

The culture of this millennium announces to be highly interactive and collaborative. Mathematicians must seize the opportunity not only to collaborate with scientists in other fields but also to reach out to the community at large. Mathematicians are uniquely qualified to articulate the value of mathematics in catalyzing major advances in science and health, in promoting powerful economic tools and efficiencies, and in proving the patterns and perhaps truths of the universe in which we live.

The trend toward interactivity is an important feature of the sciences in our time. We have just observed that, within mathematics and throughout the sciences, much of the most productive work is being done at the frontiers between subfields, fields, and disciplines. Unfortunately, some institutions have been slow to adapt to this reality. Mathematics loses a lot when it is isolated or fragmented according to various paradigms. Universities around the world, as well as many industries and government agencies, stand to gain much by removing barriers to collaborations. In particular, much can be done to enhance powerful and diverse interactions between academic and industrial mathematicians. Of course the primary missions of academia and industries are different; however, the two cultures have much to learn from one another and can gain from collaboration.

[8] The seven Millennium Problems named by the Clay Mathematics Institute of Cambridge, Massachusetts, are: Birch and Swinnerton-Dyer Conjecture, Hodge Conjecture, Navier-Stokes Equations, P versus NP, Poincaré Conjecture, Riemann Hypothesis, and the Yang-Mills Theory.

6. Concluding Thoughts

In conclusion, the scientific enterprise can function at full potential only if there is a fast flow of knowledge between the creators and users of mathematics. This is something mathematics education can and should facilitate, especially since the subject area of mathematics is currently so active and vital both in research and applications. Well-schooled mathematicians are in demand. While *mathematician* is a top job in the US today, it is no longer possible for a mathematician to remain aloof from the passing needs of the world or to continue working in an ivory tower. As funds get scarce, the future of our profession is at stake.

Acknowledgments. The authors are grateful to Professor Peter D.Taylor for his interest in this paper.

References

[1] http://www.fmee2007.org/

[2] http://www.claymath.org/millennium/

[3] K. Appel and W. Haken, Every planar map is four colorable, *Contemp. Math.*, vol. 98, Amer. Math. Soc., Providence, RI, 1989.

[4] J. Ewing, Toward Excellence: *Leading a Doctoral Mathematics*, *Department in the 21st Century*, American Mathematical Society, Providence, RI, 1999.

[5] B. Glymour, Stable models and causal explanation in evolutionary biology, *Philos. Sci.* 75(2008), 571-583.

[6] K. Gödel, Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme. I. [On formally undecidable propositions of Principia Mathematica and related systems. I] *Monatsh. Math. Phys.* 38(1931), 173-198.

[7] P. Halmos, The Problem of Learning to Teach, *Amer. Math. Monthly* 82(1975), 466-476.

[HALMOS] P. Halmos, Has Progress in Mathematics Slowed Down? *Amer. Math. Monthly* 9(1990), 561-588.

[9] C. Kiselman, The Swedish Mathematics Delegation, *Stockholm Intelligencer* 2004, pp. 8-9.

[10] M. Matthen, What sort of science is evolutionary biology? *Dialogue* 30(1991), 129-141

[11] G. McColm, A Metaphor for Mathematics Education, *Notices Amer. Math. Soc.* 54(2007), 499-502.

[12] S.E. Needleman, Doing the Math to Find the Good Jobs. Mathematicians Land Top Spot in New Ranking of Best and Worst Occupations in the U.S., *The Wall Street Journal*, Jan. 26, 2009, p. D2http://online.wsj.com/article_email/SB12311923611 7055127lMyQjAxMDI5MzAxNjEwOTYyWj.htm

[13] N. Robertson, D. Sanders, P. Seymour, and R. Thomas, A new proof of the four-colour theorem, Electron. Res. Announc. Amer. Math. Soc. 2(1996), 17-25 (electronic).

[14] P. Taylor, W. Whiteley, and E. Barbeau, Why the Sky Is Still Falling, *Notices Amer. Math. Soc.* 56(2009), 4.

[15] A. Turing, On Computable Numbers, with an Application to the Entscheidungsproblem, *Proceedings of the London Mathematical Society*, ser. 2, vol. 42, 1937.

[16] A.N. Whitehead, *The Aims of Education and Other Essays*, Macmillan Company, 1929 (reprinted in Education in the Age of Science, edited by Brand Blanshard, New York, Basic Books, 1959.)

I, Mathematician II

Further Introspections on the Mathematical Life

What do mathematicians think of themselves, and what do others think of them? These questions were the theme of a special session at the San Jose MathFest of 2007. And they resonated with a large and diverse group of mathematicians and students. For these are the queries that govern the way that mathematicians live.

A large and diverse group of mathematicians and mathematical people were assembled to offer their view of these matters. The contributions represent a vast array of perspectives on "The Psychology of the Mathematician." It is hoped that readers will find the thoughts assembled here to be stimulating and to be cause for further rumination. It is a complex and rewarding world that we live in, and one that can only benefit from some introspection

10: 1-933223-98-7
ISBN 13: 978-1-933223-98-8

COMAP

The Consortium for Mathematics and Its Applications
www.comap.com